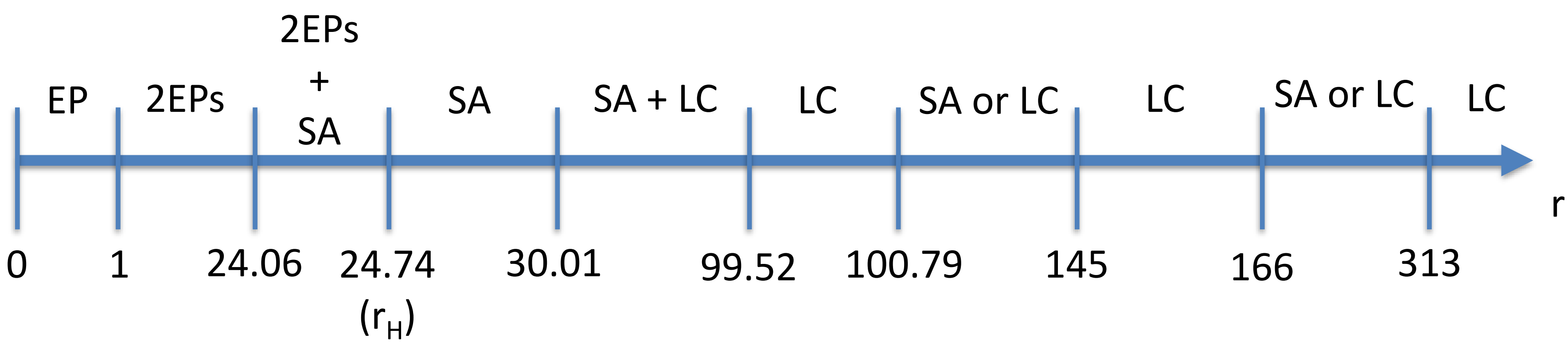
3.1.4 Exploring the parameter space

- The behavior of the Lorenz system as the parameter(s) vary is the subject of much contemporary research
- Lorenz look at the parameters σ=10, b=8/3, r=28, and we mostly explored the dynamics around these values. What happens when they are varied ?
- The dynamics has ben fully characterized for σ=10, b=8/3, as r varies, and shows an interesting behavior :



EP: stable equilibrium point
SA: strange attractor
LC: stable limit cycle

3.1.4 Exploring the parameter space

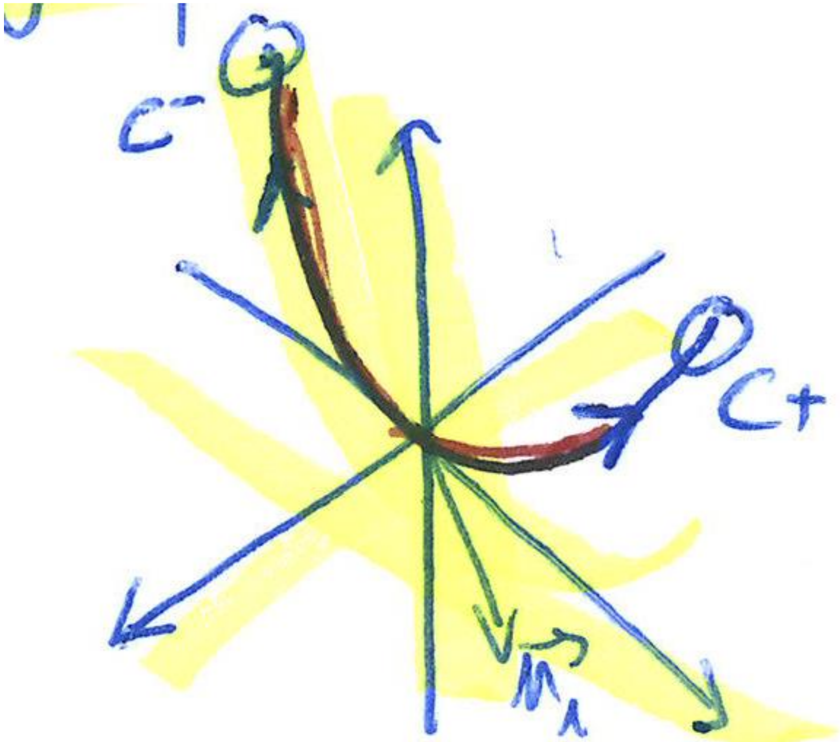
$1 < r < r_H$:

- When $r > 1$, the origin becomes unstable. One can identify the stable and unstable manifolds. The 1D unstable manifold is a curve tangent to the first eigenvector ($\mathbf{u_1}$), the 2D stable manifold is a surface tangent to the two eigenvectors ($\mathbf{u_2}$ and $\mathbf{u_3}$)
  The stable and unstable manifolds cannot be crossed by an orbit, so the 2D stable manifolds divide the orbits in two sets

3.1.4 Exploring the parameter space

$1 < r < r_H$:

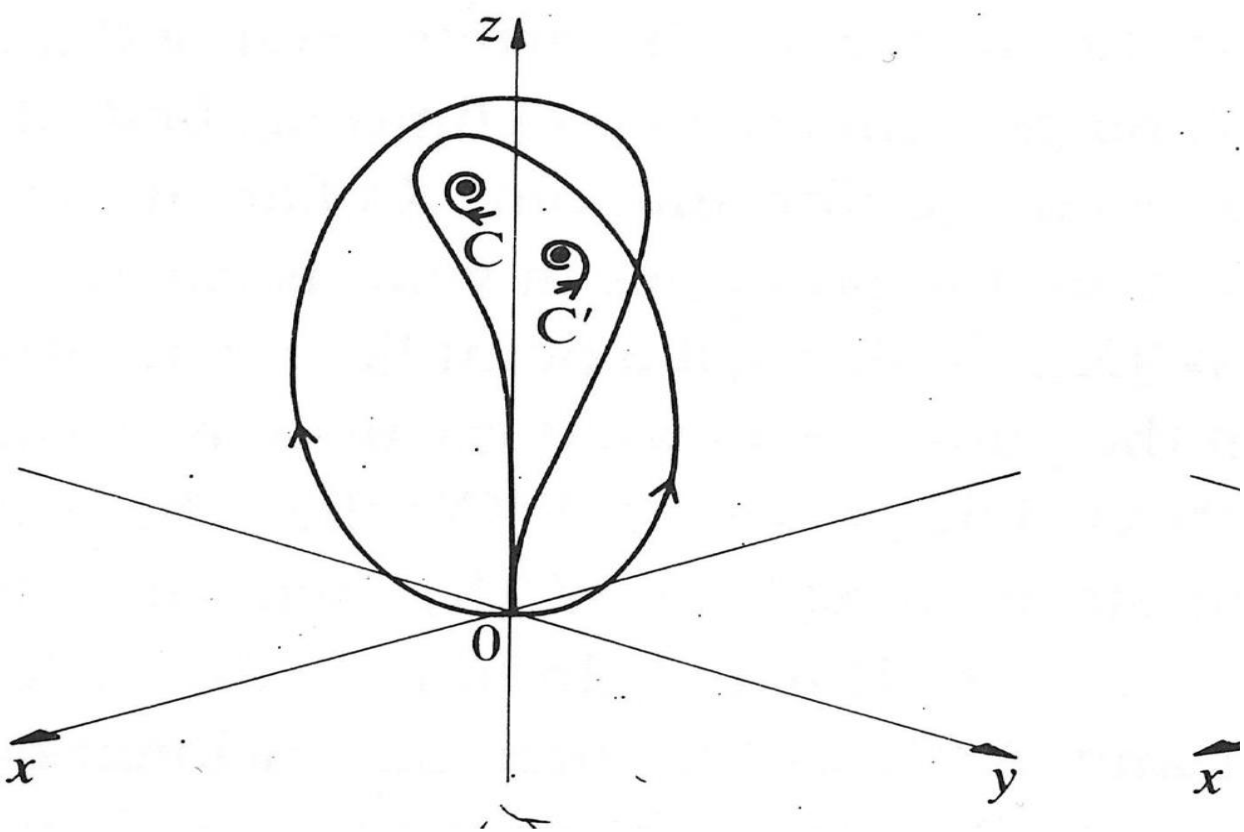
- For $1 < r < r_0 \approx 13{,}93$ (numerical)
    - The unstable manifold of 0 leaves the origin and ends of $C^{+/-}$ depending if we consider the unstable manifold parallel or anti-parallel to $\mathbf{u_1}$



3.1.4 Exploring the parameter space

$1 < r < r_H$:

- For $r = r_0 \approx 13.93$ (numerical)
    - Two homoclinic orbits appear, between 0 and itself. These orbits leave 0 for the unstable manifold and come back along the plan of the stable manifold.



3.1.4 Exploring the parameter space

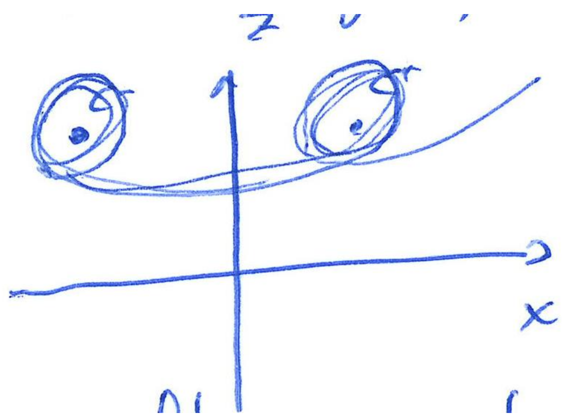
$1 < r <$ r_H :

- For $r_0 \approx 13.93 < r < r_1 \approx 24.06$

  Two unstable limit cycles (of finite period) arise from the homoclinic orbts (homoclinic bifurcation). The limit cycles are the ones that we find at the Hopf bifurcation that occurs at $r=r_1$

  As r increases, the orbits cross-over from C+ to C- and back, an increasing number of times, before spiraling into either C+ or C-. This is sometimes called **pre-chaos** or **transient chaos**. The orbits that tend to C+ or C- are very close to each others, the basins of attraction of the two stable equilibrium points extend very complex, complementary, interwoven domains.

  This is a bit like rolling a dice : the system ends-up in one of the 6 possible equilibrium positions, but after a phase of crazy motion that makes it impossible to predict the final position.



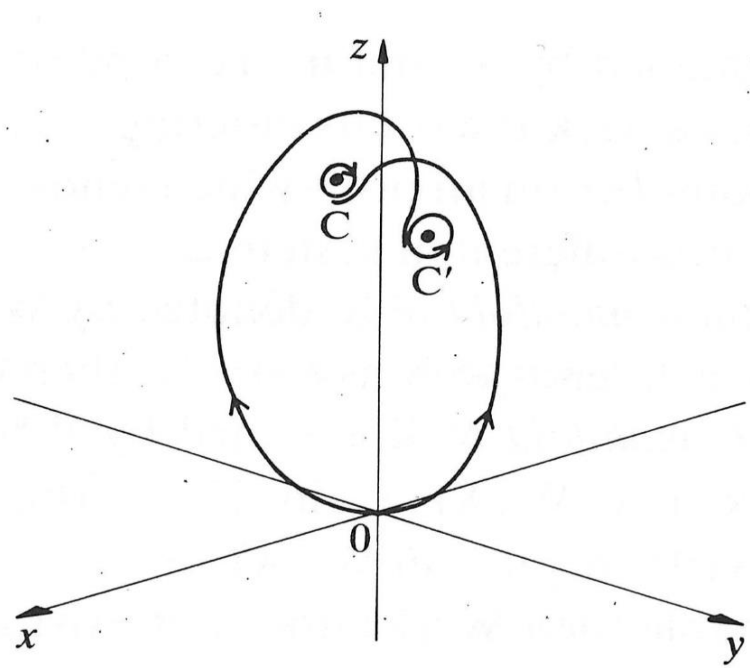
3.1.4 Exploring the parameter space

1 < r < r_H :

- $r = r_1 \approx 24.06$

  The unstable manifold of O leaves O and enters the saddle limit cycles around $C^{+/-}$.
  The two orbits that are created are called heteroclinic between O and the limit cycle (heteroclinic: an orbit that joints two different invariant sets)

  The creation of the heteroclinic orbits is what defines the r_1 point and marks the onset of chaos.



(b)

3.1.4 Exploring the parameter space

$1 < r < r_H$:

- $r_1 < r < r_H$

  The meandering time around $C^{+/-}$ becomes infinite, indeed from the heteroclinic orbits the strange attractor is born. This coexists with the attractors $C^{+/-}$,