Introduction to Particle Accelerator Physics

Final Exam

Numerical results without derivation are not sufficient. Do symbolic derivations first and insert numbers only at the end (unless stated otherwise). You may use books, notes and a calculator (without communication features).

I. Transverse Beam Dynamics (11 points)

Part I: FODO cell stability

Assume a synchrotron with a circumference of $C = 3\,\text{km}$ composed *only* of symmetric FODO cells. The horizontal tune is $Q_x = 7.28$ and the total number of FODO cells is $N_{\text{FODO}} = 48$.

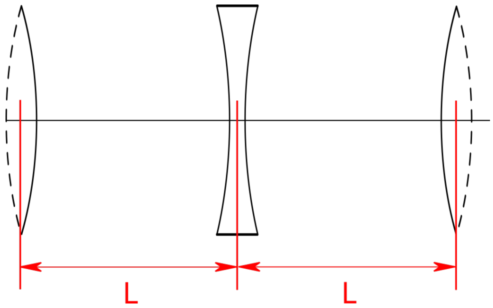

Figure 1: FODO cell, part I.

(a) Calculate the horizontal phase advance per FODO cell μ_{FODO}. (1 point)

(b) Knowing μ_{FODO}, calculate the focal length f of the quadrupoles using the thin-lens approximation. Recall that the FODO cell transport matrix between the centers of two consecutive focusing quadrupoles reads (see Fig. 1) (3 points)

$$M_{\text{FODO, I}} = \begin{pmatrix} 1 - \frac{L^2}{2f^2} & 2L + \frac{L^2}{f} \\ -\frac{L}{2f^2}\left(1 - \frac{L}{2f}\right) & 1 - \frac{L^2}{2f^2} \end{pmatrix}.$$

(c) Is the transverse particle motion stable for the given lattice? Explain why. (1 point)

Part II: FODO cell equivalence

We consider again a FODO cell with focal lengths $\mp f$ and equal drift lengths L between the focusing and defocusing quadrupoles as illustrated in Fig. 2, left. However, other than in part I of the exercise we now define the transport matrix M_{FODO} starting just before the focusing quadrupole instead of at its center

$$M_{\mathrm{FODO,\,II}} = \begin{pmatrix} 1 - \frac{L}{f} - \frac{L^2}{f^2} & 2L + \frac{L^2}{f} \\ -\frac{L}{f^2} & 1 + \frac{L}{f} \end{pmatrix}.$$

We are going to study the transport matrix M_{OFO} of the drift-lens-drift arrangement shown in Fig. 2, right. It is defined by a single focusing quadrupole of focal length $-\tilde{f}$ between two drift sections of possibly different lengths L_1 and L_2.

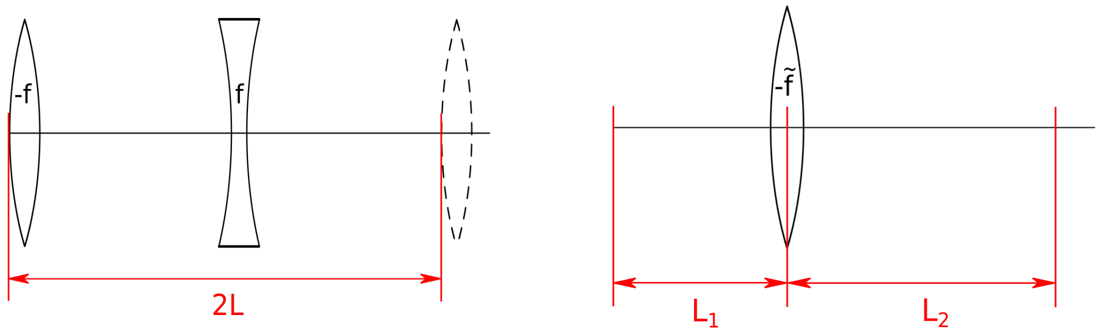

Figure 2: *Left:* FODO cell, part II. *Right:* Drift-lens-drift (OFO) structure.

(a) Compute the transport matrix M_{OFO} of the drift-lens-drift system analytically. (2 points)

(b) Show that M_{OFO} is indeed equivalent to M_{FODO} and express $\tilde{f}$, L_1, and L_2 in terms of f and L. (4 points)

II. Longitudinal Beam Dynamics (9 points)

Assume a proton storage ring with a circumference of $C = 650\,\mathrm{m}$. The stored protons have a kinetic energy of $E_{\mathrm{kin}} = 4.5\,\mathrm{GeV}$. In this storage ring the bending magnets account for 30 % of the circumference. Furthermore, the average value of the dispersion in the bending magnets, which are all assumed to have identical field strengths, is $\langle D_x \rangle = 0.8\,\mathrm{m}$.

(a) Calculate the total energy E_{tot} of the particles, the relativistic β and γ, the revolution period T_0 of the beam, its momentum p and the magnetic field B in the dipoles. (3 points)

(b) Calculate the momentum compaction factor α_c of the storage ring as well as the absolute path length change ΔC for a particle with a relative momentum deviation of 1.5 %. (2 points)

(c) Explain the concept of transition crossing *(max. 5 sentences)*. (2 points)

(d) Is the beam above or below transition energy? Based on your answer: what is the possible range for the synchronous phase ϕ_s to guarantee phase stability in the longitudinal plane and to ensure particle acceleration? (2 points)

III. Luminosity (11 points)

The Cornell Electron Storage Ring (CESR) is an electron-positron e^-e^+ collider at Cornell University in Ithaca, New York (USA). It has a physical circumference of 768 m. Assume that the machine operates at a beam momentum of 4.5 GeV/c and that the two counter-rotating beams each contain 9 colliding bunches of e^+ and e^-, respectively. The current per bunch is initially 7 mA and their normalized emittances are 100 µm rad and 500 µm rad in the horizontal and the vertical plane, respectively. The β-functions at the interaction point (IP) have values of $\beta^*_{x,y} = 0.3$ m.

(a) Calculate the center-of-mass energy of the e^-e^+ collisions for this machine. (1 point)

(b) What is the desired value of dispersion at the IP? Explain why. (1 point)

(c) Assume that the machine is set up to have the desired dispersion at the IP. Calculate the root mean square (rms) horizontal and vertical beam sizes at the collision point. (2 points)

(d) Calculate the luminosity $\mathcal{L}_0$ for the given parameters. (2 points)
If you did not manage to solve c), use $\sigma_x = 40\,\mu m$ and $\sigma_y = 200\,\mu m$ to continue.

(e) To avoid multiple collision points near the IP we now introduce a crossing angle in the horizontal plane $\phi_x = 600$ µrad. Given the rms bunch length of $\sigma_s = 6.5$ cm, calculate by what percentage this reduces the luminosity obtained in d). (2 points)

(f) Assume that the bunch intensities all decay exponentially with a time constant of $\tau = 3$ h (so-called beam lifetime). Given that all the other beam parameters remain constant – what is the luminosity $\mathcal{L}(t)$ after $t = 1.5$ h, including the reduction caused by the crossing angle? (3 points)

IV. Synchrotron Radiation (11 points)

The Diamond Light Source located near Oxford (UK) stores an electron beam with a total energy of 3 GeV at a current of 300 mA. The Diamond dipole bending radius is 7.1 m.

(a) Compute the energy loss per turn due to synchrotron radiation. (1 point)

(b) The total power consumption of Diamond is $P_{\text{tot}} = 2\,\text{MW}$. What fraction of P_{tot} is required to compensate for the synchrotron radiation losses? *(Hint: compute the total average power lost by the stored beam.)* (3 points)

(c) Let us consider operating the Diamond light source with protons (assume the same bending radius). What would be the total energy of a proton beam with a current of 300 mA that radiates the same amount of synchrotron power? (2 points)

(d) Imagine that the radio-frequency (RF) system suddenly stops restoring the energy lost due to synchrotron radiation. Explain what happens to the radius of the beam orbit? (1 point)

(e) Knowing that the maximum horizontal dispersion is 25 cm and the horizontal aperture at this location is ± 2 cm, compute the number of turns the beam survives in the ring without the RF system before it crashes into the wall. For simplicity, assume a point-like beam and that the energy lost per turn is constant over time. (4 points)

Useful constants:

- $e = 1.602 \times 10^{-19}\,\text{C}$
- $m_e = 0.511\,\text{MeV}/c^2$
- $m_p = 0.938\,\text{GeV}/c^2$
- $c = 299\,792\,458\,\text{m}\,\text{s}^{-1}$
- $\alpha = 1/137$
- $\hbar c = 197\,\text{MeV fm} \quad (1\,\text{fm} = 1 \times 10^{-15}\,\text{m})$
- $1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$