Mechanical effects of light

Idea: . photons have momentum

. "Effective" theory

$$\xrightarrow{\hspace{6cm}} E,$$

$\underbrace{\text{motion of atoms}}_{\text{centre-of-mass}}$ $\quad$ $\underbrace{\text{motion of } e^-}_{\text{internal degrees of freedom}}$

"effective" action $\longleftarrow$ ↑ addressed by laser

I. Formulation

1) Hilbert space

$$\mathcal{H} = \mathcal{H}_{at} \otimes \mathcal{H}_{field} = \mathcal{H}_{c.o.m} \otimes \mathcal{H}_{internal} \otimes \mathcal{H}_{laser\,mode} \otimes \mathcal{H}_{vac}$$

- $\mathcal{H}_{com} = Sp\{ |\vec{p}\rangle, \vec{p} \in \mathbb{R}^3 \}$ atom in free space
- $\mathcal{H}_{internal} = Sp\{ |n L J m_J \ldots\rangle \ldots \}$ energy levels of the atom
- $\mathcal{H}_{laser} = Sp\{ |n\rangle, n \in \mathbb{N} \}$ one particular mode addressed by the laser
- $\mathcal{H}_{vac} = \mathcal{F}$ Fock space

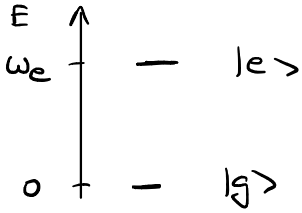
2) Simplifications

- Reduce $\mathcal{H}_{int}$ to a 2.d Hilbert space $= Sp\{ |g\rangle, |e\rangle \}$

  ↳ cycling transition

- Laser mode is in a coherent state → classical field
- Vacuum as a reservoir (Markov approximation) → Lindblad equation
- C.o.m motion is slow coupled with internal dynamics.

3) Hamiltonian



acts on $\mathcal{H}_{com} \otimes \mathcal{H}_{int}$

$$\hat{H} = \frac{\hat{\vec{p}}^2}{2m} \otimes \hat{1} + \hat{1} \otimes \omega_e |e\rangle\langle e| + \frac{\Omega(\hat{\vec{r}})}{2}\left[e^{i(\omega_0 t - \vec{k}_L \hat{\vec{r}})} \otimes |g\rangle\langle e| + h.c. \right]$$

key point: Ω and $\phi = \omega_0 t - \vec{k}_L \hat{\vec{r}}$ are operators on $\mathcal{H}_{com}$

- within dipole approximation : "size" of the atom $\ll \lambda$
- the atom "feels" Ω and ϕ at the position of its c.o.m

$\Omega(\hat{\vec{r}})$: $\langle \Psi_{com} | \Omega(\hat{\vec{r}}) | \Psi_{com} \rangle$ for c.o.m in state $|\Psi_{com}\rangle$

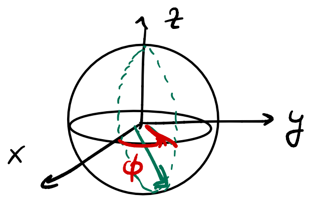
$\Omega(\hat{\vec{r}}) = - \vec{E}(\hat{\vec{r}}) d_0$ $\qquad \phi(\hat{\vec{r}}) = \omega_0 t - \vec{k}_L \cdot \hat{\vec{r}}$

II. Recoil and Doppler effects

1) Phase imprinting

reminder :

$$\hat{H}_{int} = \frac{\Omega}{2}\left(e^{-i\phi} |e\rangle\langle g| + h.c.\right) = \frac{|\Omega|}{2} \vec{n} \cdot \hat{\vec{\sigma}}$$



here $\phi(\vec{r})$

Consider: $|\Psi_{at}\rangle = |\vec{p}\rangle \otimes |g\rangle$

$$\hat{H}_{int} |\Psi_{at}\rangle = \frac{\Omega}{2}\left(\underbrace{e^{-i\phi(\hat{\vec{r}})} |\vec{p}\rangle}_{\int d^3\vec{R}\; |\vec{R}\rangle\langle\vec{R}|\, e^{-i\phi(\hat{\vec{r}})} |\vec{p}\rangle} \right) \otimes |e\rangle$$

$$= \int d^3R \; e^{-i\phi(\vec{R})} |\vec{R}\rangle \underbrace{\langle\vec{R}|\vec{p}\rangle}_{e^{i\vec{p}\cdot\vec{R}}}$$

$$\rightarrow \quad = \int d^3R \; e^{-i\phi(\vec{R}) + i\vec{p}\vec{R}} |\vec{R}\rangle$$

Plane wave: $\phi(\vec R) = \omega_0 t - \hbar\vec k_L\cdot\vec R$ then:

$$\hat H_{int}|\Psi_{at}\rangle = \frac{\hbar\Omega}{2} e^{-i\omega_0 t}\underbrace{\int d^3R\, e^{i(\hbar\vec k_L\vec R + \vec p\vec R)}|\vec R\rangle}_{|\vec p+\hbar\vec k_L\rangle}$$

So $$\hat H_{int} = \frac{\hbar\Omega}{2}\sum_{\vec p} e^{i\omega_0 t}\,|\vec p\, g\rangle\langle\vec p+\hbar\vec k_L, e| + h.c.$$

2) Shifts

Move to the rest frame of the atom (from the laboratory frame where momentum $\vec p$)

unitary transformation: $$\hat U(t) = e^{i\frac{\vec p^2}{2m}t} \qquad \hat H \to (i\partial_t\hat U)\hat U^\dagger + \hat U\hat H\hat U^\dagger$$

in this frame:

$$\hat H = \frac{\hbar\Omega}{2}\sum_{\vec p} e^{i\omega_0 t}\underbrace{\hat U|\vec p\, g\rangle\langle\vec p+\hbar\vec k_L\, e|\hat U^\dagger}_{e^{i\frac{\vec p^2}{2m}t}|\vec p\, g\rangle\langle\vec p+\hbar\vec k_L\, e|e^{-i\frac{(\vec p+\hbar\vec k_L)^2}{2m}t}} + h.c.$$

$$= \frac{\hbar\Omega}{2}\sum_{\vec p} e^{i\left(\omega_0 - \frac{\vec p\,\hbar\vec k_L}{m} - \frac{k_L^2}{2m}\right)t}|\vec p\, g\rangle\langle\vec p+\hbar\vec k_L\, e| + h.c.$$

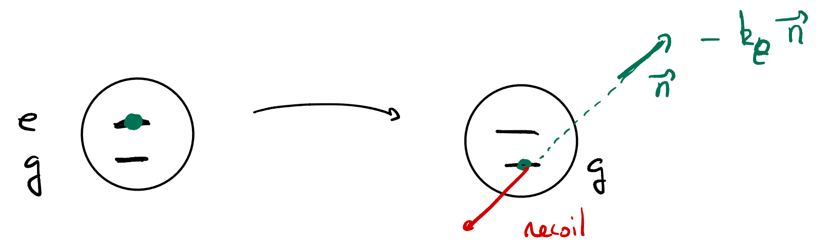
$$\omega_0 \longrightarrow \omega_0 - \frac{\vec p\cdot\hbar\vec k_L}{m} - \frac{k_L^2}{2m}$$

- Doppler shift: $\frac{\vec p\cdot\hbar\vec k_L}{m} = \vec p\cdot\vec v_{rec}$ $\qquad v_{rec} = \frac{\hbar\vec k_L}{m}$

  Ex: ^6Li at 300K: 1 GHz $\qquad$ Ex: ^6Li $\sim 0,1\ ms^{-1}$

- Recoil shift: $E_R = \frac{\hbar^2 k_L^2}{2m} = h\cdot 77\,kHz$ for ^6Li

3) Jump operators



$k_L = \frac{\omega_e}{c}$

Family of jump operators: $\hat{L}(\vec{n}) = \sum_{\vec{p}} |\vec{p}+k_e\vec{n}, g\rangle\langle \vec{p}\, e|$

$= e^{i k_e \vec{n}\cdot\vec{r}} \otimes |g\rangle\langle e|$

rate: $\Gamma_{\vec{n}}$ $\quad \left(\hat{M}(\vec{n}) = \sqrt{\Gamma_{\vec{n}}}\cdot \hat{L}(\vec{n}) \right)$

4) Lindblad equation:

$$\dot{\hat{\rho}} = -i[\hat{H},\hat{\rho}] + \sum_{\vec{n}} \Gamma_{\vec{n}} \left(\hat{L}(\vec{n})\hat{\rho}\hat{L}(\vec{n})^{+} - \frac{1}{2}\hat{L}^{+}\hat{L}(\vec{n})\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{L}^{+}\hat{L}(\vec{n}) \right)$$

note that: $\hat{L}^{+}(\vec{n})\hat{L}(\vec{n}) = |e\rangle\langle e|$ $\quad$ introduce $\Gamma = \sum_{\vec{n}} \Gamma_{\vec{n}}$

$$\dot{\hat{\rho}} = -i[\hat{H},\hat{\rho}] \underbrace{- \frac{\Gamma}{2}\left(|e\rangle\langle e|\hat{\rho} + \hat{\rho}|e\rangle\langle e| \right)}_{\text{non Hermitian "decay" of } |e\rangle} + \underbrace{\sum_{\vec{n}} \Gamma_{\vec{n}} \hat{L}(\vec{n})\hat{\rho}\hat{L}^{+}(\vec{n})}_{\text{"kicks" upon spont. emission}}$$

Last term: $\quad \hat{\rho} = |e\vec{p}\rangle\langle e\vec{p}|$

then $\quad \hat{L}(\vec{n})\hat{\rho}\hat{L}^{+}(\vec{n}) = |g, \vec{p}+k_e\vec{n}\rangle\langle g, \vec{p}+k_e\vec{n}|$

III. Semi-classical Force

1) Further simplifications

- the momentum "spread" of the atom is large $\quad \Delta p \gg k_e, k_L ..$
  $\Leftrightarrow$ position spread is small $\Delta R \ll \lambda$
  $\Rightarrow$ k_e is small $\quad$ (momentum exchanges with em field are continuous)

- internal dynamics is much faster than motion of com.
  $\Rightarrow$ steady state description of the internal state for each position
  $\Gamma, \Omega \gg E_R$

$\Rightarrow$ Ansatz for the density matrix: $\quad \hat{\rho} = \hat{\rho}_{com} \otimes \hat{\rho}_{int}^{st} \qquad \hat{\rho}_{int}^{st}(\vec{R}(t))$

2) Spontaneous emission

$$\sum_{\vec{n}} \Gamma_{\vec{n}}\, e^{i k_e \vec{n}\cdot\hat{\vec{r}}} |g\rangle\langle e| \hat{\rho} |e\rangle\langle g| e^{-i k_e \vec{n}\cdot\hat{\vec{r}}}$$

since k_e is small: $e^{i k_e \vec{n}\cdot\hat{\vec{r}}} \sim 1 + i k_e \vec{n}\cdot\hat{\vec{r}} - \frac{k_e^2}{2}(\vec{n}\cdot\hat{\vec{r}})^2 + \dots$

So:

$$= \underbrace{\sum_{\vec{n}} \Gamma_{\vec{n}}}_{\Gamma} |g\rangle\langle e|\hat{\rho}|e\rangle\langle g| + \underbrace{\sum_{\vec{n}} i k_e \vec{n}\cdot\hat{\vec{r}}\, |g\rangle\langle e|\hat{\rho}|e\rangle\langle g|}_{=0} + \sum_{\vec{n}} |g\rangle\langle e|\hat{\rho}|e\rangle\langle g| (-i k_e \vec{n}\cdot\hat{\vec{r}})$$

$$+ O(k_e^2)$$

up to $O(k_e^2)$: no net effect of spont. emission of the center of mass motion!

now of to $O(k_e^2)$:

$$\dot{\hat{\rho}} = -i[\hat{H},\hat{\rho}] + \Gamma |g\rangle\langle e|\hat{\rho}|e\rangle\langle g| - \frac{\Gamma}{2}\left(|e\rangle\langle e|\hat{\rho} + \hat{\rho}|e\rangle\langle e|\right)$$

the dissipative part of the dividend eq is only affecting the internal state!!

3) Force:

$$\vec{F} = \frac{d}{dt}\langle\hat{\vec{p}}\rangle = \frac{d}{dt}\mathrm{Tr}(\hat{\rho}\hat{\vec{p}}) = \mathrm{Tr}(\dot{\hat{\rho}}\cdot\hat{\vec{p}})$$

$$= -i\,\mathrm{Tr}\left([\hat{H},\hat{\rho}]\cdot\hat{\vec{p}}\right)$$

$\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$

$$= -i\,\mathrm{Tr}\left(\hat{\rho}\cdot[\hat{\vec{p}},\hat{H}]\right)$$

$$\vec{F} = -i\langle[\hat{\vec{p}},\hat{H}]\rangle$$

$\hat{H}$ depends on $\hat{\vec{r}}$: move to position representation $\hat{\vec{p}} = -i\vec{\nabla}_r$

(via $\Omega(\vec{r})$, $\phi(\vec{r})$ ...)

then $\langle[\hat{\vec{p}},\hat{H}]\rangle = -i\langle\vec{\nabla}_r\hat{H}\rangle$

Proof: for any state $|\alpha\rangle$: $\langle\alpha|\vec{\nabla}_r(\hat{H}|\alpha\rangle)$

$$= \langle\alpha|(\vec{\nabla}_r\hat{H})|\alpha\rangle + \langle\alpha|\hat{H}(\vec{\nabla}_r|\alpha\rangle)$$

So:

$$\boxed{\vec{F} = -\langle\vec{\nabla}_r\hat{H}\rangle}$$

For an atom at rest: $\hat{H} = \omega_e |e\rangle\langle e| + \frac{\hbar\Omega(\vec{r})}{2}\left(e^{i(\omega_0 t - \vec{k}_L\cdot\vec{r})}|g\rangle\langle e| + hc\right)$

in a frame rotating at ω_0 (for internal dof):

$$\hat{H} = \Delta |e\rangle\langle e| + \frac{\hbar\Omega(\vec{r})}{2}\left(e^{-i\vec{k}_L\cdot\vec{r}}|g\rangle\langle e| + hc\right)$$

$$\langle -\vec{\nabla}_r \hat{H}\rangle = -\frac{\vec{\nabla}\hbar\Omega(\vec{r})}{2}\left(\underbrace{\langle e^{-i\vec{k}_L\cdot\vec{r}}|g\rangle\langle e| + hc\rangle}_{\text{internal dof} \to \text{OBE!}}\right) + i\vec{k}_L\frac{\hbar\Omega}{2}\left(\langle e^{-i\vec{k}_L\cdot\vec{r}}|g\rangle\langle e| + \dots\right)$$

$$\boxed{\vec{F} = \frac{\vec{\nabla}\hbar\Omega}{\hbar\Omega}\frac{\Delta\cdot s(\vec{r})}{1+s(\vec{r})} + \vec{k}_L\cdot\frac{\Gamma}{2}\frac{s(\vec{r})}{1+s(\vec{r})}} \qquad s(\vec{r}) = \frac{\hbar\Omega^2(\vec{r})/2}{\Delta^2 + \Gamma^2/4}$$

$$= \vec{F}_{dip} + \vec{F}_{rad}$$

- $\vec{F}_{dip}$: dipole force $= \vec{F}_{dip} = \frac{\Delta}{2}\frac{\vec{\nabla}s}{1+s}$
- $\vec{F}_{rad}$: radiation pressure $= \vec{F}_{rad} = \vec{k}_L\cdot\Gamma\cdot\langle \rho_{ee}\rangle_{st}$

IV - Interpretation

1) Dipole force

reexpressed as: $\vec{F}_{dip} = -\vec{\nabla}\left(-\frac{\Delta}{2}\ln(1+s(\vec{r}))\right)$

$\to$ dipole force is conservative and $U_{dip} = -\frac{\Delta}{2}\ln(1+s)$

limiting case: $\Delta \gg \hbar\Omega, \Gamma$

$s \ll 1 \Rightarrow \langle\rho_{ee}\rangle \ll 1$

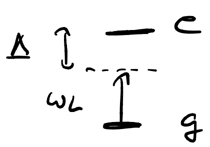
then $U_{dip} = -\frac{s}{2}\Delta = -\frac{\hbar\Omega^2(\vec{r})}{4\Delta}$

- $\hbar\Omega \propto \mathcal{E}$ E-field amplitude

  $\Rightarrow U_{dip} \propto I$ light intensity

- U_{dip} is also independent of Γ

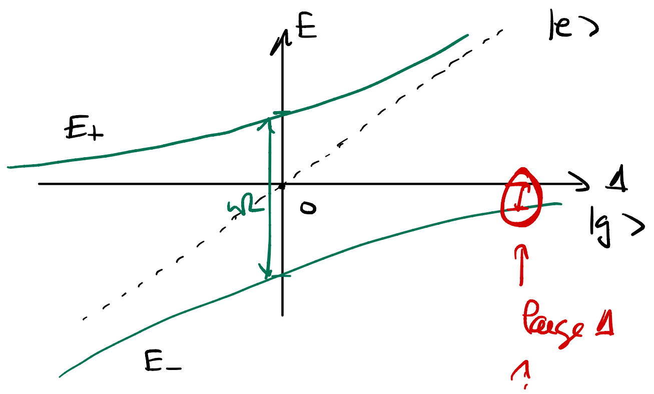
Recovering the dipole potential : 2-level atom in external field



$$\hat{H} = \omega_e|e\rangle\langle e| + \frac{\hbar\Omega}{2}\left(e^{-i\omega_L t}|e\rangle\langle g| + hc\right)$$

in a rotating frame: $\hat{H} = \Delta |e\rangle\langle e| + \frac{\Omega}{2} \left(|e\rangle\langle g| + h.c. \right)$

energy spectrum:



large Δ: $E_\pm = \frac{1}{2} \left(\Delta \pm \sqrt{\Delta^2 + \Omega^2} \right)$ $\quad E_- \to -\frac{\Omega^2}{4\Delta}$

and $|-\rangle \to |g\rangle$

Now if $\Omega(\vec{r}) \to E_-(\vec{r})$

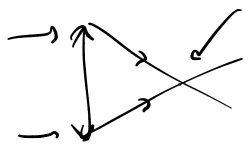
Remarks:

- $U_{dip} \Longleftrightarrow$ AC-Stark shift

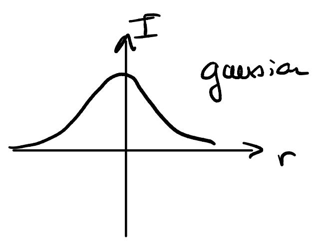
  (can also be obtained directly using 2^{nd} order perturbation theory)

- Also possible for DC fields: $\Delta \to \omega_e$

- Optical dipole trap:

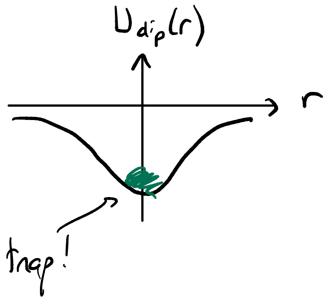


at the focus:

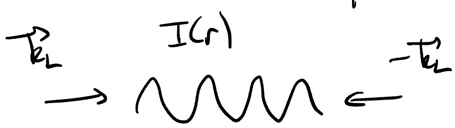


$\Rightarrow$ for $\Delta > 0$: $(\omega_e > \omega_L)$

" optical tweezer "



Optical lattice:

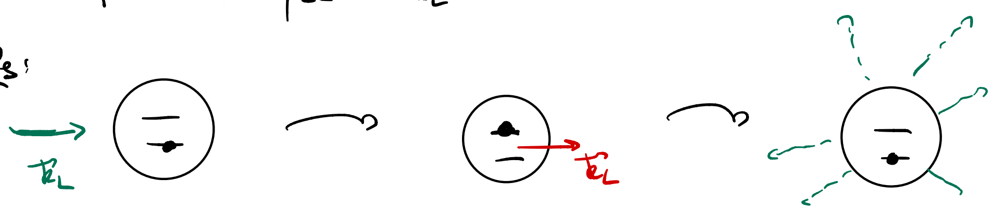


$\Rightarrow U_{dip} = U_0 \cos^2 k_L r \Rightarrow$ band structure

2) Radiation pressure

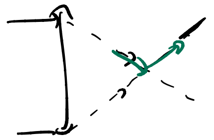
$$\vec{F} = \Gamma \langle p_{ee} \rangle^{st} \vec{k}_L$$

cycles:



Remarks:

- intrinsically related to spontaneous emission $\Leftrightarrow$ incoherent
- dipole force: stimulated emission

tweezer 

- radiation pressure is related to photon absorption:

3) Doppler cooling

an atom moving with momentum $\vec{p}$

$$\Delta_0 \rightarrow \Delta_0 + \vec{p}\cdot\vec{v}_{rec} - E_{rec}$$

here: E_{rec} is negligible

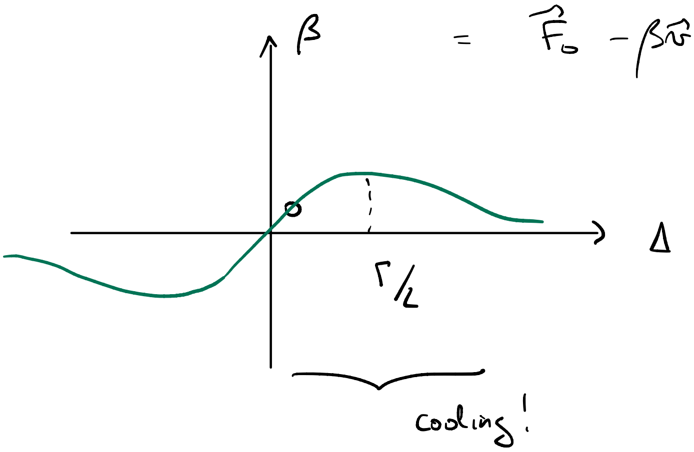
$$\vec{F}_{rad} = \hbar k_L \frac{\Gamma\Omega^2}{4} \frac{1}{\Delta_0^2 + \Gamma'^2} \qquad \Big\{ \; \Gamma' : \text{power broadened linewidth}$$

$$= \hbar k_L \frac{\Gamma\Omega^2}{4} \frac{1}{(\Delta_0 + \vec{p}\cdot\vec{v}_{rec})^2 + \frac{\Gamma'^2}{2}}$$

First order expansion for Doppler shift:

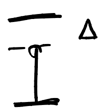
$$\vec{F}_{rad} = \hbar k_L \frac{\Gamma\Omega^2}{4} \left(\frac{1}{\Delta_0^2 + \Gamma'^2} - \vec{p}\cdot\vec{v}_{rec} \frac{2\Delta_0}{(\Delta_0^2+\Gamma'^2)^2} + \dots \right)$$

$$= \vec{F}_0 - \beta\vec{v} \qquad \beta : \text{friction coefficient.}$$



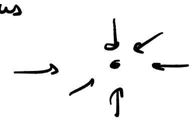
Interpretation:

- Doppler shift bring the atom closer to resonance when it moves against the beam



Remarks:

- to cool in all directions: 6 laser beams

"optical molasses" 

4) Limitation and extensions

- limit to Doppler cooling : terms $O(k_L^2)$ in the Lindblad equation

  random walk in k space $\langle \Delta p^2 \rangle \sim 2 m \hbar \Gamma$

  $\rightarrow$ limit temperature: $kT \sim \hbar \Gamma$ $\left(\begin{array}{l} {}^{6}Li = 280\,\mu K \\ {}^{88}Sr : 400\,\mu K \end{array} \right.$

- the finite k_L $\Rightarrow$ cannot reach $\bar{v} = 0$ in free space

  $\rightarrow$ E_{rec} is the hard limit

  For trapped atoms $\rightarrow$ reaching the ground state is possible

- other mechanism to cool in free space : Sisyphus cooling