Synthesizing arbitrary quantum states in a superconducting resonator

JunZhe Chen

EPFL/Aalto

2024.11.14

Synthesizing arbitrary quantum states in a superconducting resonator

Max Hofheinz[1], H. Wang[1], M. Ansmann[1], Radoslaw C. Bialczak[1], Erik Lucero[1], M. Neeley[1], A. D. O'Connell[1], D. Sank[1], J. Wenner[1], John M. Martinis[1] & A. N. Cleland[1]

Generation of Fock states in a superconducting quantum circuit

Max Hofheinz[1], E. M. Weig[1]†, M. Ansmann[1], Radoslaw C. Bialczak[1], Erik Lucero[1], M. Neeley[1], A. D. O'Connell[1], H. Wang[1], John M. Martinis[1] & A. N. Cleland[1]

Introduction

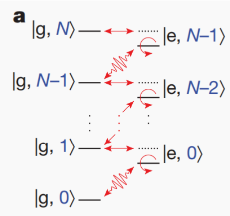
> A significant difference between these systems (non-linear and linear quantum systems) is that a two-level spin can be prepared in an arbitrary quantum state using classical excitations, whereas classical excitations applied to an oscillator generate a coherent state, nearly indistinguishable from a classical state.
>
> — *Max Hofheinz et al.(2008)*

- Having reached the strong-coupling regime, the next...
- Preparation of classical & quantum state
- Interplay between non-linear & linear quantum system
- Many types of Hamiltonian manipulation.

Jaynes-Cummings model

The engineered Hamiltonian can be extremely complicated, while the simplest model of our interest is:

$$H_{JC}/\hbar = \omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\omega_q}{2}\sigma_z + g(a\sigma_+ + a^\dagger\sigma_-) = \bigoplus_{n=0}^{\infty} H_n \quad (1)$$



$$H_n = \begin{pmatrix} n\omega_r & g\sqrt{n} \\ g\sqrt{n} & (n-1)\omega_r + \omega_q \end{pmatrix} \quad \begin{matrix} |n,g\rangle \\ |n-1,e\rangle \end{matrix} \quad (2)$$

(bare basis)

- The detuning $\Delta = \omega_q - \omega_r$ is the rate of phase accumulation between dressed state $|n,+\rangle$ and $|n,-\rangle$.

Target: N-photon Fock state

resonant-SWAP gate:

$$H_n^{int} = \begin{pmatrix} 0 & g\sqrt{n} \\ g\sqrt{n} & 0 \end{pmatrix}, \quad S_n = e^{-iH_n^{int}t} = \begin{pmatrix} \cos(gt\sqrt{n}) & -i\sin(gt\sqrt{n}) \\ -i\sin(gt\sqrt{n}) & \cos(gt\sqrt{n}) \end{pmatrix} \tag{3}$$

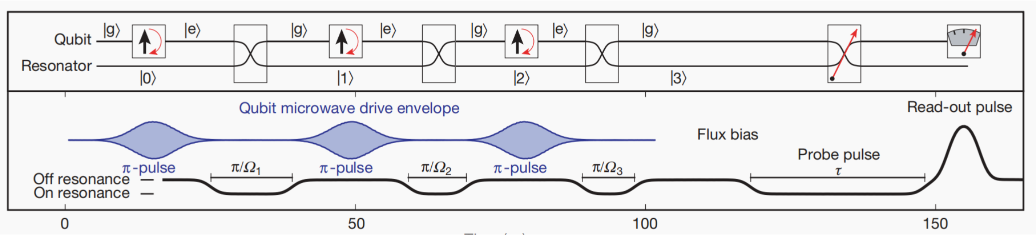


Figure: Pulse sequence for preparing n-Fock state [1]

Device and control setup

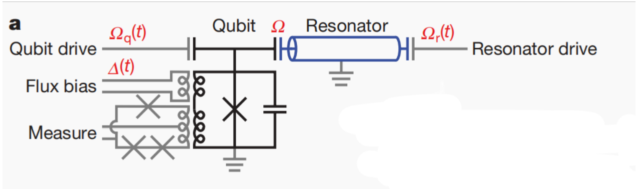


Figure: Device circuit diagram [2]

T_e	$T_{1,r}$	$T_{1,q}$	$T^*_{2,q}$
25mK	3.5us	650ns	150ns
$g/2\pi$	$\omega_r/2\pi$	$\Delta_{off}/2\pi$	
38MHz	6.57GHz	-463MHz	

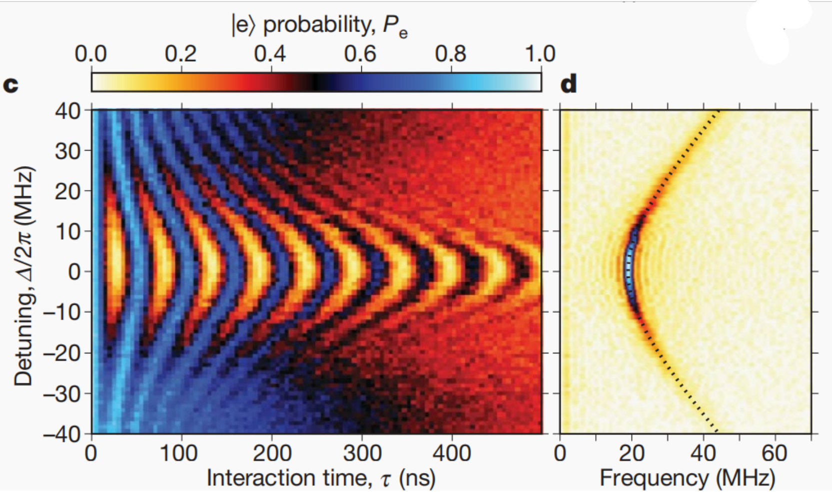
Table: Device parameters

Hamiltonian

$$H/\hbar = \Delta(t)\sigma_+\sigma_- + \left(g\sigma_+ a + g^*\sigma_- a^\dagger\right) + (\Omega_q(t)\sigma_+ + \Omega_q^*(t)\sigma_-) + (\Omega_r(t)a^\dagger + \Omega_r^*(t)a), \quad (4)$$

where $\Delta(t) = \omega_q(t) - \omega_r$, and the rotating transformation and RWA were applied.

Calibration: Rabi oscillation



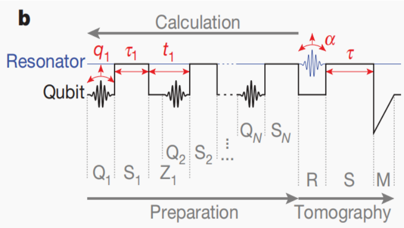
Target: arbitrary N-photon state

Target state and decomposition (Law and Eberly)

$$\left|\psi_{target}\right\rangle = |g\rangle \otimes \sum_{n=0}^{N} |c_n| e^{i\phi_n} |n\rangle$$
$$= S_N Q_N S_{N-1} Q_{N-1} ... S_2 Q_2 S_1 Q_1$$
$$= U(T) |g, 0\rangle, \quad (5)$$

where the target operation $U(T)$ is complete in total evolution time T and decomposed into 2N pieces. Solving the equation of inverse evolution:

$$U^\dagger(T) = Q_1^\dagger S_1^\dagger Q_2^\dagger S_2^\dagger ... S_{N-1}^\dagger Q_{N-1}^\dagger S_N^\dagger \quad (6)$$



- Highest N is limited by the bandwidth of generator
- Analysis starts from the highest number portion
- Additional phase gate Z_j may be inserted where we need

Target: arbitrary N-photon state and decomposition

Interacting Hamiltonian revisited ($\Omega_r = 0$):

$$H_I(t) = [\Omega_q(t) + ga]\sigma_+ + [\Omega_q^*(t) + g^*a^\dagger]\sigma_- \tag{7}$$

U Decomposition

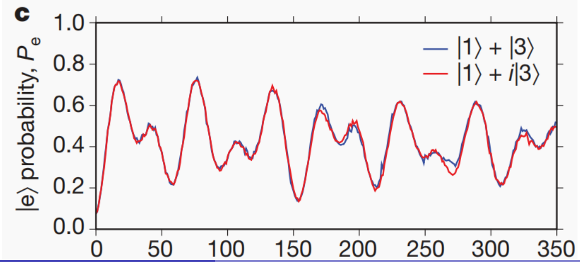
$$S_j^{(n)} = \begin{pmatrix} \cos|g|\tau_j\sqrt{n} & -ie^{i\phi}\frac{\sin|g|\tau_j\sqrt{n}}{\sqrt{n}} \\ -ie^{-i\phi}\frac{\sin g\tau_j\sqrt{n}}{\sqrt{n}} & \cos|g|\tau_j\sqrt{n} \end{pmatrix}, \quad Q_j = \begin{pmatrix} \cos|\Omega_j|\tau_j & -ie^{i\theta_j}\sin|\Omega_j|\tau_j \\ -ie^{-i\theta_j}\sin|\Omega_j|\tau_j & \cos|\Omega_j|\tau_j \end{pmatrix} \tag{8}$$

where j=1,2,...N, the $g = |g|e^{i\phi}$, and the Ω_j is supposed to be constant over each period $\Omega_j = |\Omega_j|e^{i\theta_j} = \Omega_q(\tau_j)$.

Target: $|0\rangle + i\,|3\rangle$ as an example

Table 1 | Sequence to generate the resonator state $|\psi\rangle = |1\rangle + i|3\rangle$

Sequence of states, operations	Operational parameter	System state, parameter value
$\lvert\psi\rangle$		$\lvert g\rangle(0.707\lvert 1\rangle+0.707i\lvert 3\rangle)$
S_3	$\tau_3\Omega$	1.81
Q_3	q_3	3.14
$\lvert\psi_2\rangle$		$\lvert g\rangle(-0.557i\lvert 0\rangle+0.707\lvert 2\rangle)+0.436\lvert e\rangle\lvert 1\rangle$
Z_2	$t_2\Delta$	4.71
S_2	$\tau_2\Omega$	1.44
Q_2	q_2	$-2.09 - 2.34i$
$\lvert\psi_1\rangle$		$(0.553-0.62i)\lvert g\rangle\lvert 1\rangle-(0.371+0.416i)\lvert e\rangle\lvert 0\rangle$
Z_1	$t_1\Delta$	3.26
S_1	$\tau_1\Omega$	1.96
Q_1	q_1	$-2.71-1.59i$
$\lvert\psi_0\rangle$		$(0.197-0.98i)\lvert g\rangle\lvert 0\rangle$



$$S_3^{(3)\dagger} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

$$S_3^{(1)\dagger} = \begin{pmatrix} \cos\left(\frac{\pi}{2\sqrt{3}}\right) & i\sin\left(\frac{\pi}{2\sqrt{3}}\right) \\ i\sin\left(\frac{\pi}{2\sqrt{3}}\right) & \cos\left(\frac{\pi}{2\sqrt{3}}\right) \end{pmatrix}$$

$$Q_3{}^\dagger = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

Wigner tomography of $|0\rangle + |N\rangle$

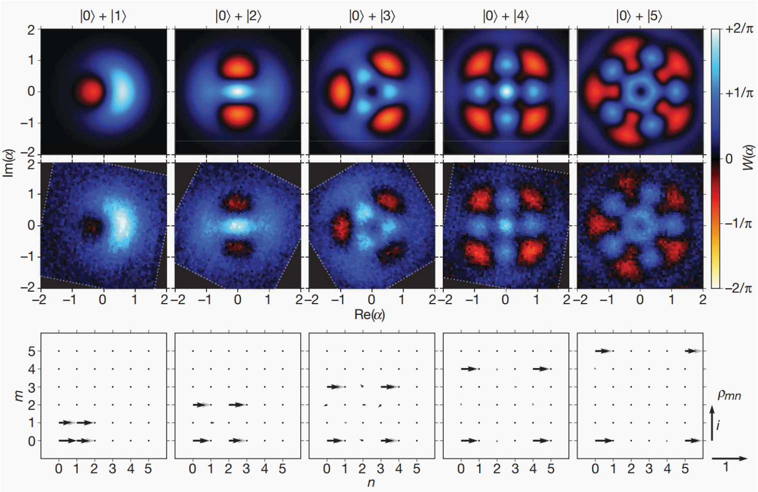


Figure: Fidelity: 0.92, 0.89, 0.88, 0.94, and 0.91.

Wigner tomography of $|0\rangle + e^{ik\pi}|3\rangle + |6\rangle$

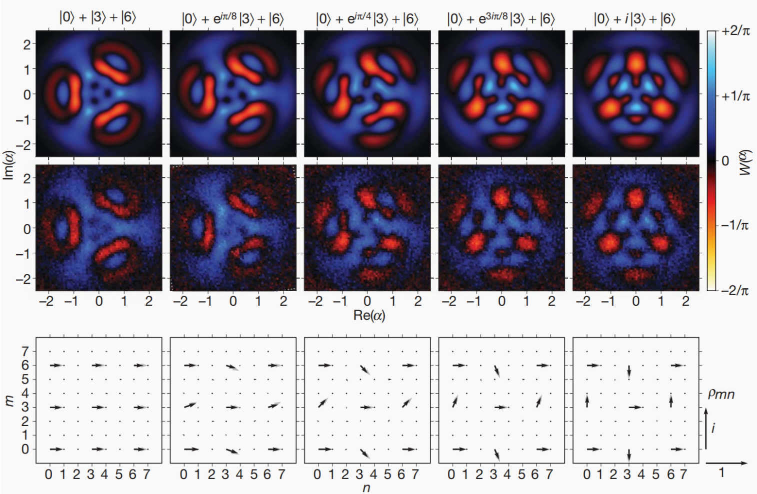


Figure: Fidelity: 0.89, 0.91, 0.91, 0.91, and 0.91

Wigner tomography: parity measurement

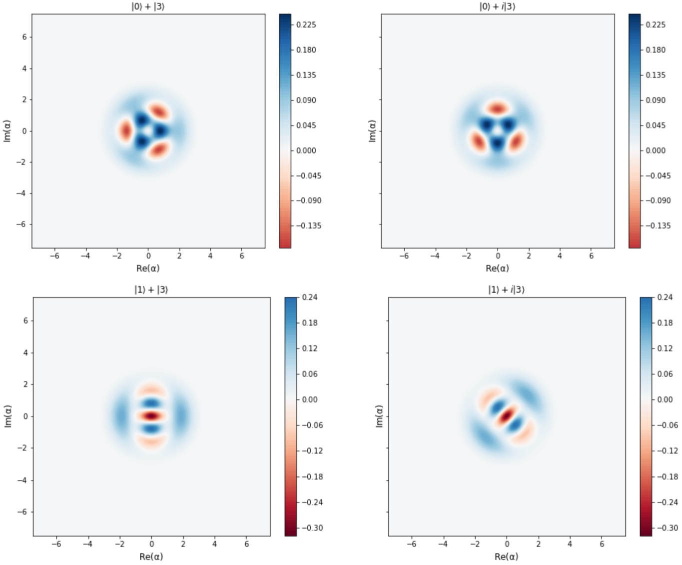
Def: Wigner function

$$
\begin{aligned}
W(\alpha) &= \frac{1}{\pi^2} \int d^2\beta e^{\alpha\beta^* - \alpha^*\beta} \chi_s(\beta) \\
&= \frac{2}{\pi} \mathrm{Tr}(\underbrace{D(-\alpha)\rho D(\alpha)}_{\rho'} \Pi) \\
&= \frac{2}{\pi} \sum_n (-1)^n \rho'_{nn}(-\alpha),
\end{aligned}
\tag{9}
$$

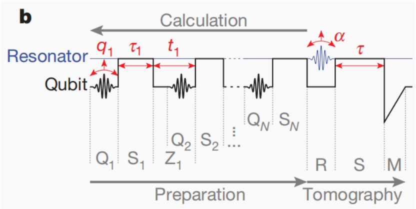
where the parity operator $\Pi = e^{i\pi a^\dagger a}$ has the eigenvalue $+1$ for even and -1 for odd resonator Fock state.

The Wigner function of ρ is obtained by preparing the shifted state $D(-\alpha)\rho D(\alpha)$ and performing parity measurement.

Wigner tomography: Fock state example



Calibration: resonator state measurement



- P_g immediately known
- Premeasured $g\sqrt{n}$
- P_n to be linear least square fitted

The state evolution over the middle period 'S' of tomography:

$$P_e(\tau) \simeq \frac{1}{2}\left(1 - P_g \sum_{n=0}^{\infty} P_n \cos\left(g\sqrt{n}\tau\right)\right), \tag{10}$$

where $P_n = |c_n|^2 = \rho'_{nn}(-\alpha)$. [3]

Main message

Summary

- Rich ideas and techniques behind the simple JC model, which can surely be extended
- For the first time, the preparation of N-fock state and arbitrary resonator state on circuit-QED platform (without projective measurement)
- Nonlinear elements like auxiliary qubits are powerful to engineer resonator states
- The relevant theoretical analysis and experimental techniques are insightful

Some outdated aspects:

- Upper limit of highest fock number
- Fidelity is relatively low
- Devices have been upgraded in recent years
- ...

References I

[1] Max Hofheinz et al. "Generation of Fock states in a superconducting quantum circuit". In: Nature 454.7202 (2008), pp. 310–314.

[2] Max Hofheinz et al. "Synthesizing arbitrary quantum states in a superconducting resonator". In: Nature 459.7246 (2009), pp. 546–549.

[3] P. Lougovski et al. "Fresnel Representation of the Wigner Function: An Operational Approach". In: Phys. Rev. Lett. 91 (1 June 2003), p. 010401. DOI: 10.1103/PhysRevLett.91.010401. URL: https://link.aps.org/doi/10.1103/PhysRevLett.91.010401.