Solid state systems for quantum information, Session 11

Assistants : franco.depalma@epfl.ch, filippo.ferrari@epfl.ch

Exercise 1 : Optimizing the measurement setup

1. **The need of attenuation:**

   The blackbody radiation present in cables depends on the temperature of a resistor R connected to them. To see this, let us consider a bath at temperature T. The temperature-dependent mean thermal photon occupation number n_{BE} of the bath at a given frequency $\omega/2\pi$ is given by the Bose-Einstein distribution:

$$n_{BE} = \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1}. \tag{1}$$

   Placing an attenuator that reduces the power of the signal by a factor A, at a temperature stage T, one relates the input noise photon occupation n_i to the output noise photon occupation n_o at frequency ω by the following rule:

$$n_o(\omega) = \frac{n_i(\omega)}{A} + \frac{A-1}{A} n_{BE}(T, \omega). \tag{2}$$

   This is to be thought of as a beam-splitter letting $1/A$ of the signal go through, and adding $(A-1)/A$ of the thermal emission from a bath at temperature T.

   (a) What is the classical limit of Eq.1?
   Hint: Consider $\hbar\omega \ll k_B T$ and apply a first-order Taylor approximation in the denominator.

   (b) Estimate the amount of total attenuation (starting from room temperature) needed to reach a noise level of $n_{th} = 10^{-3}$ at a frequency of 6 GHz at the input of the sample, which has a temperature of 20 mK. Assume that the noise at room temperature is dominated by thermal noise.

   (c) What limits the amount of attenuation that can be used in practical experiments? We typically distribute 60 dB of attenuation between the 4 K stage, the cold plate at 100 mK, and the base temperature stage at 20 mK. What is the noise photon number you get for a 20/20/20 dB attenuator distribution at a frequency of 6 GHz?

   (d) For each of the attenuators, plot the noise photon occupation number at a frequency of 6 GHz at the input of the sample as a function of the attenuation if the other two attenuators are fixed to 20 dB? Explain your observations.

2. **Noise in the amplification chain:**

We consider the amplification chain sketched in Fig. 1, which is used to amplify signals on their way from the superconducting chip towards the room temperature acquisition device. The chain consists of 1) an effective attenuator with attenuation constant A taking into account cable losses, 2) a Josephson Parametric Amplifier (JPA), 3) a High-Electron Mobility Transistor (HEMT) amplifier, and 4) a room temperature (RT) amplifier. Each amplifier has a gain G_i, as well as an added noise N_i specifying the effective number of noise photons at the input of the amplifier (the JPA's added noise is often negligibly small). The noise is given in terms of photons according to the table below.

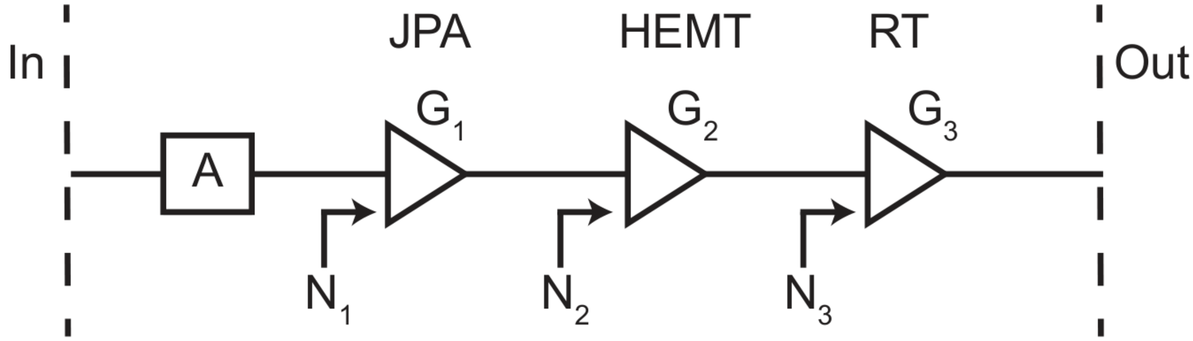

Figure 1: Sketch of an amplification chain with 3 stages.

Amplifier	Gain(dB)	Noise added
JPA	$G_1 = 20$	$N_1 \ll 1$
HEMT	$G_2 = 40$	$N_2 = 14$
RT amp.	$G_3 = 40$	$N_3 = 174$

The recorded signal is of the form $S = a + h^\dagger$. Without any input signal, the noise measured at the output of the chain is $\langle hh^\dagger \rangle = 1 + N_{\text{eff}}$.

(a) Write an equivalent single amplifier model with the effective gain G_{eff} and the effective input noise N_{eff} of the whole chain. Demonstrate that in the limit of large gain of the first amplifier, the effective noise figure is dominated by the noise of the first amplifier. Hint: Assume that the attenuator A adds one noise photon due to vacuum noise.

(b) Estimate the chain efficiency $\eta = \frac{1}{1+N_{\text{eff}}}$ for $A = 1$ dB. How close is it to being quantum limited?

Exercise 2 : Microwave pulse generation by frequency upconversion

As discussed in the lecture and some of the previous problem sets, single-qubit manipulation is achieved by applying a voltage pulse $V_p(t)$ to the qubit drive line which oscillates at a frequency resonant with the qubit transition frequency ω_{ge}. A typical functional form for such pulses reads

$$V_p(t) = V_0 e^{-(t/\tau)^2} \cos(\omega_{ge} t + \phi), \tag{3}$$

where $\omega_{ge} = 2\pi \cdot 6$ GHz and $\tau = 5$ ns. Voltage pulses couple to the qubit via dipole-field interactions where the phase ϕ of the driving field is used to control the axis about which the Bloch vector of the qubit is rotated.

1. To generate voltage pulses with a controlled envelope, e.g. the Gaussian envelope $V_p(t)$, we use an arbitrary waveform generator (AWG) with a sampling rate of 1.2 GS/s. What is the maximum bandwidth of a signal that can be generated by such an instrument?

2. To generate pulses in the GHz regime, one typically uses a frequency mixer, which can multiply two signals. Usually, one multiplies the signal generated by the AWG at an intermediate frequency (IF) with a continuous local oscillator (LO) field generated by a microwave generator running at GHz frequency, see schematic Fig. 2(a).

   Derive the output signal when applying a continuous frequency of $\omega_{\rm LO} = \omega_{ge} + \omega_{\rm IF} = 2\pi \cdot 6.2$ GHz to the LO port and the pulse $V_p(t) = V_0 e^{-(t/\tau)2} \cos(\omega_{\rm IF} t + \phi)$ with $\omega_{\rm IF} = 2\pi \cdot 200$ MHz to the IF port. Calculate the Fourier transform, and plot its absolute value in the frequency domain. What issue do you see when using the up-converted signal to drive the qubit?

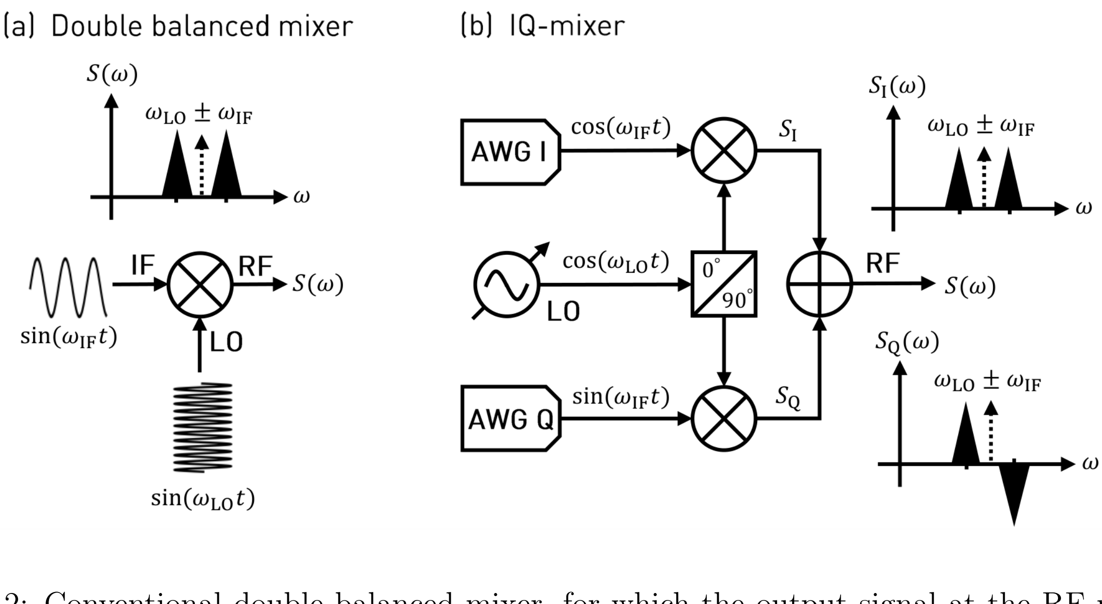

Figure 2: Conventional double balanced mixer, for which the output signal at the RF port is the product of the signals at the IF and LO port. (b) IQ-mixer, which is composed of two double balanced mixers, a 90° hybrid splitter, and a microwave combiner. The hybrid splitter divides the incoming LO signal equally and adds a 90° phase shift for the LO to the quadrature mixer, such that the signal results in $\cos(\omega_{\rm LO} t - \pi/2)$.

3. To avoid the generation of two sidebands at frequencies $\omega_{\mathrm{LO}} \pm \omega_{\mathrm{IF}}$, we perform two up-conversion processes, according to the IQ-mixing scheme shown in Fig. 2(b). Show that one of the two sidebands is effectively eliminated in the signal $S(\omega)$. What is the motivation for letting the experimentalist provide the two IF signals I and Q independently instead of generating the Q input internally in the IQ-mixer by phase-shifting the I input?

4. A drive pulse at the same frequency ω_{ge} can also be created with $\omega_{\mathrm{LO}} = 2\pi \cdot 5.8$ GHz. How do the I and Q input signals need to be modified in this case?

5. We now consider the acquisition of the microwave pulses for the readout of the qubit state. In the readout output line, the bandwidth of the signal acquisition is limited by the sampling rate of the analog to digital converter (ADC). Describe briefly how the IQ-mixer from Fig. 2(b) can be used to convert the readout signal from the GHz band down to frequencies within the bandwidth of the ADC?