PHYS-448 Introduction to Particle Accelerators
Solution of Tutorial 10

EPFL – *Teaching Assistants:*

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Exercise 1. The design peak luminosity of the LHC is $L = 10^{34}\,\mathrm{cm^{-2}s^{-1}}$.

1. Knowing that at $E_{\text{c.o.m.}} = 14\,\mathrm{TeV}$ and that the total event rate is $\frac{dN}{dt} = 1.115 \times 10^9\,\mathrm{s^{-1}}$, calculate the total cross section σ_{tot} in barn ($1\,\mathrm{b} = 10^{-28}\,\mathrm{m^2}$).
2. At the same $E_{\text{c.o.m.}}$, the Higgs cross section is about $\sigma_{\text{Higgs}} \approx 57\,\mathrm{pb}$. How many Higgs bosons are produced per second?

Solution:

- The event rate and cross section are related with luminosity through:

$$\frac{dN_{\text{tot}}}{dt} = \sigma_{\text{tot}} \times L \quad \Rightarrow \quad \sigma_{\text{tot}} = \frac{\frac{dN_{\text{tot}}}{dt}}{L} \approx \frac{1.115 \times 10^9}{10^{34}} = 1.115 \times 10^{-25}\,\mathrm{cm^2} = 111.5\,\mathrm{mb}.$$

- Using the same expression for Higgs boson production:

$$\frac{dN_{\text{Higgs}}}{dt} = \sigma_{\text{Higgs}} \times L \approx 57 \times 10^{-36} \times 10^{34} = 0.57\,\mathrm{s^{-1}}.$$

Exercise 2. The LHC collides 7 TeV proton beams. Calculate the center-of-mass energy relation for two beams:

$$E_{\text{C.O.M.}} = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2},$$

for the following cases:

1. Two colliding proton beams in the LHC.
2. An LHC proton beam hitting a fixed target.

What is the required proton beam energy hitting a fixed target to have the same $E_{\text{C.O.M.}}$ as the LHC?

Solution:

- Collider: $E_{\text{C.O.M.}} = E_1 + E_2 = 7 + 7 = 14\,\text{TeV}$.
- Fixed target: Using $\vec{p}_2 = 0$, we have:

$$E_{\text{C.O.M.}} = \sqrt{(E_1 + m_p)^2 - E_1^2 + m_p^2} = \sqrt{2E_1 m_p + 2m_p^2} = 114.6\,\text{GeV}.$$

- For equivalent $E_{\text{C.O.M.}}$ as LHC:

$$E_{\text{C.O.M.}} = \sqrt{2E_1 m_p + 2m_p^2} \Rightarrow E_1 = \frac{E_{\text{C.O.M.}}^2}{2m_p} - m_p = \frac{(14 \times 10^{12})^2}{2 \times 0.938 \times 10^9} - 0.938 \times 10^9 = 1.045 \times 10^{17}\,\text{eV}.$$

Exercise 3. There are many future proposals for large particle accelerators. An interesting 10 TeV muon collider proposal is presented in *"10 TeV Center of Mass Energy Muon Collider"* K. Skoufaris [1]. Although the LHC tunnel could be potentially used for this collider, it would be expensive to change all the magnets — not to mention the muon beam cooling system...

Instead, as an intellectual exercise, let's inject some single muons from this proposed collider beam in the present LHC lattice for protons, neglecting any lifetime issues. Performing a Twiss command with the MAD-X simulation code results in the optics functions shown in Figure 1.

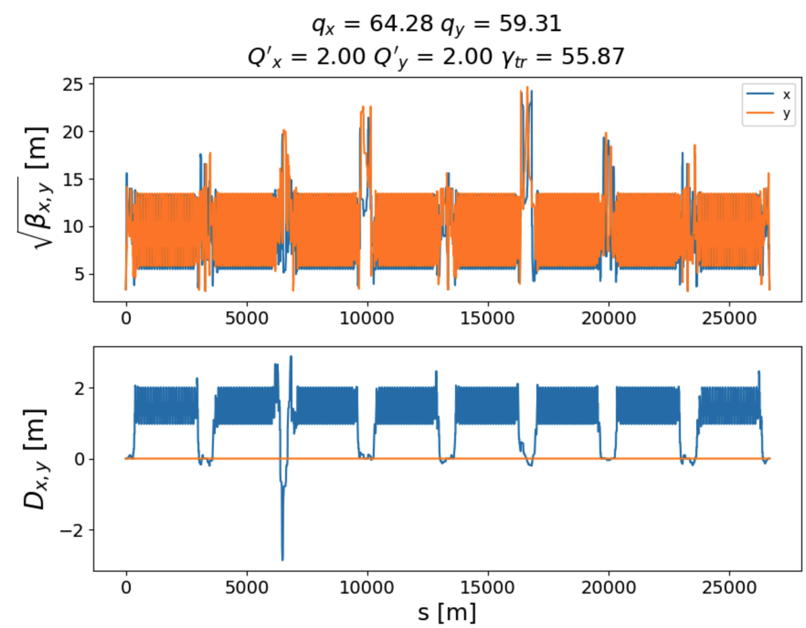

Figure 1: $\beta_{x,y}$ functions (top) and dispersion functions $D_{x,y}$ (bottom) for the muon collider in the LHC.

The $5\sigma_{x,y}$ beam envelope in the horizontal and vertical planes over the whole LHC can be seen in Figure 2. Recall the equation for the one-sigma ($n\sigma_x$, where $n = 1$) horizontal beam size:

$$\sigma_x(s) = \sqrt{\beta_x(s)\epsilon_x + \left(D_x(s)\frac{\Delta p}{p}\right)^2}. \tag{1}$$

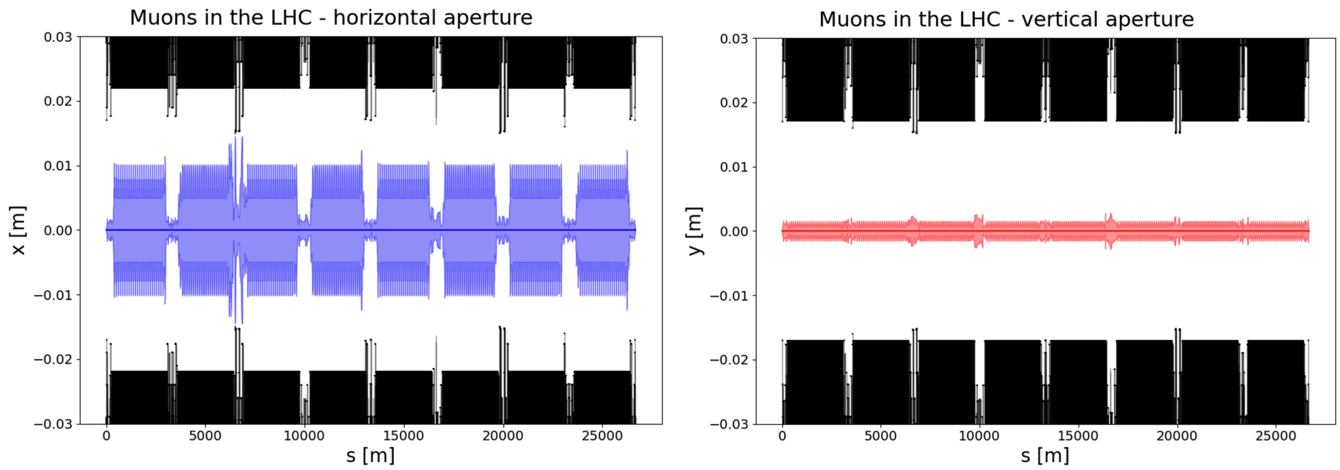

Figure 2: Horizontal and vertical beam envelope for the muon beam.

D_x is the horizontal dispersion, $\beta_{x,y}$ is the beta function, $\epsilon_{x,y}$ is the emittance, and $\frac{\Delta p}{p}$ is the momentum spread, which we assume to be 0.05%. Let us see what can be deduced from these plots and the provided information:

1. What is the muon beam energy for this collider?
2. What is (very) approximately the horizontal emittance ϵ_x of this beam? (Hint: check the max values of the plots at an easy location, such as $s = 1500$ m.)
3. How would Equation (1) look like in the vertical plane? Why is the blue envelope so much bigger than the red one?
4. What is approximately the vertical beam size at the same location where you checked horizontally?
5. What is the main advantage of using muons instead of electrons in a circular collider?

Solution:

- 10 TeV collider means 5 TeV beam energy.
- We use Equation (1) very approximately. In x, we find the emittance from:

$$\epsilon_x = \frac{1}{\beta_x}\left(\sigma_x^2 - \left(D_x \frac{\Delta p}{p}\right)^2\right).$$

  We check at an easy location $s = 1500$ m what the peak values are — it will not be exact but will give an idea. From Figure 1, we assume $\beta_x \approx 14^2 = 196$ m, $D_x = 2$ m, and $\frac{\Delta p}{p} = 0.0005$. From Figure 2 at $s = 1500$ m, one can see that $5\sigma_x = 0.01$ m, so $\sigma_x = 0.01/5 = 0.002$ m. Plugging this in, we see:

$$\epsilon_x = \frac{1}{196}\left(0.002^2 - (2 \times 0.0005)^2\right) = 1.5 \times 10^{-8}\,\text{m rad}.$$

- As the vertical dispersion $D_y = 0$ at this location, we would simply have:

$$\epsilon_y = \frac{\sigma_y^2}{\beta_y}.$$

The blue envelope is much larger than the red one because the horizontal dispersion term $D_x \frac{\Delta p}{p}$ contributes significantly in the horizontal plane, while $D_y = 0$ in the vertical plane.

- Assuming $\beta_y = 196$ m and $\sigma_y = 0.001/5$ m at $s = 1500$ m, we get:

$$\epsilon_y = \frac{(0.001/5)^2}{196} = 2.0 \times 10^{-10}\,\text{m rad}.$$

- Muons are about 200 times heavier than electrons and have the same charge, thus emitting much less synchrotron radiation than electrons per turn.

References

1. K. Skoufaris, C. Carli, and D. Schulte, *"10 TeV center of mass energy muon collider,"* JACoW IPAC, vol. 2022, pp. 515–518, 2022.