Lecture 2: Single electron tunneling and Coulomb blockade

March 2025

1a. What is the relevant energy-scale for Coulomb blockade?
The Coulomb blockade effect in quantum dots is characterized by an interplay of several energy scales, like- Coulomb energy, Confinement energy, Comparison of Coulomb energy and quantization energy, Source-drain coupling and temperature.

1b. How large is the capacitance (to infinity) of a sphere with radius r = 1 nm. Compare this to k$_B$T at room temperature and at T = 50 mK.

The capacitance C of a conducting sphere of radius r in a vacuum (or air, approximated as vacuum) is given by:

$$C = 4\pi\epsilon_0 r$$

where:

- ϵ_0 is the vacuum permittivity ($8.854 \times 10^{-12}\,\mathrm{F/m}$),
- r is the radius of the sphere.

For a sphere with radius $r = 1\,\mathrm{nm} = 1 \times 10^{-9}\,\mathrm{m}$, the capacitance is:

$$C = 4\pi(8.854 \times 10^{-12}\,\mathrm{F/m})(1 \times 10^{-9}\,\mathrm{m}) \approx 1.11 \times 10^{-19}\,\mathrm{F}.$$

Charging Energy E_C

The charging energy E_C is given by:

$$E_C = \frac{e^2}{2C}$$

where e is the elementary charge ($1.602 \times 10^{-19}\,\mathrm{C}$). Substituting the capacitance:

$$E_C = \frac{(1.602 \times 10^{-19}\,\text{C})^2}{2 \times 1.11 \times 10^{-19}\,\text{F}} \approx 1.15 \times 10^{-19}\,\text{J}.$$

To convert this to electron volts (eV), divide by $1.602 \times 10^{-19}\,\text{J/eV}$:

$$E_C \approx 0.72\,\text{eV}.$$

Comparison to $k_B T$

The thermal energy $k_B T$ is given by:

- k_B is the Boltzmann constant ($1.381 \times 10^{-23}\,\text{J/K}$),
- T is the temperature.

At Room Temperature ($T = 300\,\mathbf{K}$):

$$k_B T = (1.381 \times 10^{-23}\,\text{J/K})(300\,\text{K}) \approx 4.14 \times 10^{-21}\,\text{J}.$$

Convert to eV:

$$k_B T \approx \frac{4.14 \times 10^{-21}\,\text{J}}{1.602 \times 10^{-19}\,\text{J/eV}} \approx 0.026\,\text{eV}.$$

At $T = 50\,\mathbf{mK} = 0.05\,\mathbf{K}$:

$$k_B T = (1.381 \times 10^{-23}\,\text{J/K})(0.05\,\text{K}) \approx 6.91 \times 10^{-25}\,\text{J}.$$

Convert to eV:

$$k_B T \approx \frac{6.91 \times 10^{-25}\,\text{J}}{1.602 \times 10^{-19}\,\text{J/eV}} \approx 4.31 \times 10^{-6}\,\text{eV}.$$

Comparison:

- At room temperature ($T = 300\,\text{K}$):
  - $E_C \approx 0.72\,\text{eV}$,
  - $k_B T \approx 0.026\,\text{eV}$.
  - E_C is much larger than $k_B T$ ($E_C \gg k_B T$), so Coulomb blockade effects are significant.
- At $T = 50\,\text{mK}$:
  - $E_C \approx 0.72\,\text{eV}$,
  - $k_B T \approx 4.31 \times 10^{-6}\,\text{eV}$.
  - E_C is vastly larger than $k_B T$, making Coulomb blockade effects even more pronounced.

2a. An amount of charge Q_0 is placed on a capacitor with capacitance C. There is a resistance R to ground. What is the time-dependent charge Q(t) on the capacitor? What is the characteristic time-scale for this problem?

To determine the time-dependent charge $Q(t)$ on the capacitor and the characteristic time-scale for this problem, we analyze the discharge of the capacitor through the resistor R.

1. Time-Dependent Charge $Q(t)$:

The system can be described by the differential equation for the discharge of a capacitor through a resistor. Using Kirchhoff's voltage law, we have:

$$\frac{Q(t)}{C} + R\frac{dQ(t)}{dt} = 0,$$

where:

- $Q(t)$ is the charge on the capacitor at time t,
- C is the capacitance,
- R is the resistance.

This is a first-order linear differential equation. Rearranging:

$$\frac{dQ(t)}{dt} = -\frac{Q(t)}{RC}.$$

The solution to this equation is an exponential decay:

$$Q(t) = Q_0 e^{-t/\tau},$$

where:

- Q_0 is the initial charge on the capacitor at $t = 0$,
- τ is the characteristic time-scale of the system (explained below).

2. Characteristic Time-Scale τ:

The characteristic time-scale for this problem is the **time constant** τ, which is given by:

$$\tau = RC.$$

This is the time it takes for the charge on the capacitor to decay to $1/e$ (approximately 36.8%) of its initial value Q_0.

3. Final Expression for $Q(t)$:

Combining the results, the time-dependent charge on the capacitor is:

$$Q(t) = Q_0 e^{-t/RC}.$$

4. Interpretation:

- At $t = 0$, $Q(t) = Q_0$, as expected.
- As $t \to \infty$, $Q(t) \to 0$, meaning the capacitor fully discharges over time.
- The rate of discharge is determined by the time constant $\tau = RC$. Larger R or C results in a slower discharge.

Summary:

- The time-dependent charge on the capacitor is $Q(t) = Q_0 e^{-t/RC}$.
- The characteristic time-scale is the time constant $\tau = RC$.

2b. The Heisenberg uncertainty principle states that the energy of an electron is ill defined when the electron stays in a state only for a short time:

$$\delta E \delta t \geq \hbar/2 \tag{1}$$

What is the uncertainty in energy for the system discussed in a?

So, uncertainty energy,

$$\delta E = \hbar/2RC \tag{2}$$

2c. This uncertainty has to be compared with the charging energy EC = e^2/2C. In which case can Coulomb blockade be observed

$$\delta E \gg E_c or \delta E \ll E_c? \tag{3}$$

:

The charging energy should be larger than uncertainty energy -

$$\delta E \ll E_c \tag{4}$$

2d. Which relation should hold for the resistance to observe Coulomb blockade? Do you recognize this value? Does it depend on the capacitance?

Charging the island with an additional charge takes the time Δ t= R_tC which is the RC-time constant of the quantum dot. If we wish to resolve the charging energy

$$\Delta E_c = \frac{e^2}{C} \tag{5}$$

the system will respect Heisenberg's uncertainty relation

$$\Delta E_c \Delta t > h \tag{6}$$

which leads to the condition

$$R_t > \frac{h}{e^2} \tag{7}$$

This result means that the tunneling resistance Rt of the quantum dot has to be significantly larger than the resistance quantum h/e^2 implying that the quantum point contacts coupling the system to source and drain have to be deep in the tunneling regime.

2e. The network is connected to a voltage source and a current meter. Draw the IV characteristics for R = 1kΩ and R = 100 k Ω.

Initially ($t = 0$) when voltage V is applied:

- The capacitor acts like a **short circuit** (zero resistance), and the initial current is high.
- Ohm's law gives the initial current as:

$$I_0 = \frac{V}{R}$$

During Charging ($t > 0$)

- The capacitor charges up over time, and the current decreases exponentially following:

$$I(t) = \frac{V}{R} e^{-t/RC}$$

- The voltage across the capacitor increases, eventually reaching V, and current approaches zero.

Steady State ($t \to \infty$)

- The capacitor is fully charged, and current stops flowing.
- The I-V curve flattens to **zero current**.

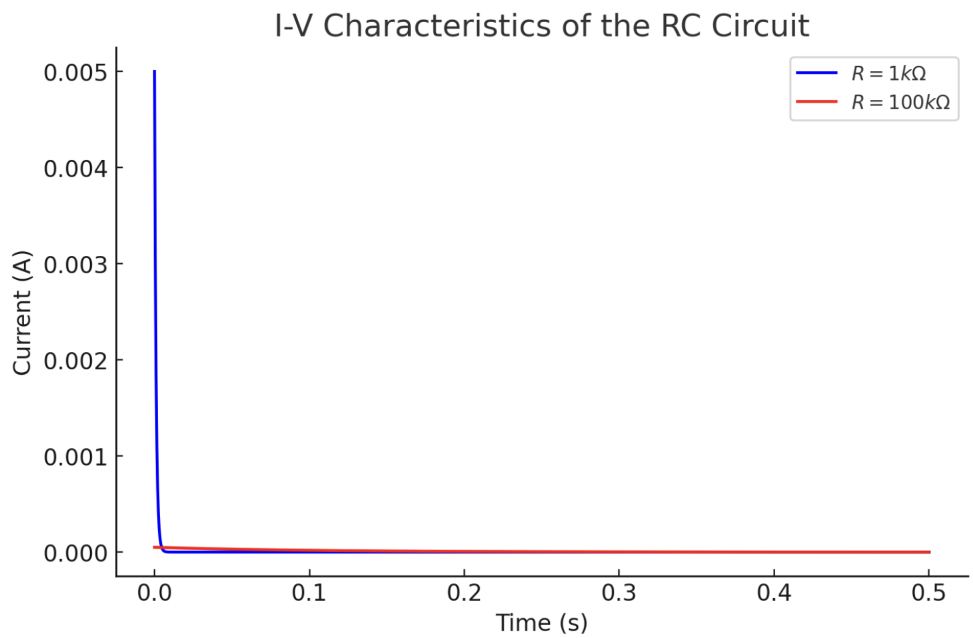

Figure 1: I-V graph (a) blue graph represents 1k Ω and (b) red graph represents 100kΩ

3a. The stability diagram shown on the right is measured on a small gold grain. (K.I. Bolotin et al., APL 84, 2004, 3154). What is the charging energy and the total capacitance of the grain?

1. Charging Energy E_C

The charging energy is related to the width of the Coulomb diamonds along the bias voltage axis at zero gate voltage: From the Coulomb diamond plot, the width in bias voltage (V_B) is approximately 180mV (-90mV to + 90mV)

$$E_C = eV_{\text{diamond}} = 90 \text{ meV}$$

Calculation of Total Capacitance C

We use the formula:

Step 1: Convert E_C to Joules

$$E_C = 90 \times 10^{-3} \times (1.6 \times 10^{-19})$$

$$E_C = 1.44 \times 10^{-20} \text{ J}$$

Step 2: Solve for C

$$C = \frac{e^2}{E_C}$$

Substituting values:

$$C = \frac{(1.6 \times 10^{-19})^2}{1.44 \times 10^{-20}}$$

$$C = \frac{2.56 \times 10^{-38}}{1.44 \times 10^{-20}}$$

$$C = 1.78 \times 10^{-18} \text{ F} = 1.78 \text{ aF}$$

Final Answer:

$$C \approx 1.78 \text{ aF}$$

3b. Use the slopes of the diamonds to find the gate coupling Cg/Ctot. What are the gate, source and drain capacitances?

Capacitance Calculations

Step 1: Gate Coupling Ratio

The gate coupling ratio is given by:

$$\frac{C_g}{C_{tot}} = \frac{\Delta V_d}{\Delta V_g}$$

Given:

$$\Delta V_g = 625 \text{ mV} = 0.500 \text{ V}, \quad \Delta V_d = 180 \text{ mV} = 0.180 \text{ V}$$

$$\frac{C_g}{C_{tot}} = \frac{0.180}{0.5} = 0.36$$

Thus,

$$C_g = C_{tot} * 0.36$$

Step 2: Total Capacitance Calculation

Using the charging energy formula:

$$E_c = \frac{e^2}{C_{tot}}$$

Rearrange for C_{tot}:

$$C_{tot} = \frac{e^2}{E_c}$$

Given that $E_c = 90 \text{ meV} = 90 \times 10^{-3}$ eV:

$$C_{tot} = \frac{(1.6 \times 10^{-19} C)^2}{(90 \times 10^{-3} \times 1.6 \times 10^{-19} J)}$$

$$C_{tot} = 1.78 \times 10^{-18} \text{ F} = 1.78 \text{ aF}$$

Step 3: Compute Individual Capacitances

- Gate capacitance:

$$C_g = 0.6408 \text{ aF}$$

- Source and Drain capacitances (assuming $C_s \approx C_d$):

$$C_s + C_d = C_{tot} - C_g = 1.78 - 0.6408 = 1.1392 \text{ aF}$$

3c. At Vg = -0.5 V a switch occurs. How much is the change in the induced (offset) charge? Is this an integer multiple of e?

so, ΔV_G = 0.25V Gate capacitance = C_G = 0.6408 aF

Use, Q = C_G * V_G = 1* e C where, e = 1.6* 10^-19 C

3d. Suppose that the switch is due to the charging of another island nearby by a single electron. What is the capacitance between the two islands?

Change in Gate Voltage:

The switch occurs at

$$V_g = -0.5V$$

This corresponds to a change in the gate voltage:

$$\Delta V_g = 0.5V$$

Change in Charge:

The change in charge on the nearby island is:

$$\Delta Q = e$$

where

$$e = 1.6 \times 10^{-19} \text{ C}$$

is the elementary charge.

Capacitance Between the Two Islands:

The capacitance C_{12} between the two islands is given by:

$$C_{12} = \frac{\Delta Q}{\Delta V_g}$$

Substituting the values:

$$C_{12} = \frac{1.6 \times 10^{-19} \text{ C}}{0.5 \text{ V}}$$

$$= 3.2 \times 10^{-19} \text{ F}$$

Final Answer:

The capacitance between the two islands is:

$$C_{12} = 3.2 \times 10^{-19}\ \mathrm{F} \quad \text{(or 0.32 aF)}.$$

4a. A metallic sphere with radius R is placed at the origin. An electron is placed at $r = a > R$. Use the method of image charges to calculate its potential energy. Sketch the charge distribution on the sphere. Calculate the difference in energy for the electron located at r = ∞ and when the electron is located on the sphere. Is the difference positive of negative?

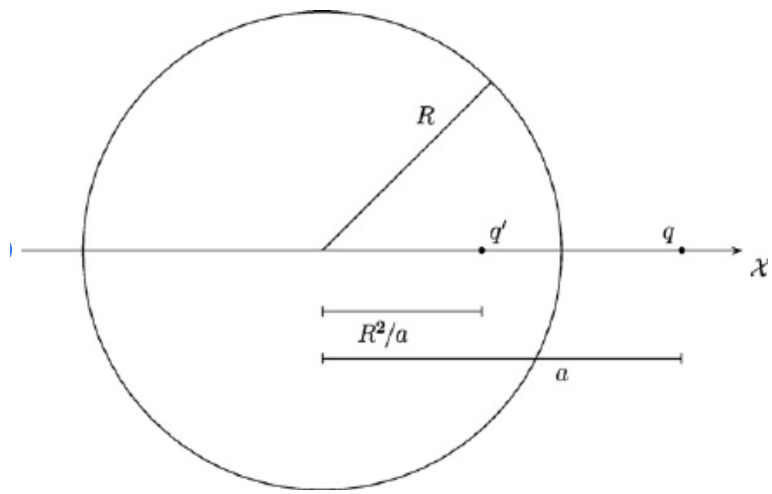

Figure 2: Image charge

Step 1: Method of Image Charges

A metallic sphere (radius R) is placed at the origin. An electron (charge $-e$) is placed at a distance $r = a > R$. To calculate the potential energy of the electron, we use the method of image charges.

The image charge q' is located at a distance

$$b = \frac{R^2}{a}$$

from the center of the sphere.

The magnitude of the image charge is

$$q' = -\frac{R}{a}e.$$

Step 2: Potential Energy of the Electron

The potential energy U of the electron at $r = a$ is due to the interaction between the electron and its image charge. The potential energy is given by:

$$U(a) = \frac{1}{4\pi\epsilon_0} \cdot \frac{(-e) \cdot q'}{a - b}.$$

Substituting $q' = -\frac{R}{a}e$ and $b = \frac{R^2}{a}$:

$$U(a) = \frac{1}{4\pi\epsilon_0} \cdot \frac{(-e) \cdot \left(-\frac{R}{a}e\right)}{a - \frac{R^2}{a}}.$$

$$U(a) = \frac{1}{4\pi\epsilon_0} \cdot \frac{Re^2}{a^2 - R^2}.$$

Step 3: Potential Energy at $r = \infty$

When the electron is at infinity ($r = \infty$), the potential energy due to the sphere is zero:

$$U(\infty) = 0.$$

Step 4: Potential Energy on the Sphere ($r = R$)

When the electron is on the surface of the sphere ($r = R$), the potential energy is:

$$U(R) = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2R}.$$

Step 5: Energy Difference

The difference in energy between the electron at $r = \infty$ and the electron on the sphere ($r = R$) is:

$$\Delta U = U(\infty) - U(R) = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2R}.$$

Since ΔU is positive, work must be done to move the electron from infinity to the sphere.

4b. What is the electrostatic potential of the sphere? Calculate its capacitance.

Electrostatic Potential and Capacitance of a Conducting Sphere

Step 1: Electrostatic Potential

The electrostatic potential V of the sphere is the potential at its surface due to the image charge. The potential at the surface of the sphere ($r = R$) is:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q'}{R}$$

Substituting $q' = \frac{R}{a}e$:

$$V = \frac{1}{4\pi\epsilon_0}\frac{R}{a}\frac{e}{R}$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{e}{a}$$

Step 2: Capacitance of the Sphere

The capacitance C of the sphere is given by:

$$C = \frac{Q}{V} = 4\pi\epsilon_0 R.$$

4c. Repeat 3a for the situation where an electron is already on the sphere.

Potential Energy of an Electron on a Conducting Sphere

If an electron is already on the sphere, the total charge on the sphere is $-e$. The potential energy of the system is:

$$U(R) = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2R}.$$

The energy difference ΔU remains the same as in part (a):

$$\Delta U = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2R}.$$

4d. Repeat 3a for the situation where the sphere is grounded.

Potential Energy of an Electron Near a Grounded Sphere

If the sphere is grounded, it is held at zero potential. The image charge q' is still:

$$q' = -\frac{R}{a}e,$$

and the potential energy of the electron at $r = a$ is:

$$U(a) = \frac{1}{4\pi\epsilon_0} \cdot \frac{Re^2}{a^2 - R^2}.$$

The potential energy on the sphere ($r = R$) is zero because the sphere is grounded:

$$U(R) = 0.$$

The energy difference ΔU is:

$$\Delta U = U(R) - U(\infty) = 0 - 0 = 0.$$

5a. What is the total energy of N electrons on a large metallic island? Take both the charging energy and gate potential into account.

The potential of the island is, however, unknown in general, but its charge is known to be an integer multiple of the elementary charge. We can therefore write

$$V_0(Q_0) = \frac{Q_0 - Q_0^{(0)}}{C_\Sigma} - \sum_{j=1}^{n} \frac{C_{0j}}{C_\Sigma} V_j,$$

where $C_\Sigma \equiv C_{00} = -\sum_{i=1}^{n} C_{0i} > 0$. The electrostatic energy needed to add N additional electrons to the quantum dot is given by

$$E_{\text{elstat}}(N) = \int_{Q_0^{(0)}}^{Q_0^{(0)} - |e|N} dQ_0 V_0(Q_0) = \frac{e^2 N^2}{2C_\Sigma} + |e| N \sum_{j=1}^{n} \frac{C_{0j}}{C_\Sigma} V_j.$$

5b. Now the size of the island is made much smaller and the level-spacing becomes important. What is the total energy U(N) in this case?

If we assume that the solution of this single-particle problem gives energy levels $\epsilon_n^{(0)}$, the total energy of the island with N additional electrons is

$$E(N) = \sum_{n=1}^{N} \epsilon_n^{(0)} + \frac{e^2 N^2}{2C_\Sigma} + |e| N \sum_{i=1}^{n} \frac{C_{0i}}{C_\Sigma} (V_i - V_i^{(0)}). \tag{8}$$

5c. What does the word "chemical potential" mean? How is it defined when only a small number of electrons is in the system? Definition of Chemical Potential

In a system with many particles, the chemical potential is defined as the change in the total energy (E) when an additional electron is added, while keeping entropy (S) and volume (V) constant:

$$\mu = \left(\frac{\partial E}{\partial N}\right)_{S,V}$$

where N is the number of electrons.

When only a small number of electrons is present in the system (such as in a quantum dot or a small metallic island), the chemical potential is influenced by both the charging energy and the discrete quantum energy levels. In such a system, the chemical potential is given by:

$$\mu(N) = E(N) - E(N-1)$$

For a small metallic island (where quantum effects are important), the energy levels are discrete, and the chemical potential takes the form:

$$\mu(N) = \epsilon_N^{(0)} + \frac{e^2 N}{C_\Sigma} + |e| \sum_{i=1}^{n} \frac{C_{0i}}{C_\Sigma}(V_i - V_i^{(0)})$$

5d. Calculate the chemical potential for the total energy in a and b. Is it the same for each electron?

The chemical potential is defined as the energy required to add an additional electron to the system:

$$\mu(N) = E(N) - E(N-1).$$

For a large metallic island, using the total energy expression from part (a), the chemical potential is:

$$\mu(N) = \frac{e^2 N}{C_\Sigma} + |e| \sum_{i=1}^{n} \frac{C_{0i}}{C_\Sigma}(V_i - V_i^{(0)}).$$

For a small island with discrete energy levels (from part b), the chemical potential includes both the charging energy and the quantum energy levels:

$$\mu(N) = \epsilon_N^{(0)} + \frac{e^2 N}{C_\Sigma} + |e| \sum_{i=1}^{n} \frac{C_{0i}}{C_\Sigma}(V_i - V_i^{(0)}).$$

Since the quantum energy levels $\epsilon_N^{(0)}$ vary with N, the chemical potential is not the same for each electron in the small island case.

5e. Calculate the gate voltages of the charge degeneracy points.

The charge degeneracy points occur when the chemical potential aligns such that the system can fluctuate between N and $N+1$ electrons. This happens when:

$$\mu(N) = \mu_{\text{ext}}$$

where μ_{ext} is the external electrostatic potential. Setting $\mu(N) = \mu(N+1)$ gives the gate voltage V_g at which the charge degeneracy point occurs:

$$V_g = V_g^{(0)} + \frac{e}{C_g}\left(N + \frac{1}{2}\right).$$

This defines the gate voltages at which transitions between charge states occur.

Proof:

The total energy of a metallic island with N electrons, considering both charging energy and gate potential, is:

$$E(N) = \frac{e^2 N^2}{2C_\Sigma} + |e| N \sum_{i=1}^{n} \frac{C_{0i}}{C_\Sigma} V_i.$$

For a system with a **single gate capacitor** C_g, the energy can be written as:

$$E(N) = \frac{e^2 N^2}{2C_\Sigma} - eN\frac{C_g}{C_\Sigma} V_g.$$

where V_g is the applied gate voltage.

The **chemical potential** $\mu(N)$ is the energy cost of adding one more electron to the system:

$$\mu(N) = E(N) - E(N-1).$$

Substituting the energy expressions:

$$\mu(N) = \left[\frac{e^2 N^2}{2C_\Sigma} - eN\frac{C_g}{C_\Sigma} V_g\right] - \left[\frac{e^2 (N-1)^2}{2C_\Sigma} - e(N-1)\frac{C_g}{C_\Sigma} V_g\right].$$

Expanding the terms:

$$\mu(N) = \frac{e^2}{2C_\Sigma}\left[N^2 - (N-1)^2\right] - e\frac{C_g}{C_\Sigma} V_g \left[N - (N-1)\right].$$

Since:

$$N^2 - (N-1)^2 = 2N - 1,$$

we get:

$$\mu(N) = \frac{e^2}{C_\Sigma}\left(N - \frac{1}{2}\right) - e\frac{C_g}{C_\Sigma}V_g.$$

Step 3: Charge Degeneracy Condition

Charge degeneracy occurs when the energy to add an electron equals the external potential influence, meaning:

$$\mu(N) = 0.$$

Setting the equation to zero:

$$\frac{e^2}{C_\Sigma}\left(N - \frac{1}{2}\right) - e\frac{C_g}{C_\Sigma}V_g = 0.$$

Solving for V_g:

$$V_g = \frac{e}{C_g}\left(N - \frac{1}{2}\right).$$

Thus, the charge degeneracy points occur at gate voltages:

$$V_g = V_g^{(0)} + \frac{e}{C_g}\left(N + \frac{1}{2}\right).$$

where $V_g^{(0)}$ accounts for any offset voltage or background charge.

6a. Electrons that tunnel to a quantum dot have to pay the charging energy and the charging energy and the level spacing, which results in diamonds in the stability diagram. The stability diagram shown below is measured here in Delft in a carbon nanotube quantum dot (Sapmaz et al.Phys. Rev. B 71, 153402, 2005). Find the addition energies for each of the four different diamonds.

$$\Delta\mu_1 = \Delta\mu_2 = Ec + dU + J \tag{9}$$

$$\Delta\mu_2 = E_c + \delta - dU \tag{10}$$

$$\Delta\mu_4 = E_c + \Delta - \delta - dU \tag{11}$$

in this pictureb three small diamonds are same size. So,

$$\delta \approx J + 2dU \tag{12}$$

So,These values we will use to calculate additional energies- $E_c = 6.6$ meV, $\Delta = 8.7$ meV $\delta = J = 2.9$ meV and $dU \approx 0$ meV.

$$\Delta\mu_1 = \Delta\mu_2 = 6.6 + 2.9 = 9.5meV \tag{13}$$

$$\Delta\mu_3 = 6.6 + 2.9 = 9.5meV \tag{14}$$

$$\Delta\mu_4 = 6.6 + 8.7 - 2.9 = 12.4meV \tag{15}$$

6b. There are many more lines visible in this stability diagram than one would expect for a simple quantum dot. Lines running parallel to the diamond edges can be used to find values for the energy difference between the ground state and excited states in a given charge state. Use a sketch of an energy diagram of the leads and the dot to explain how this works.

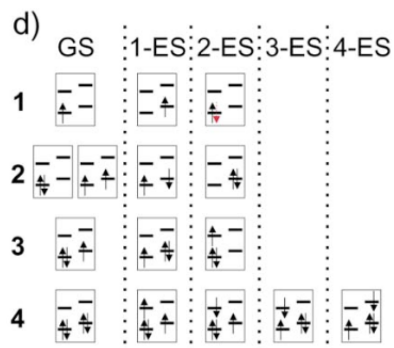

Figure 3: Energy states

In this system, we observe a fourfold degeneracy with four electrons, leading to a fourfold periodicity. Each Coulomb diamond in the diagram represents the addition of a single electron. Due to the non-uniform level spacing, the energy required to add each electron varies, resulting in distinct heights for the Coulomb diamonds. The diagram illustrates the sequential filling of electron states, clearly demonstrating how electrons are added to the system one by one.

6c. The band structure in a metallic nanotube is linear and given by E(k) = $v_F$hk/2 π. Calculate the level spacing for a nanotube with length L = 350 nm. Which lines would correspond to this energy?

Energy Dispersion Relation

The energy dispersion relation for a metallic nanotube is given by:

$$E(k) = \frac{v_F hk}{2\pi} \tag{16}$$

where:

- v_F is the Fermi velocity ($v_F \approx 10^6$ m/s for graphene-based nanotubes),
- h is Planck's constant ($h \approx 6.626 \times 10^{-34}$ J·s),
- k is the wavevector.

Step 1: Quantization of k

For a nanotube of length L, the allowed wavevectors are quantized due to the boundary conditions:

$$k_n = \frac{2\pi}{L} n, \quad n = 0, \pm 1, \pm 2, \ldots \tag{17}$$

Thus, the energy levels are:

$$E_n = \frac{v_F h}{2\pi} \cdot \frac{2\pi}{L} n = \frac{v_F h}{L} n \tag{18}$$

Step 2: Energy Level Spacing

The level spacing is the energy difference between two consecutive energy levels:

$$\Delta E = E_{n+1} - E_n = \frac{v_F h}{L} \tag{19}$$

Substituting values:

$$\Delta E = \frac{(10^6 \text{ m/s})(6.626 \times 10^{-34} \text{ J·s})}{350 \times 10^{-9} \text{ m}} \tag{20}$$

$$\Delta E \approx 1.89 \times 10^{-3} \text{ eV} = 1.89 \text{ meV} \tag{21}$$

Step 3: Corresponding Spectral Lines

The corresponding spectral line wavelength is given by:

$$E = \frac{hc}{\lambda} \tag{22}$$

Solving for λ:

$$\lambda = \frac{hc}{\Delta E} \tag{23}$$

Substituting values:

$$\lambda = \frac{(6.626 \times 10^{-34} \text{ J·s})(3.0 \times 10^8 \text{ m/s})}{(1.89 \times 10^{-3} \times 1.6 \times 10^{-19} \text{ J})} \tag{24}$$

$$\lambda \approx 656\,\mu m = 0.656 \text{ mm} \quad (25)$$

This places the spectral line in the **far-infrared (terahertz) region** of the electromagnetic spectrum.

6d. When the effect of interactions between the electrons is on the dot neglected, each level has a four fold degeneracy (2x due to spin, 2x due to clockwise/anticlockwise). Find the charging energy of this nanotube from the sizes of the diamonds.

Charging energy, E_c , If we consider one electron is in one level then, [12.1+7.1+5.1+2.1] /4 =6.6 meV

6e. What is the gate capacitance of the tube? Calculate the length of the tube using the equation for the gate capacitance in the slides. Is this in agreement with the length found from the level spacing?

The gate capacitance C_g can be calculated using the relation:

$$C_g = \frac{e}{\Delta V_g}$$

= 3.2 aF

Length calculation:

$$C_g = \frac{2\pi\epsilon_0\epsilon_r L}{\ln(4h/d)}$$

where:

- ϵ_0 is the permittivity of free space, - ϵ_r is the relative permittivity of the dielectric, - h is the distance between the nanotube and the gate, - d is the diameter of the nanotube.

Rearranging for L:

$$L = \frac{C_g \ln(4h/d)}{2\pi\epsilon_0\epsilon_r}$$