

# Introduction to Particle Accelerators

# LPAP

## Laboratory for Particle Accelerator Physics

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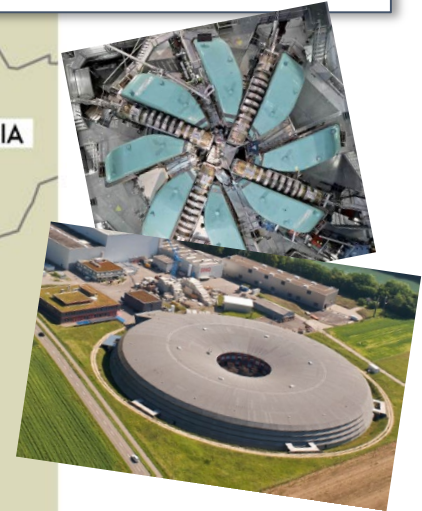
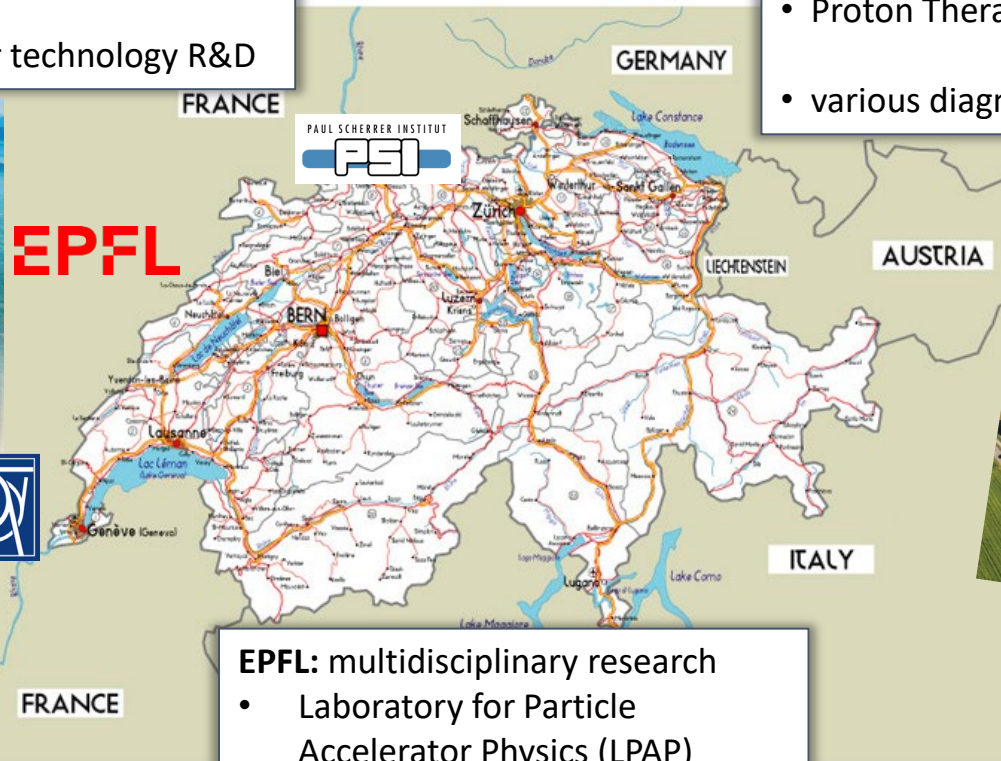
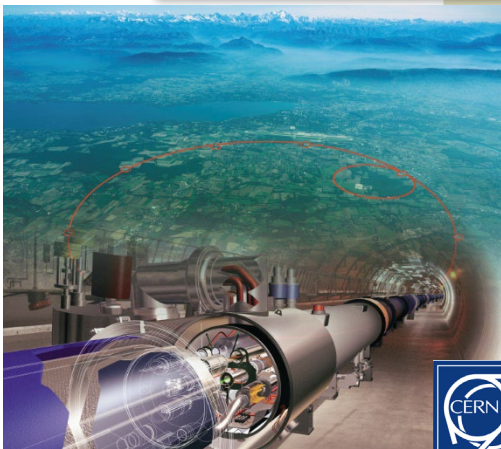
# Accelerators connect CERN, PSI and EPFL

## CERN

- Large Hadron Collider + upgrade
- FCC studies (hh, ee, eh)
- CLIC linear collider study
- various accelerator technology R&D

## PSI

- Synchrotron Light Source
- Free Electron Laser
- High Intensity Proton Accelerator
- Proton Therapy
- various diagnostic methods at RIs



## EPFL: multidisciplinary research

- Laboratory for Particle Accelerator Physics (LPAP)
- Swiss Plasma Center
- advanced manufacturing
- computer science

# Accelerator Concepts, their History and their Applications

Laboratory for Particle Accelerator Physics, EPFL

# World Accelerators: instruments for science and industry

>30'000  
accelerators

- 2 G\$ market
- 500 G\$ of goods produced with accelerators

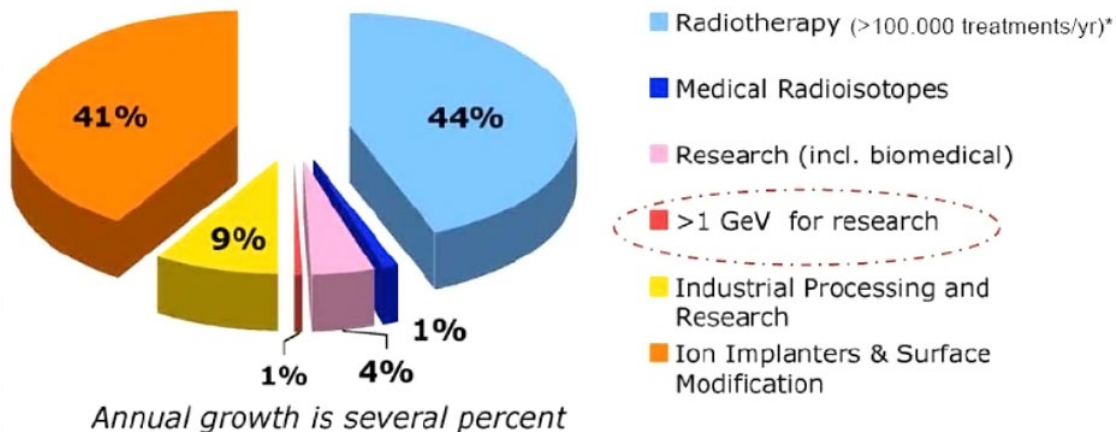


# Accelerators: Applications

>30.000 accelerators worldwide

Figure: distribution by the number

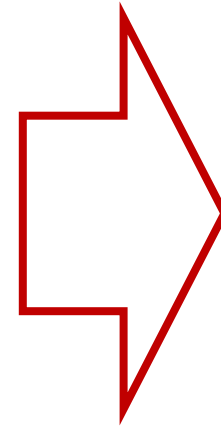
large research infrastructures (LHC, E-XFEL, FAIR, ESS, HERA) can cost Billions ...



[Ph.LeBrun, JUAS]

# Accelerators for Research: Motivation to build and develop them ...

- particle colliders
- synchrotron light sources (brightness)
- free electron lasers (time resolution)
- neutron sources
- other particle sources (neutrinos, muons ..)
- cancer therapy accelerators ( $\gamma$ , p, C)



particle physics

material science

biology

medicine

chemistry,  
pharmacy, energy  
research ..

→ in most applications the accelerator is needed to generate secondary radiation: exotic particles, X-rays, neutrons ...

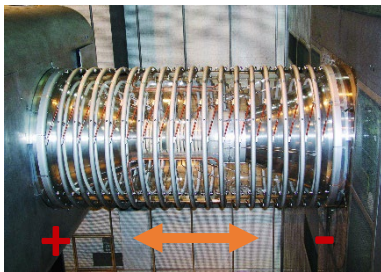
# Acceleration of Particles?

	Mass	Acceleration	Velocity	
Motorcycle	300 kg	1 g	0.085 km/s	
Bullet	0.004 kg	$10^5$ g	1 km/s	
Proton	$1,7 \times 10^{-27}$ kg	$10^{13}$ g	12.000 km/s (4.1 % $\times c$ )	} 810 kV
Electron	$9,1 \times 10^{-31}$ kg	$2 \times 10^{16}$ g	275.000 km/s (92 % $\times c$ )	



factor 2000

example: acceleration of particles in Cockcroft Walton at PSI



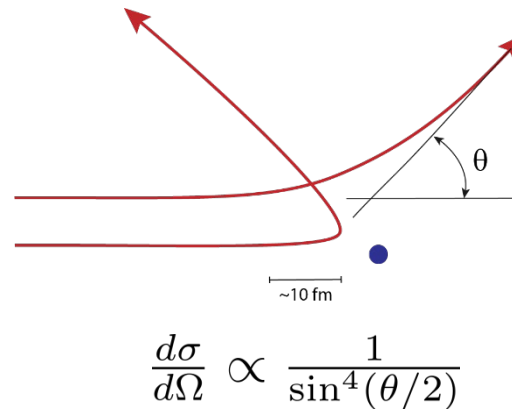
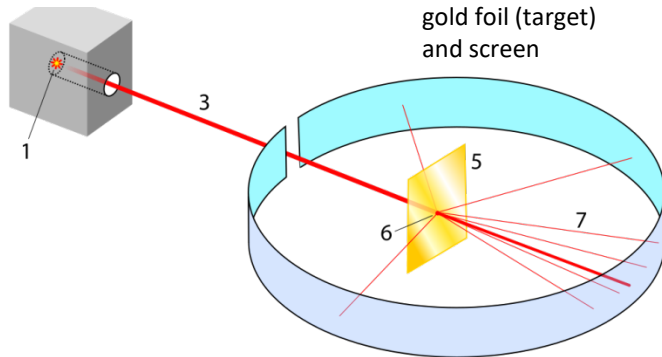
0.8 m, 810 kV

- particles are quickly «relativistic»
- electrons and protons behave quite differently

# Rutherford scattering

## - a model for accelerator applications

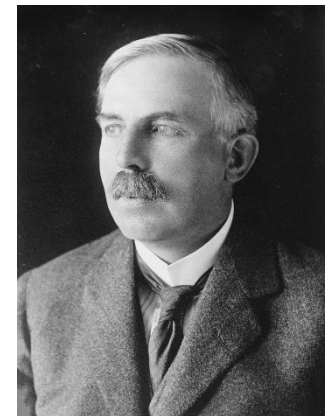
radioactive alpha source



... results showed that around 1 in 8000 alpha particles were deflected by very large angles (over 90°) ... [Wikipedia]

**Lord Rutherford (1927 @ Royal Society):**

“I have long hoped for a source of positive particles more energetic than those emitted from natural radioactive substances”.



[Ernest Rutherford, Nobel Prize 1908]

→ today a main application for accelerators is to generate tailored radiation for scattering experiments

# examples for probing matter at different energies

	Energy	equivalent wavelength
alpha from rad.decay	4.9 MeV	6.4 fm
thermal neutrons	0.025 eV	1.8 Å
photons, SR	12.4 keV	1 Å
high energy electrons	100 GeV	$10^{-17}$ m
LHC protons	7 TeV	$2 \times 10^{-19}$ m



(1Å =  $10^{-10}$ m, 1fm =  $10^{-15}$ m)

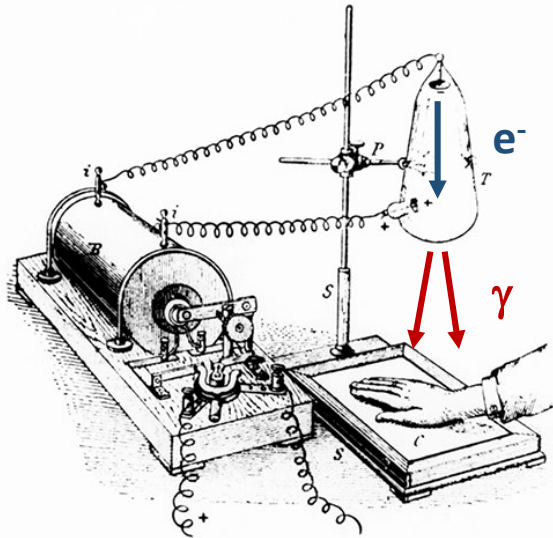
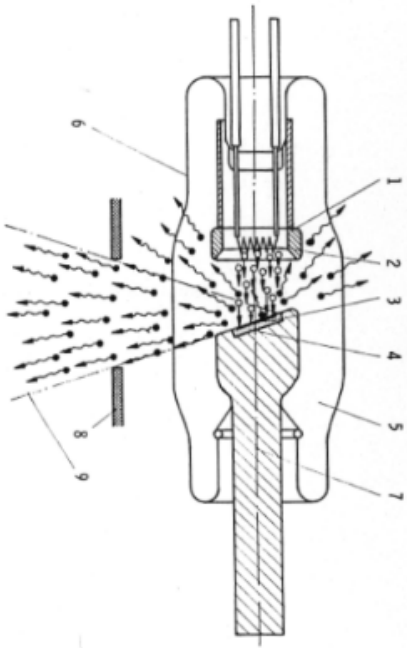
the **de Broglie wavelength**  
sets the length scale for objects  
under study using scattering

$$\lambda = \frac{h}{p} = \begin{cases} \frac{hc}{E_k} & \text{for } v \approx c, \\ \frac{hc}{\sqrt{2m_0 c^2 E_k}} & \text{for } v \ll c. \end{cases}$$

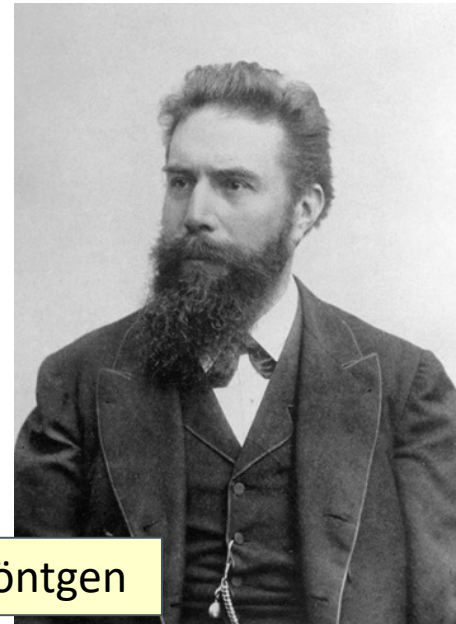
$$hc = 1.24 \text{ GeVfm}$$

$$hc = 12.4 \text{ keVÅ}$$

# Roentgen Image (1896)



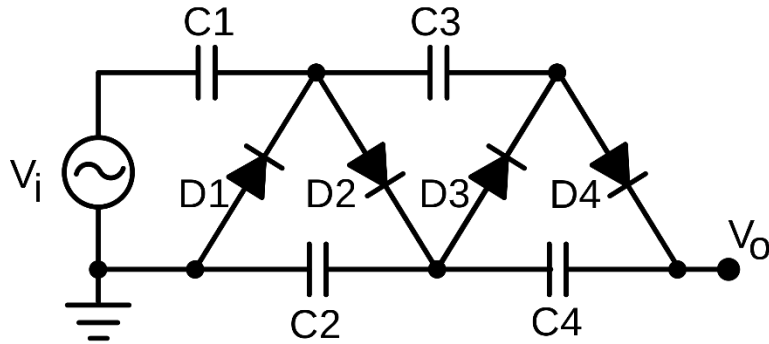
X-rays: secondary radiation generated by an „accelerator“



W.C.Röntgen



# Cockcroft Walton (DC) Accelerator



cascade circuit to  
generate high voltages



CW accelerator @ PSI  
 $U = 870 \text{ kV}$

In 1932 Cockcroft and Walton used this circuit design to power their particle accelerator, performing the first artificial nuclear disintegration.

## The Nobel Prize in Physics 1951



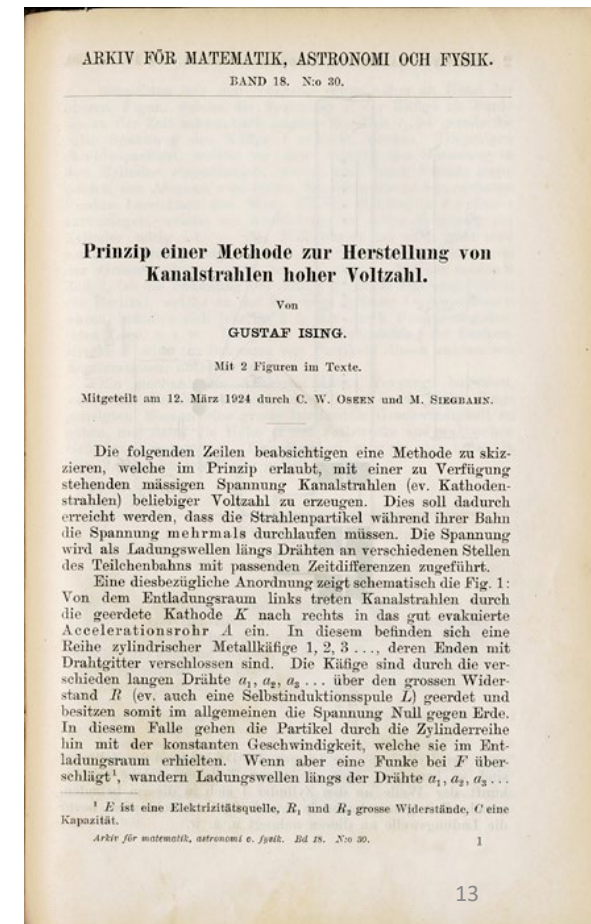
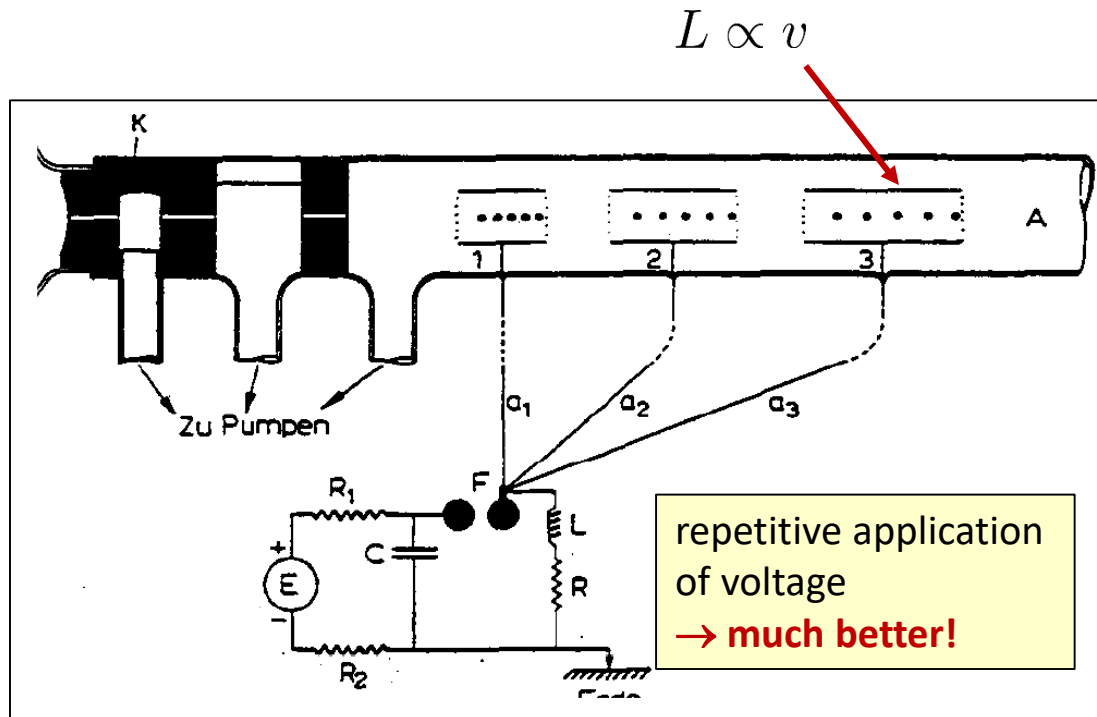
Photo from the Nobel Foundation archive.  
Sir John Douglas  
Cockcroft



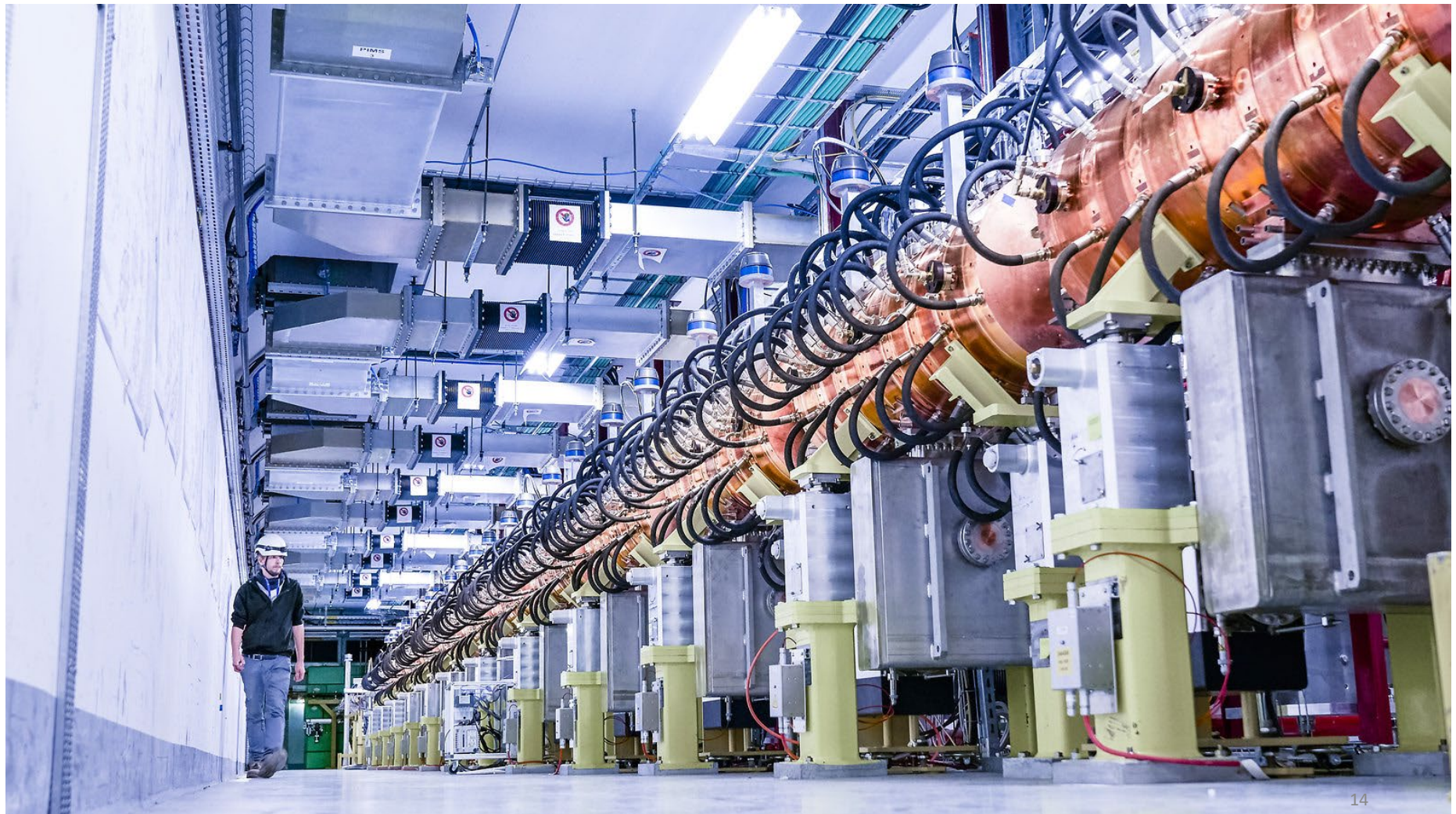
Photo from the Nobel Foundation archive.  
Ernest Thomas Sinton  
Walton

# First proposal for a linear accelerator

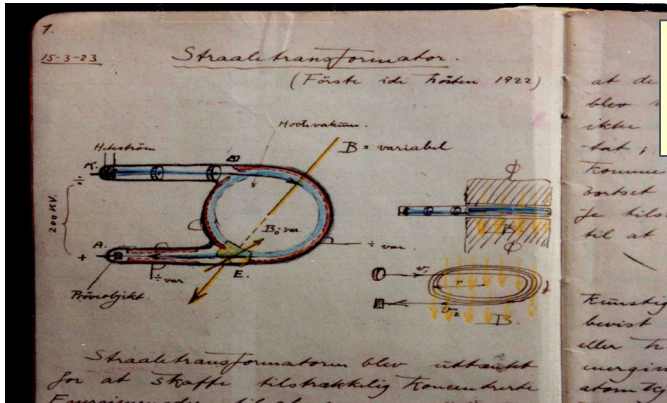
1924: Gustav Ising, Prof. at the technical university Stockholm



# CERN LINAC 4 160 MeV (90 meter linac)



# Rolf Wideröe: Betatron (1927)



Wideröes  
Notebook



Dissertation Aachen,  
27 pages!

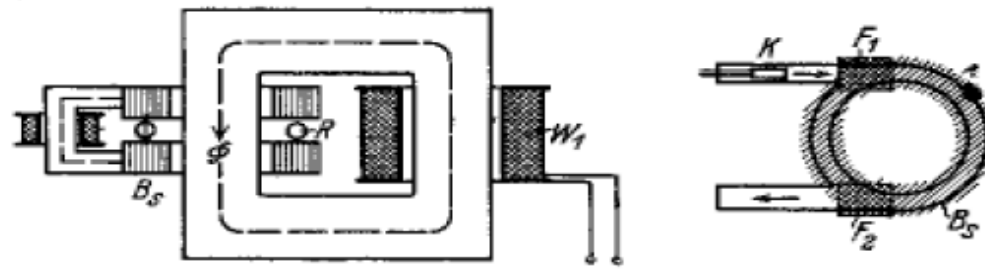


Bild 11. Wirkungsweise des Strahltransformators.

Wideröe calls the idea Beams-Transformer, meaning the beam replaces the secondary winding in a transformer

$$\vec{B} = -\nabla \times \vec{E}$$

## Über ein neues Prinzip zur Herstellung hoher Spannungen

Von der Fakultät für Maschinenwirtschaft der Technischen Hochschule zu Aachen

zur Erlangung der Würde eines Doktor-Ingenieurs

genehmigte

Dissertation

verlegt von

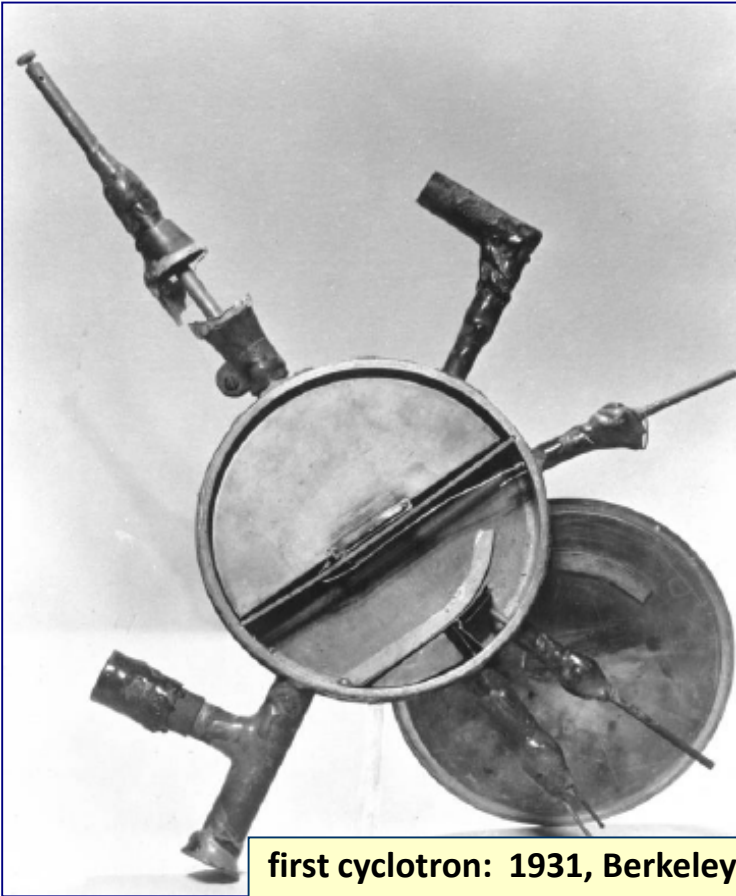
Rolf Wideröe, Oslo

Referent: Professor Dr.-Ing. W. Rogowski

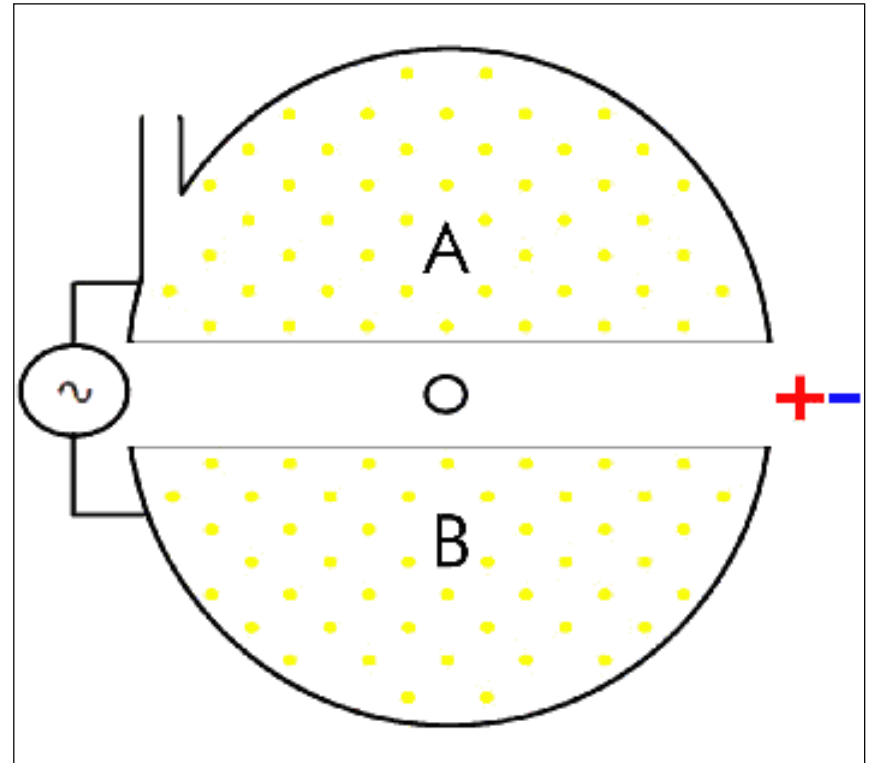
Korreferent: Professor Dr. L. Finzi

Tag der mündlichen Prüfung: 28. November 1927

# Resonant Acceleration: Cyclotron



first cyclotron: 1931, Berkeley  
1kV gap voltage, 80keV Protons

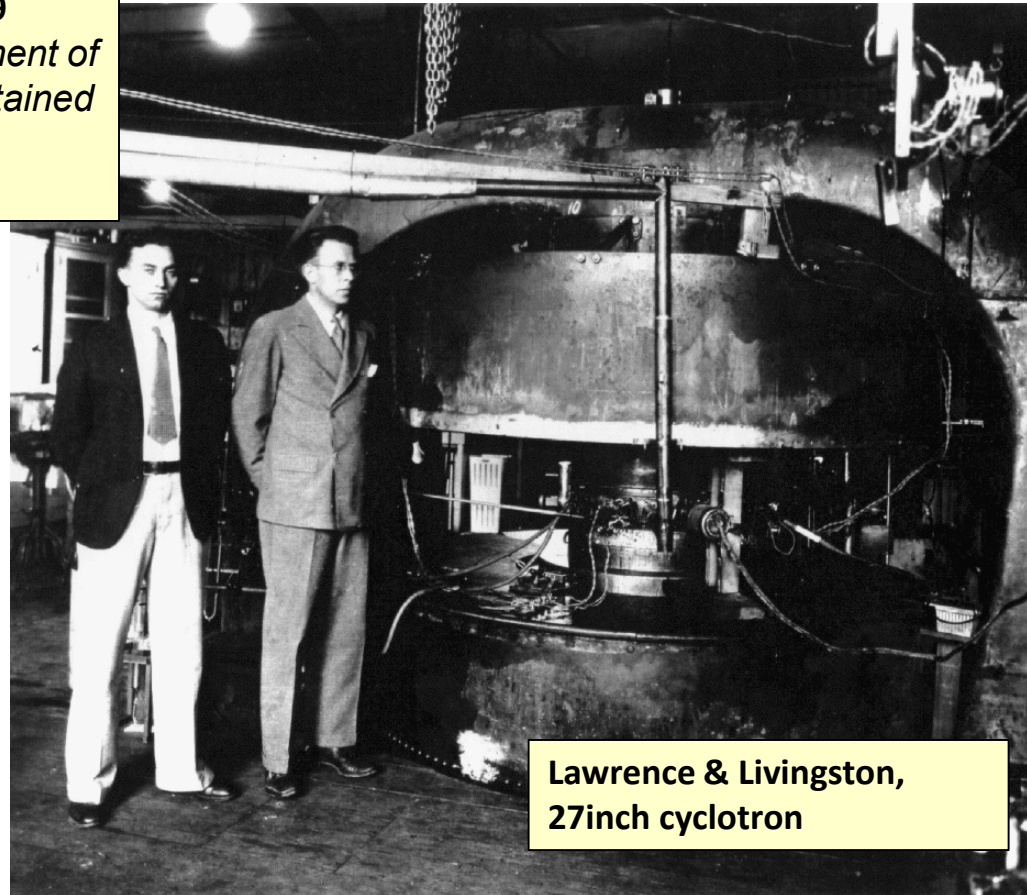


Principle: alternating RF voltage  
multiple acceleration steps

# History of the Cyclotron

**Ernest Lawrence, Nobelpreis 1939**

*"for the invention and development of the cyclotron and for results obtained with it, especially with regard to artificial radioactive elements"*



**Lawrence & Livingston,  
27inch cyclotron**



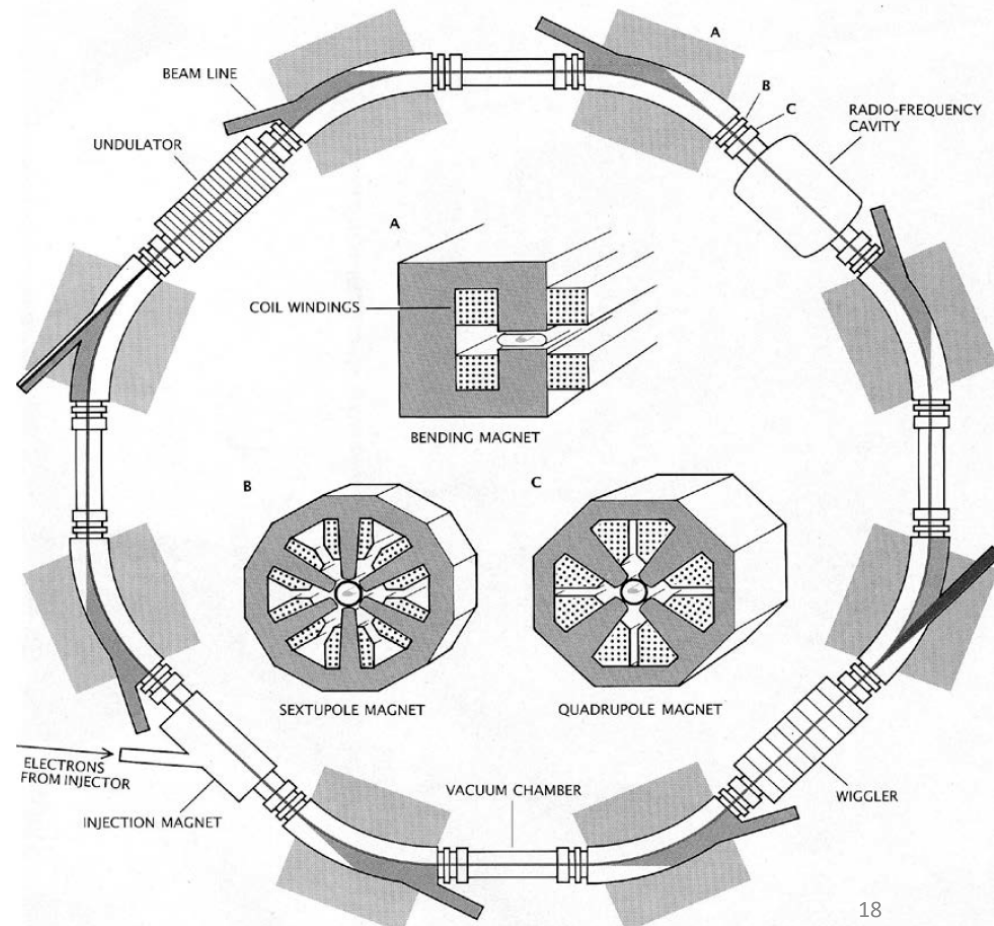
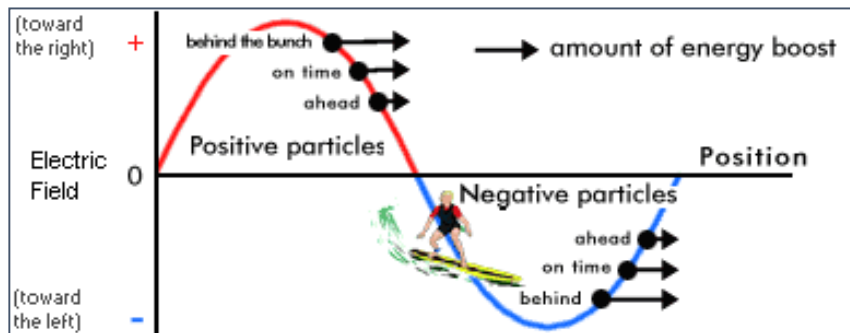
**John Lawrence (center), 1940'ies**

*first medical application: treatment of cancer with neutrons using the 60inch Zyklotron*

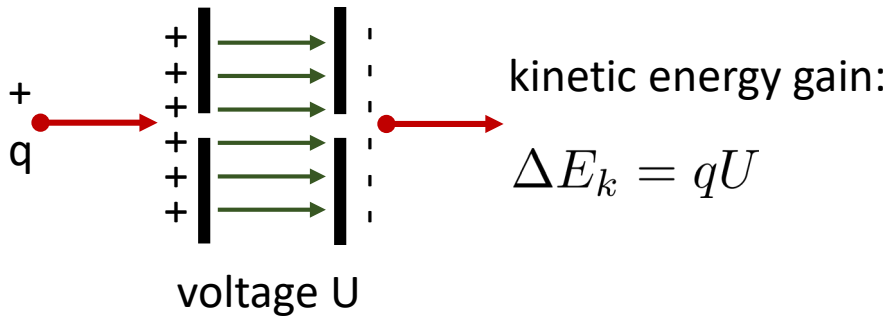
# Synchrotron

- McMillan (USA) and Veksler (UdSSR) independently in 1945
- concepts of phase stability (longitudinal focusing) and alternating gradient focusing

phase stability:  
“surfing on the RF-wave”



# Energy and Current

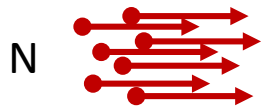


**units:**

$$1\text{eV} = 1.602 \times 10^{-19} \text{ Joule}$$

$$1\text{MeV} = 10^6 \text{eV}$$

$$1\text{GeV} = 10^9 \text{eV}$$



current =  
 charge  $\times$  #particles / time

$$I = qN/\tau$$

**units:**

$$1\text{A} = 1 \text{ C/s} = 6.2 \times 10^{18} \text{ particles/s}$$

$$10\text{nA} = 10^{-8}\text{A} = 6.2 \times 10^{10} \text{ particles/s}$$

(e.g. cancer therapy)

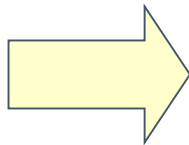
# basic relativistic relations

A.Einstein  
1879-1955

relativistic energy-  
momentum relation:

$$E = \sqrt{m_0^2 c^4 + c^2 p^2}$$

$$E = m_0 c^2 + E_k$$



$$v = c \sqrt{1 - m_0^2 c^4 / E^2}$$
$$= \beta c$$

$$p = \beta c \cdot \gamma m_0$$

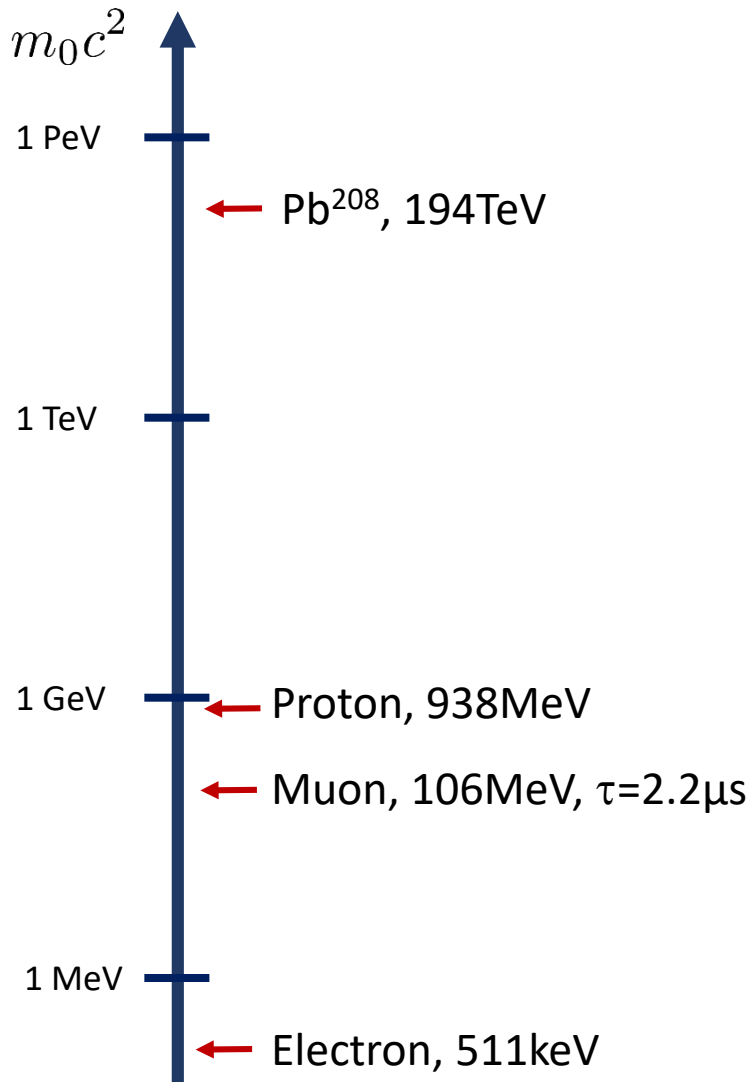
in the limit

$$E \rightarrow \infty: \quad v \rightarrow c, \quad m_{\text{eff}} \rightarrow \infty$$



$$\gamma = \frac{E}{m_0 c^2} = 1 + \frac{E_k}{m_0 c^2}$$
$$\beta = \sqrt{1 - 1/\gamma^2}$$

# Particles to Accelerate



Wide range of rest masses from electron to heavy ions

The accelerators differ vastly, e.g.

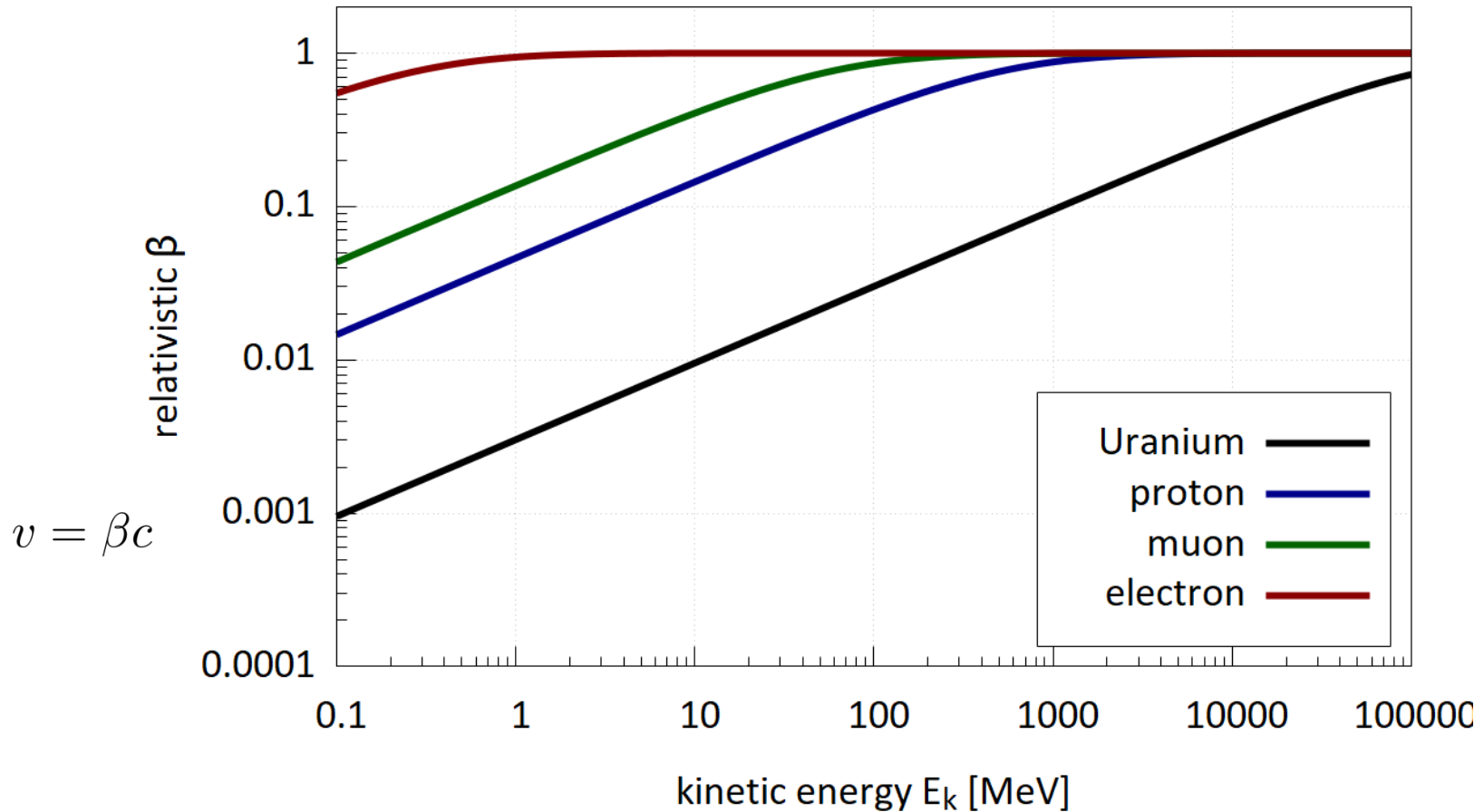
- particle speed in cavities
- synchrotron radiation power
- activation by losses
- requirements for vacuum

The muon as a heavy lepton is interesting for particle physics

$$1\text{eV} = 1.602 \times 10^{-19} \text{ Joule}$$

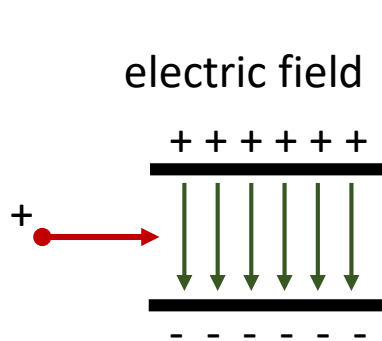
$$1\text{eV} \cong 1.783 \times 10^{-36} \text{ kg}$$

# Speed of different particles vs energy

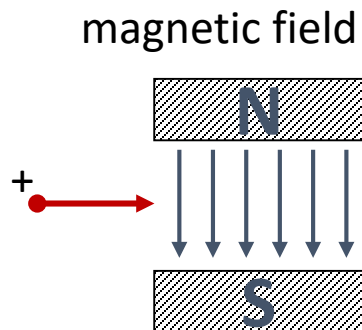


# Charged Particles in EM Fields?

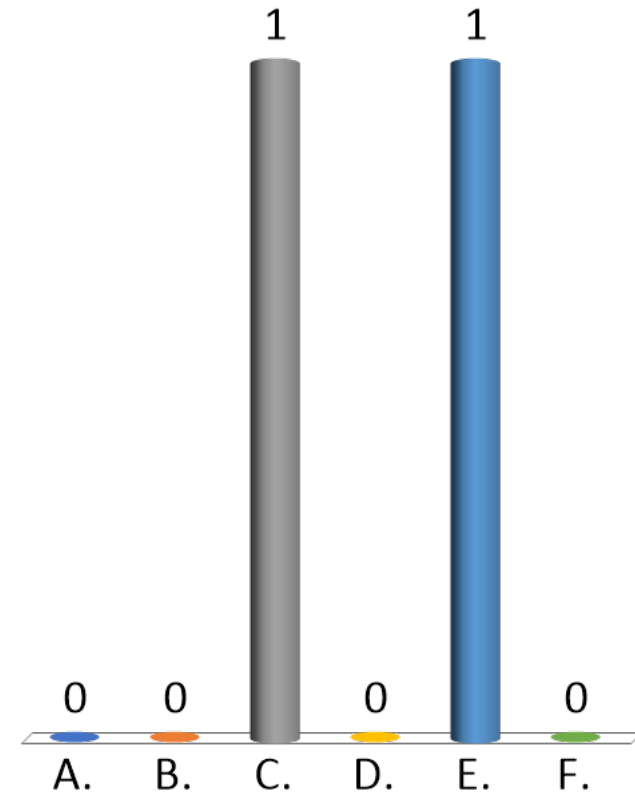
call: [responseware.eu](https://responseware.eu), session ID: **accel21**



- A. particle moves straight
- B. path is bent, energy unchanged
- C. path is bent, energy increased



- D. particle moves straight
- E. path is bent, energy unchanged
- F. path is bent, energy increased



# Lorentz Force

$$\vec{F} = e\vec{E} + e\vec{v} \times \vec{B} \quad (\text{charge} = e)$$



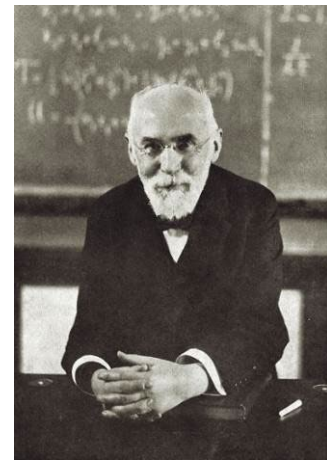
**electric field**

energy gain:  $\Delta E_k = eU$

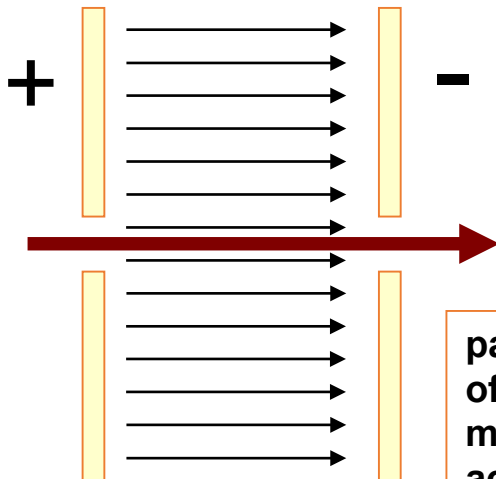


**magnetic field**

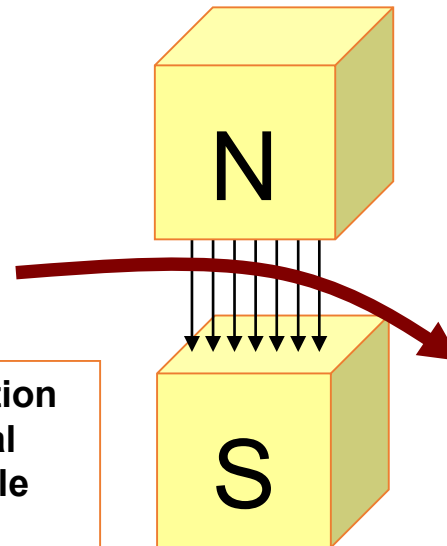
bending:  $B\rho = p/e$ ,  $\Delta E_k = 0$



H.A.Lorentz  
1853-1928



path bending and acceleration  
of particles are fundamental  
mechanisms of each particle  
accelerator



in practical units:

$$B\rho = 3.3356 p$$

[Tm]      [GeV/c]

[see appendix]

# Next: Medical and Industrial Applications

- Overview on industrial applications
- Particle Therapy
- Isotopes

# Accelerators: Essential Tools in Industry

## Ion Implantation

- Accelerators can precisely deposit ions modifying materials and electrical properties (boron, phosphorus)

## Semi Conductors

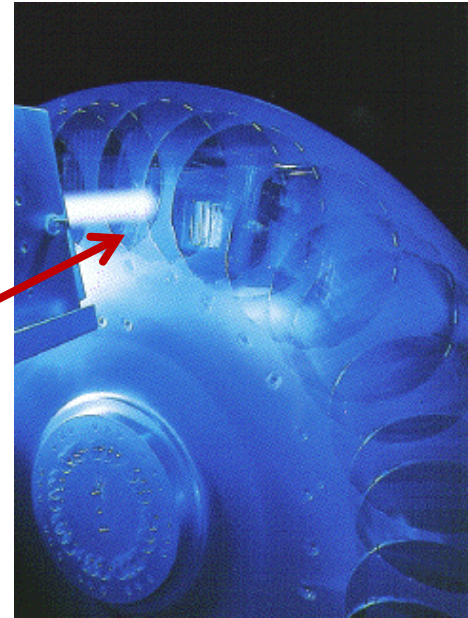
- CMOS transistor fabrication of essentially all IC's
- CCD & CMOS imagers for digital cameras
- Cleaving silicon for photovoltaic solar cells
- Typical IC may have 25 implant steps

## Metals

- Harden cutting tools
- Reducing friction
- Biomaterials for implants

## Ceramics and Glasses

- Harden surfaces
- Modify optics
- Color in Gem stones!



N<sub>2</sub> ions reduce wear and corrosion in this artificial femur



# X-Ray radiotherapy: using electron beams to produce X-Rays

Varian brothers started at Stanford



1947, 2 MeV/m  
One meter long

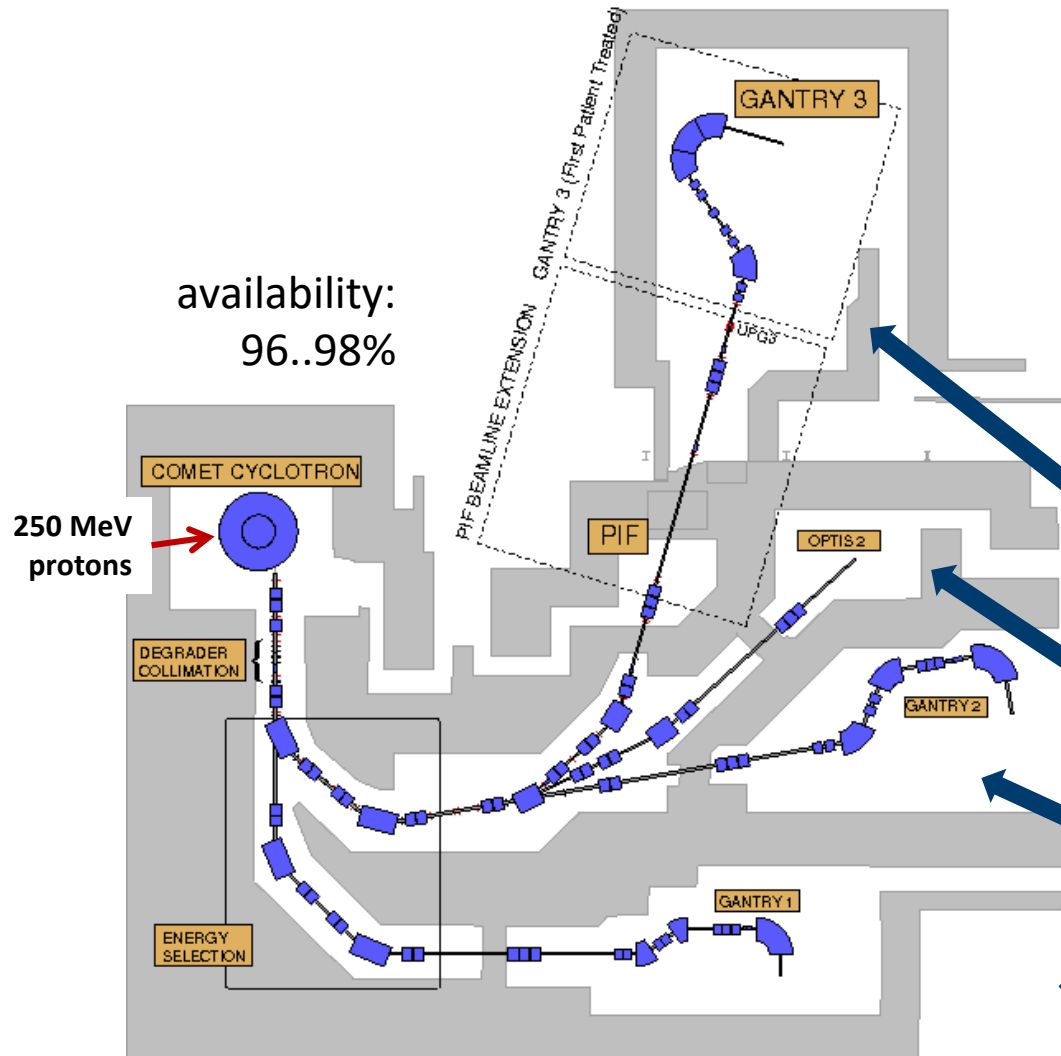
50,000,000

patients treated with  
photons



[VARIAN]

# PSI Proton Therapy Facility PROSCAN



July 2018, first patient on Gantry 3 [PSI/CPT]

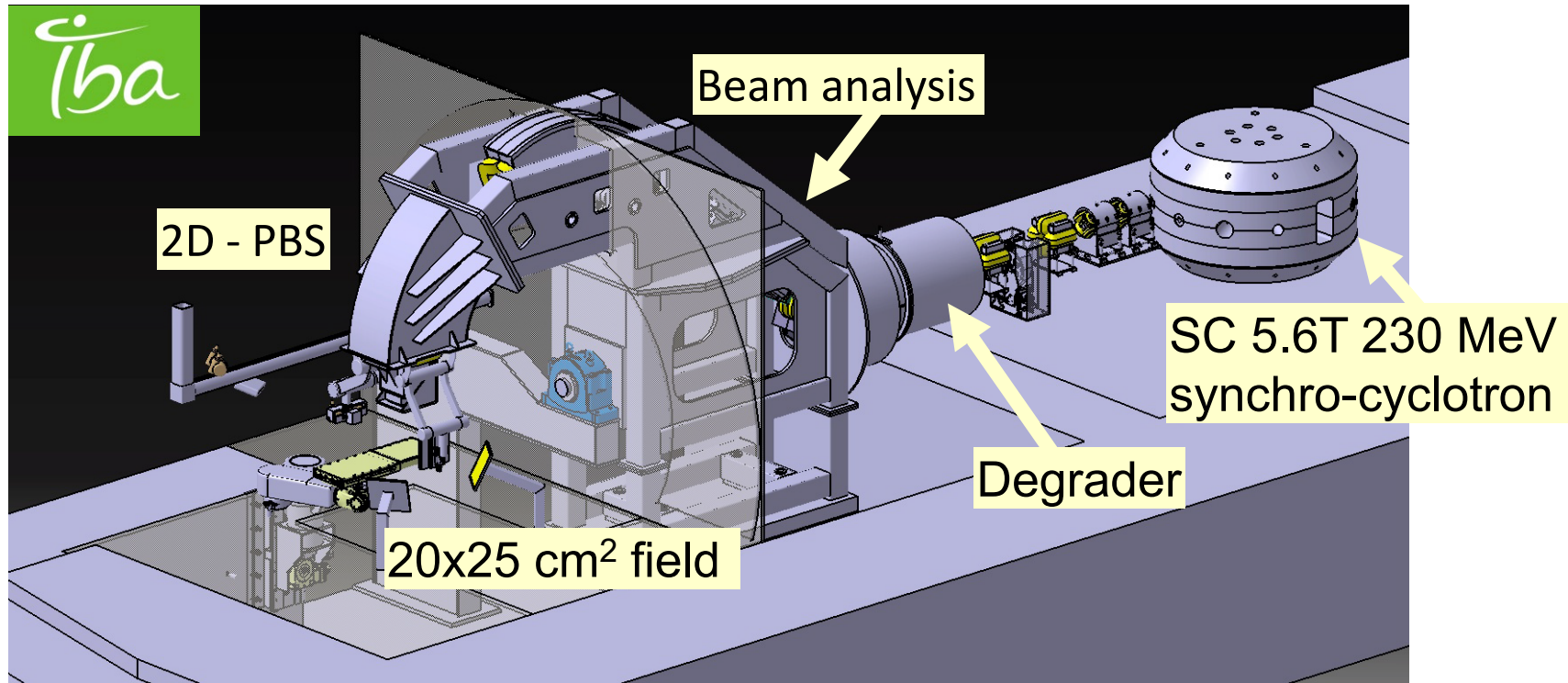
**Gantry 3**  
(up to 12 Patienten pro Tag)

**OPTIS**  
(eye melanoma treatment)

**Gantry 2**

**Gantry 1**

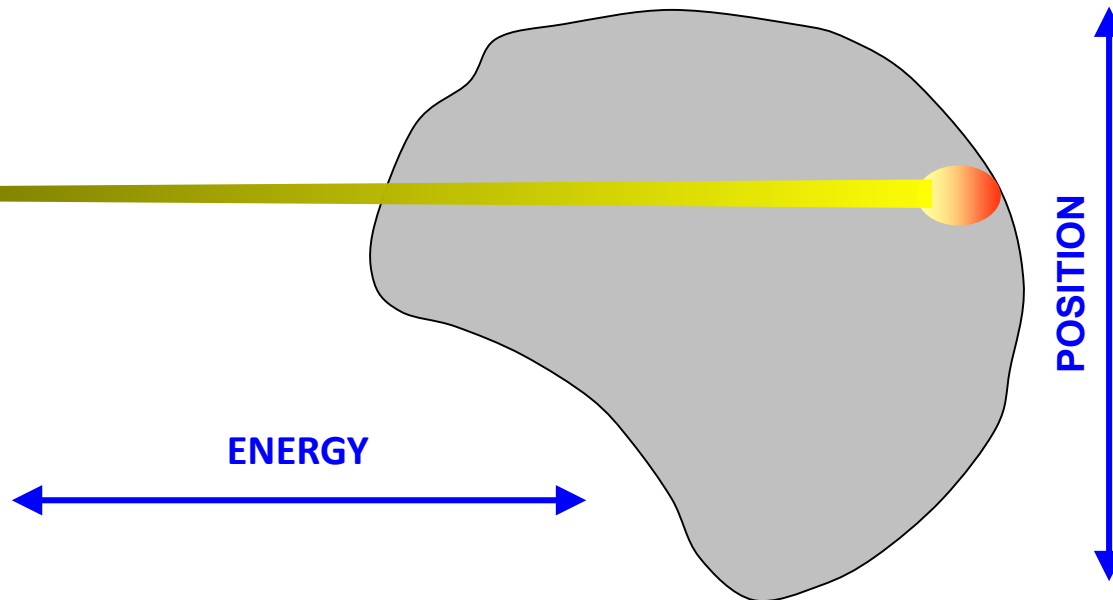
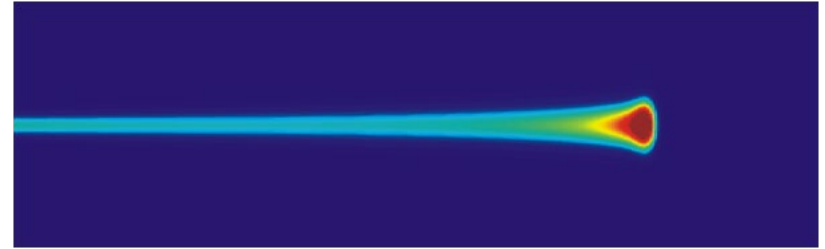
# IBA: compact treatment facility using high field synchro-cyclotron



- required area: 24x13.5m<sup>2</sup> (indeed small)
- 2-dim pencil beam scanning

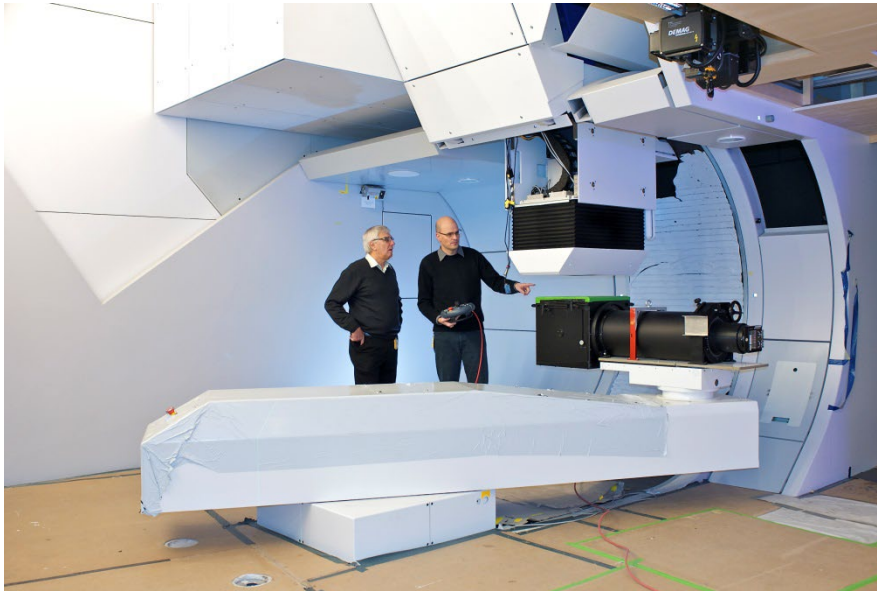
# Particle Therapy with Protons / Ions

Treatment of deep seated tumors with best protection of the surrounding



# Beam Delivery

## example: PSI Gantry II



Gantry 2 during commissioning (2010)

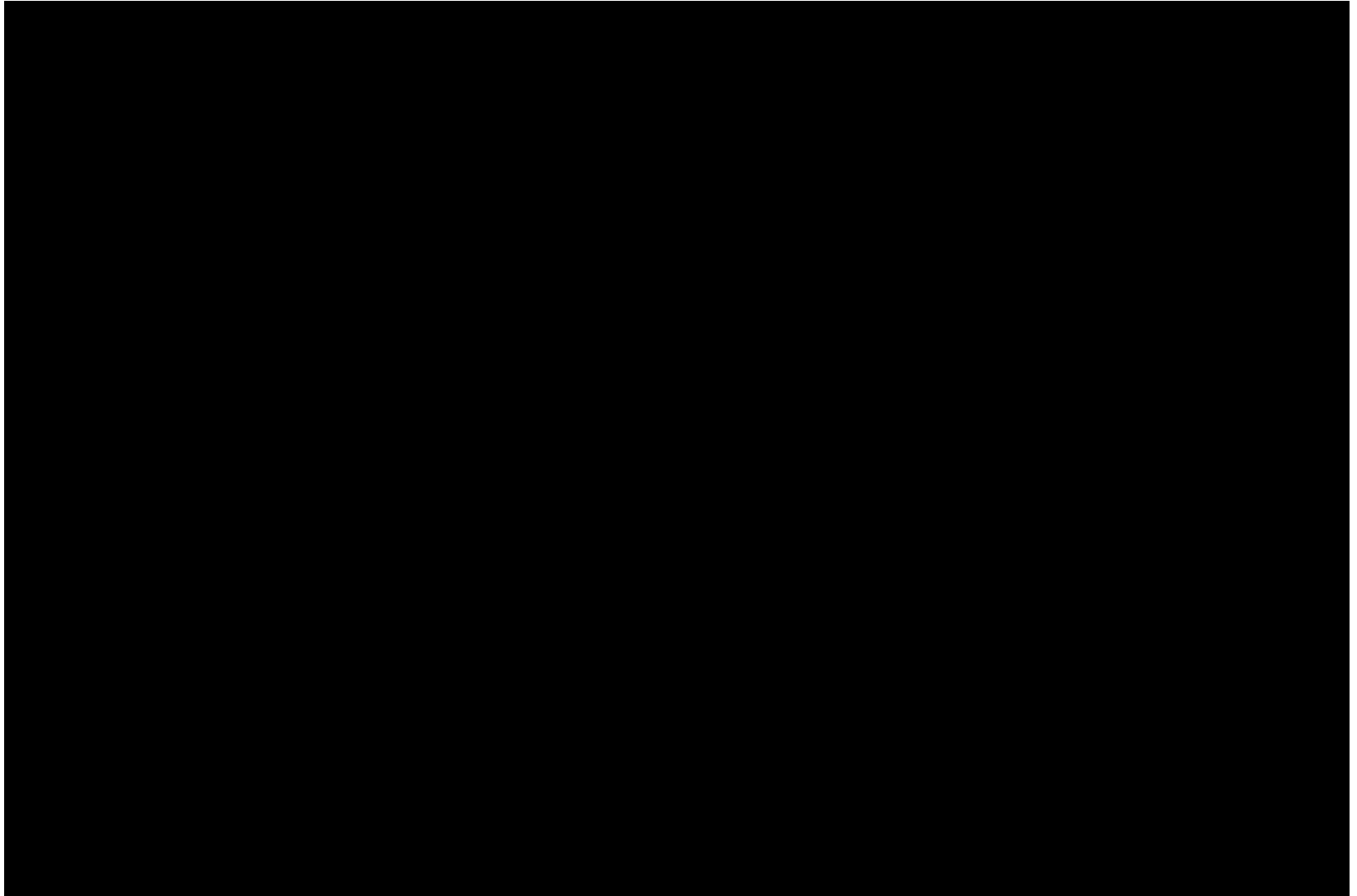


- radius 4 m, weight 180 t, magnet 40t
- precision 2D scanning,  $\delta \approx 1\text{mm}$
- fast energy change, 100 ms/step
- modern in-situ imaging (CT, X-ray)

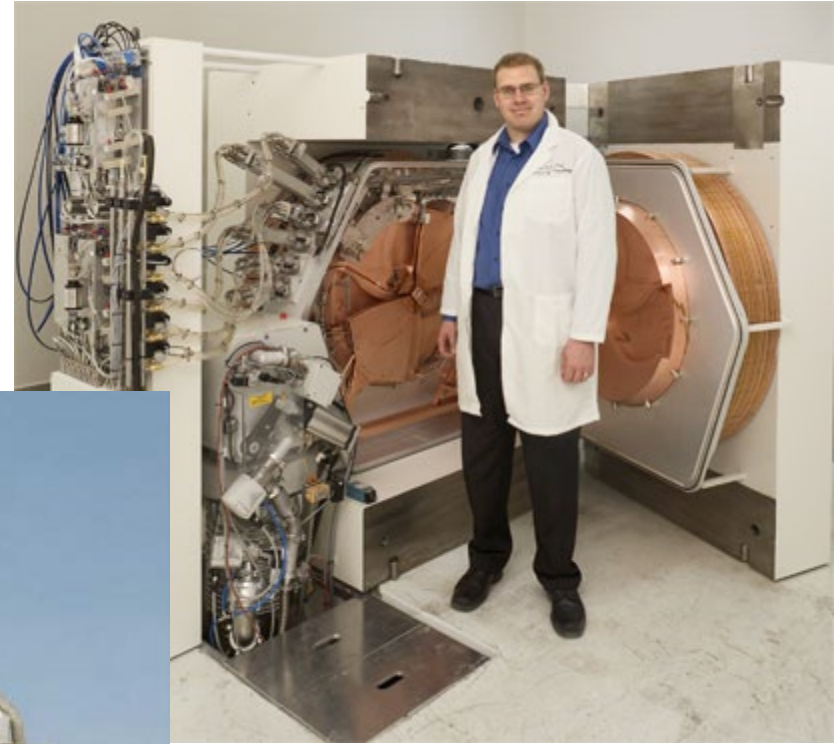
### accelerator physics challenges for gantries:

- minimize chromatic and geometric beam aberrations
- new concepts with light s.c. magnets

# Fast Energy Change with Gantry II



# compact cyclotrons for Isotope production



**CYCLONE 30 (IBA) : H<sup>-</sup> 15 à 30 MeV**

Vertical setup

# Medical Isotopes

radioactive isotopes are used for variety of diagnostic and therapeutic purposes:

- imaging of cancers and neurological problems
- cancer therapy by enriching radioactive material around tumor cells

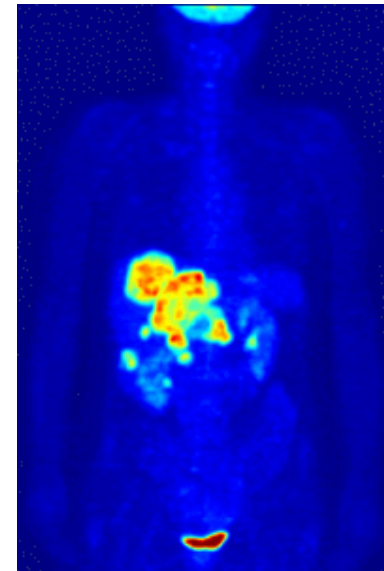
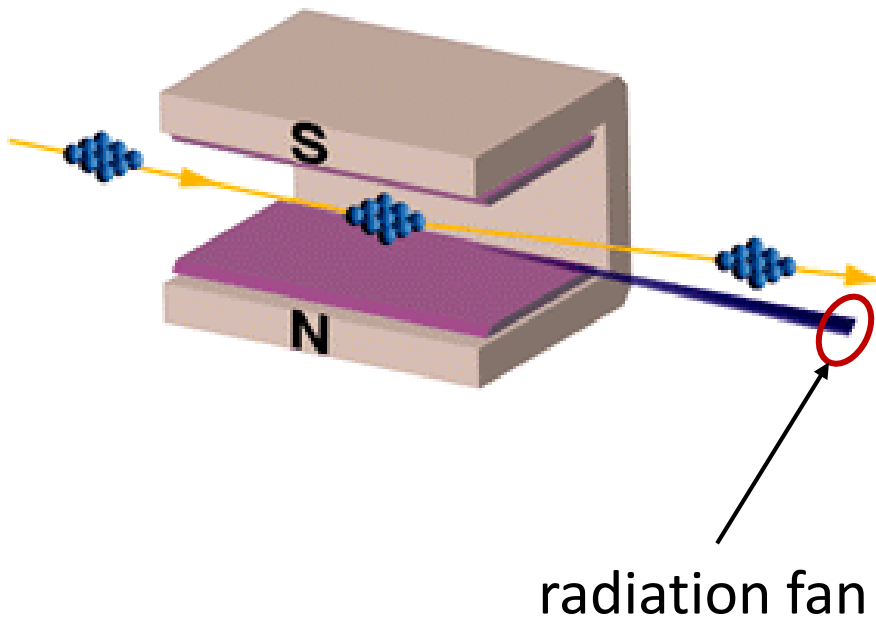


image of a PET scan using  $^{18}\text{F}$

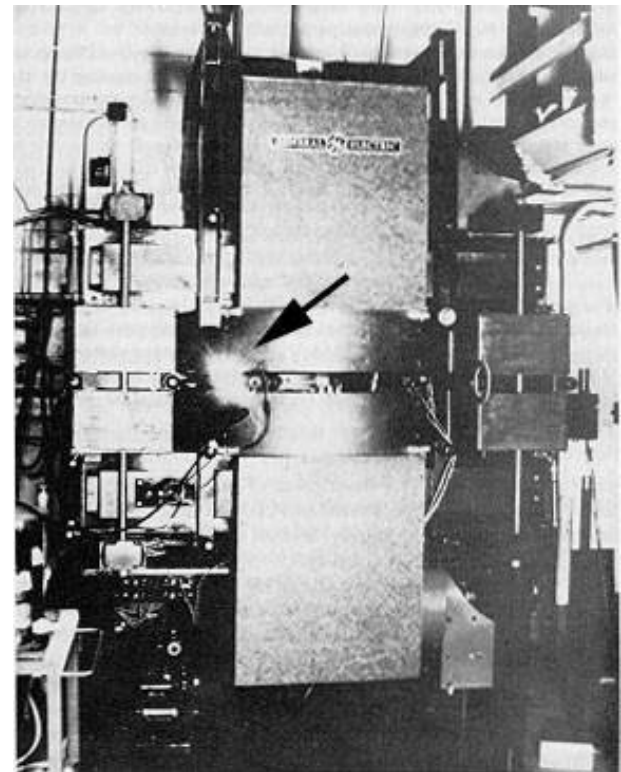
# Next: Accelerators for Condensed Matter Research

- Synchrotron Light Sources
- Free Electron Lasers
- Neutron Sources

# Synchrotron radiation from electrons in a bending magnet

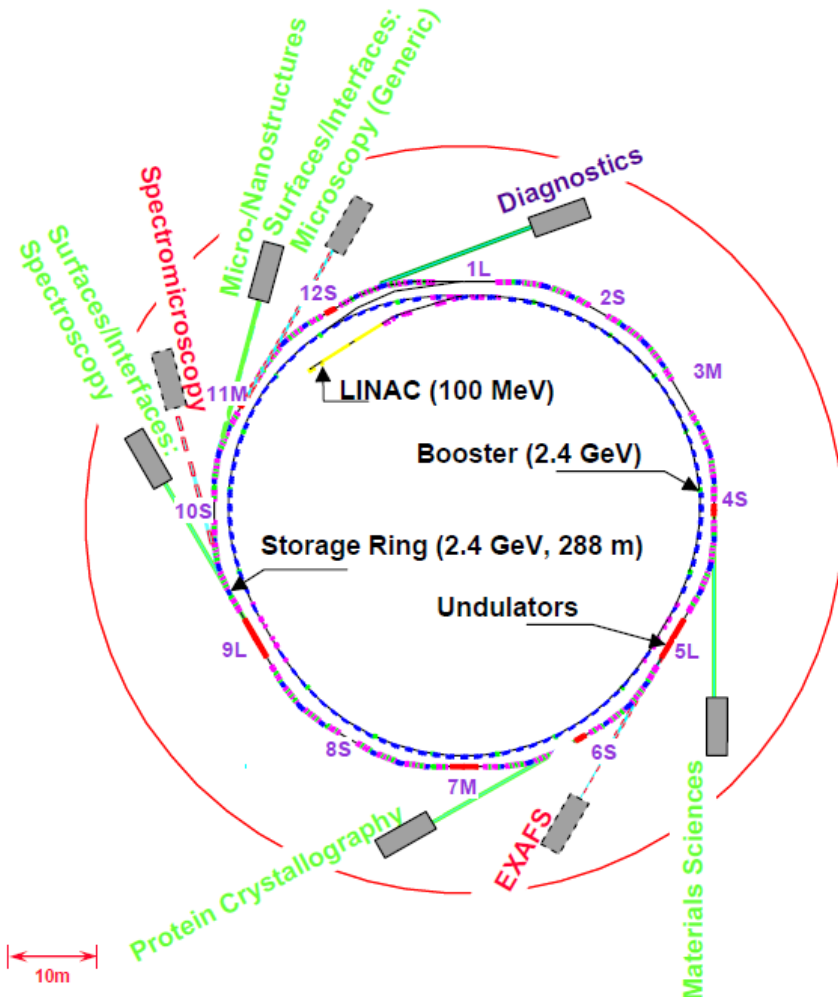


## GE Synchrotron New York State



First light observed 1947

# Swiss Light Source (SLS) – a synchrotron



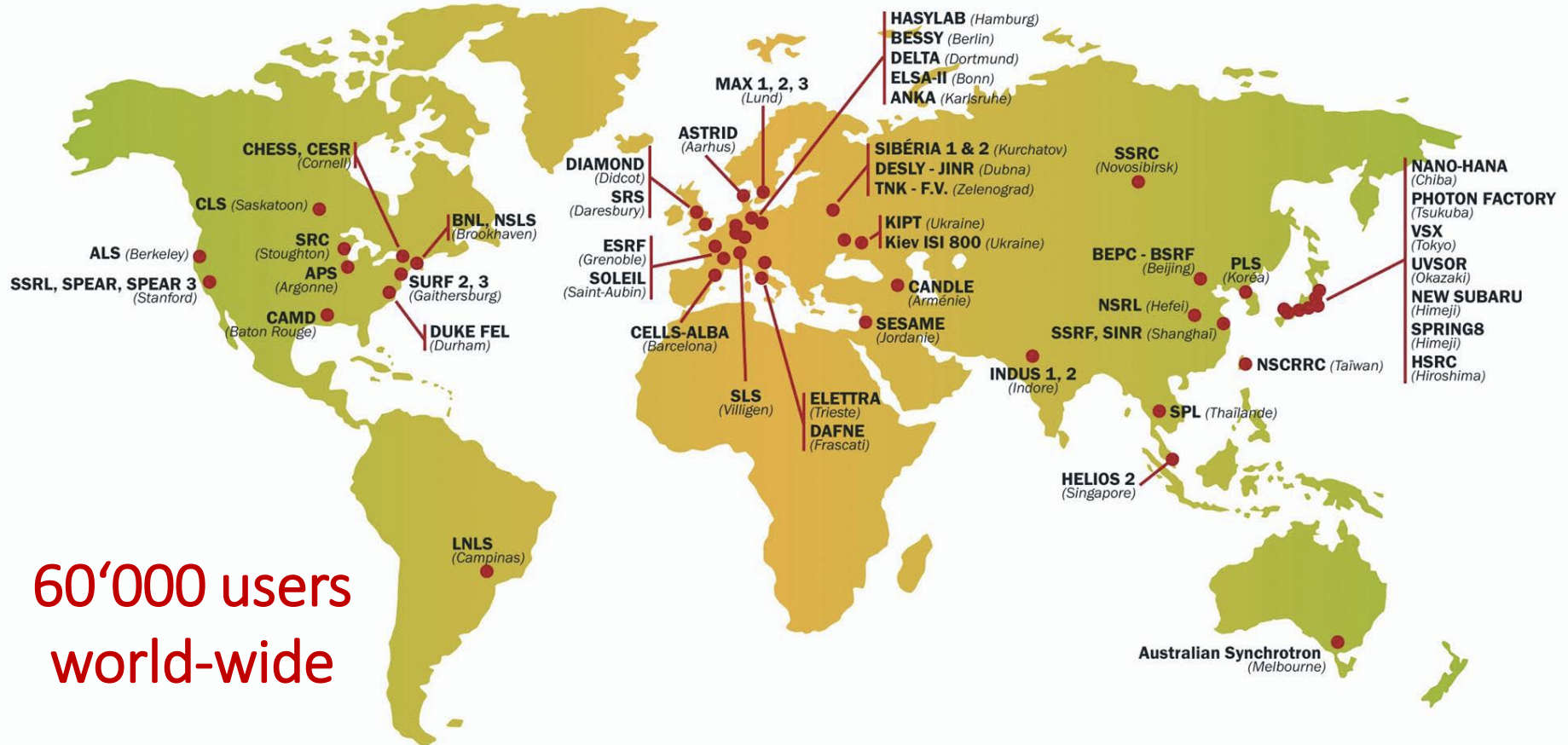
circumference:	288 m
energy:	2.4 GeV
beam current:	400 mA
beam lines:	21

integrated booster ring →  
Top-Up operation

**magnet lattice:**  
12 TBA cells  
(triple bend achromat)

12 straight sections for  
undulators

# Synchrotron Light Sources: about 50 storage ring based

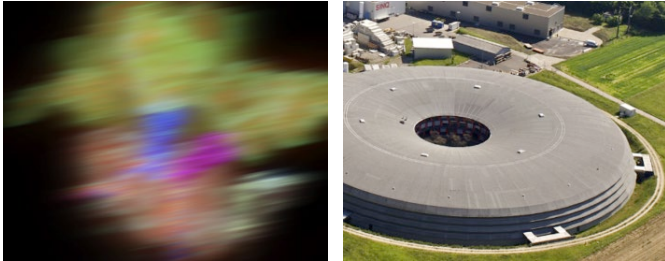


60'000 users  
world-wide

# SwissFEL – a new accelerator based Research Infrastructure

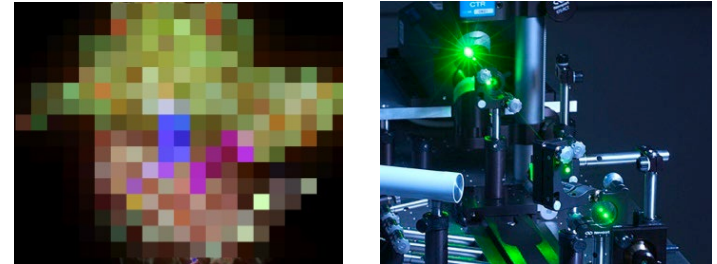
## 3<sup>rd</sup> gen. synchrotron

fine, slow

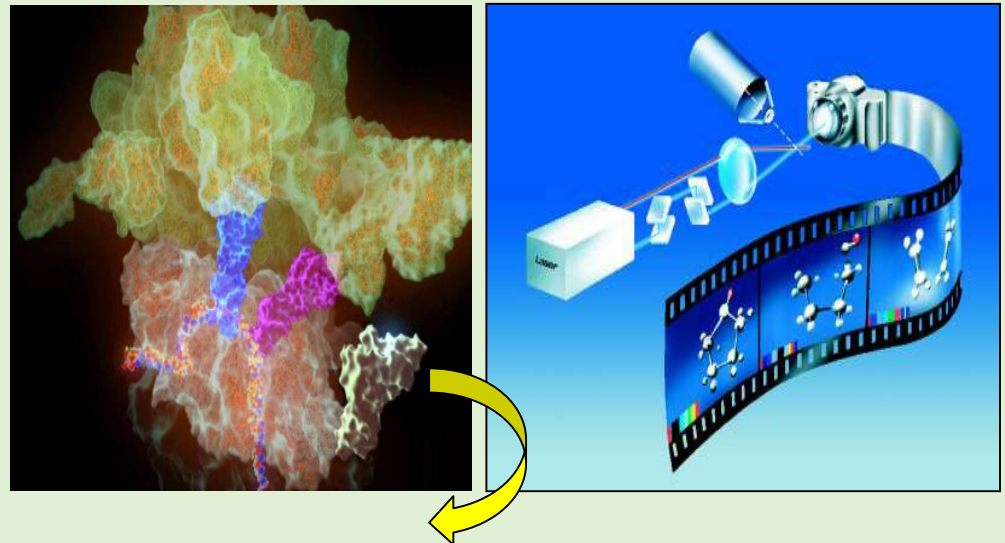


## optical lasers

fast, coarse

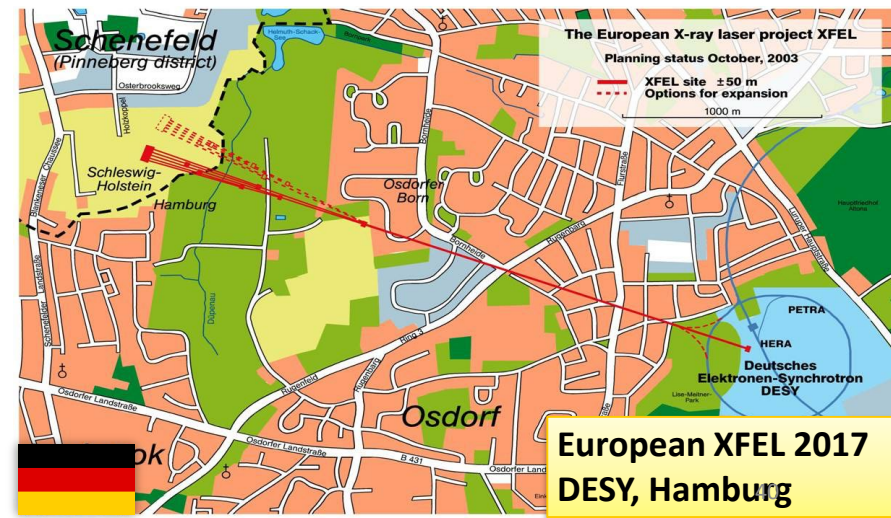


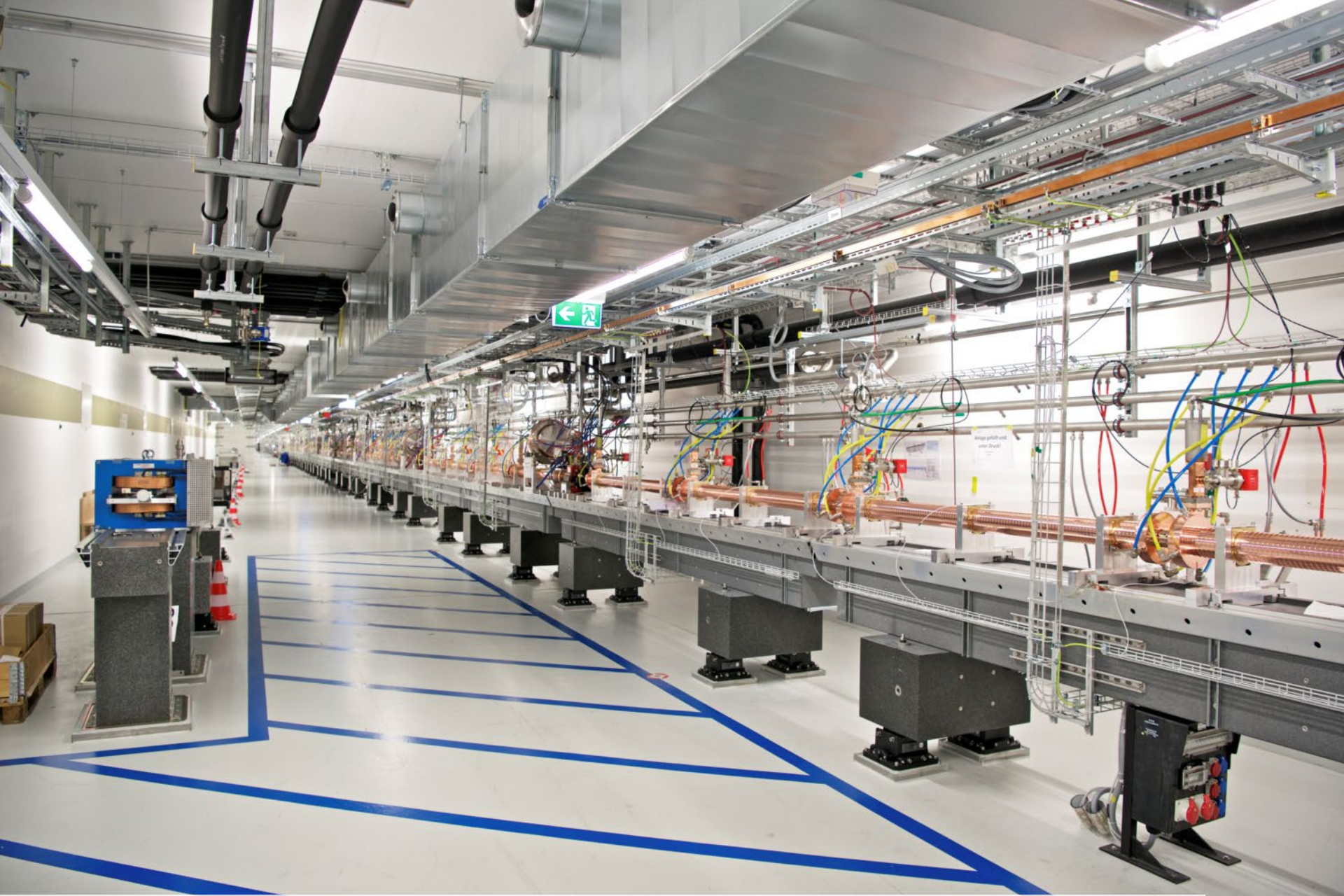
**SwissFEL** fine **and** fast  
at extreme high intensity



new direct insights into chemical, physical,  
biological mechanisms governing our daily-life <sup>39</sup>

# X-Ray Free Electron Lasers



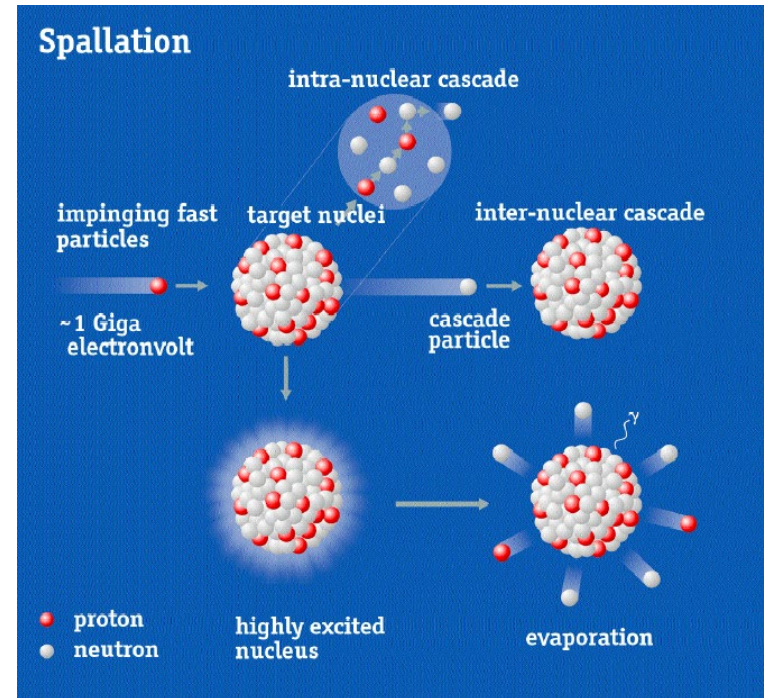


SwissFEL Linac (C-Band Frequency, 6GHz)

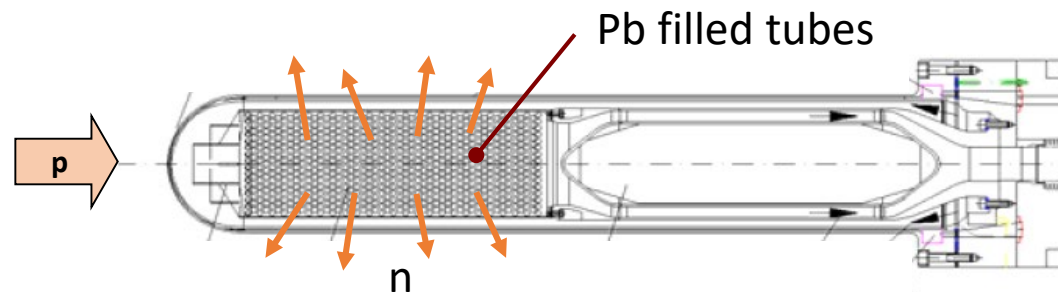
# Spallation Neutron Sources

use **proton beam** on a target → safe process but need high power beam, **MW class accelerators**

neutron energy around 1 MeV → too high  
→ use moderators:  
for thermal spectrum D<sub>2</sub>O/H<sub>2</sub>O, (ambient temperature): 0.025 eV

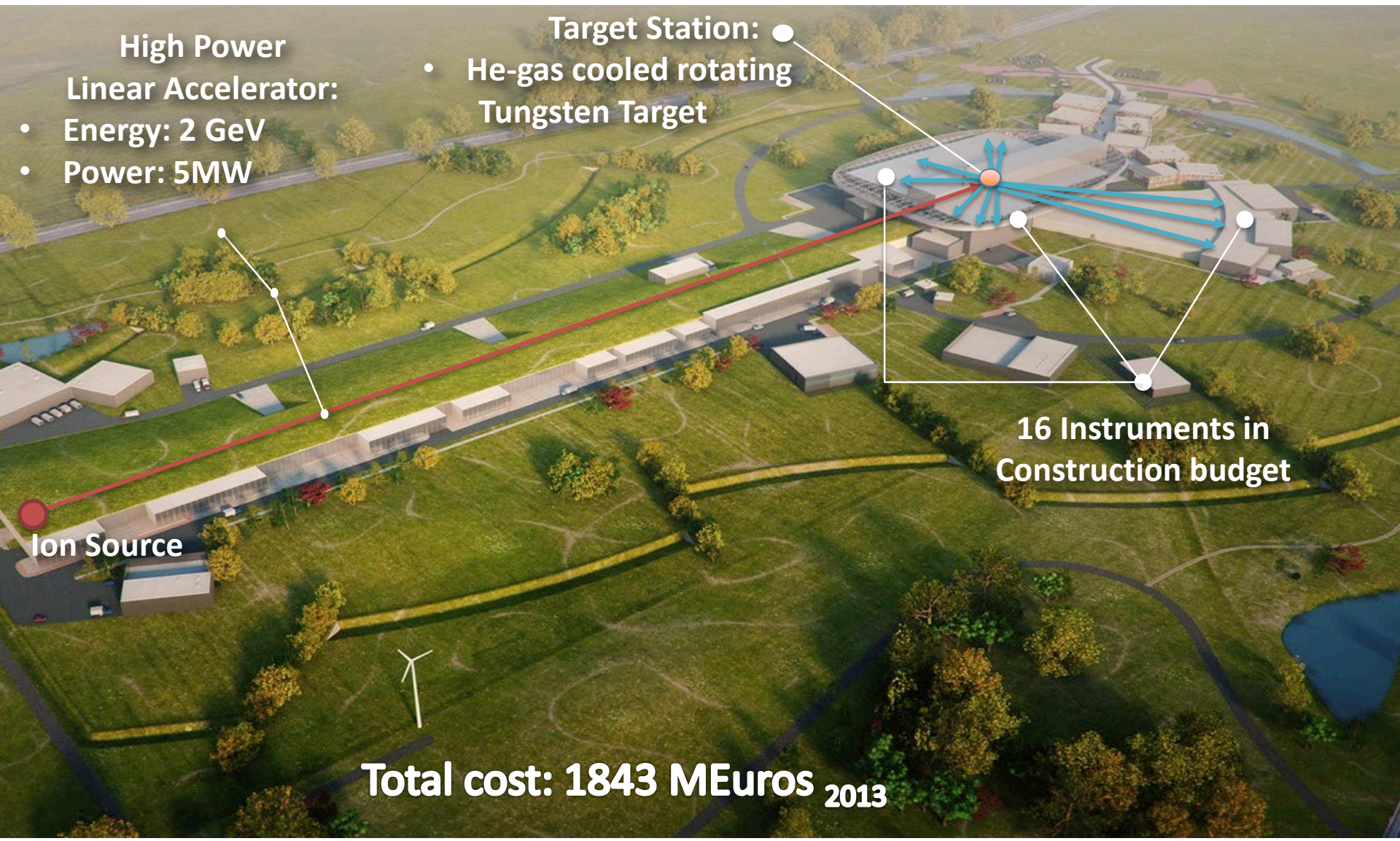


example: PSI-HIPA target

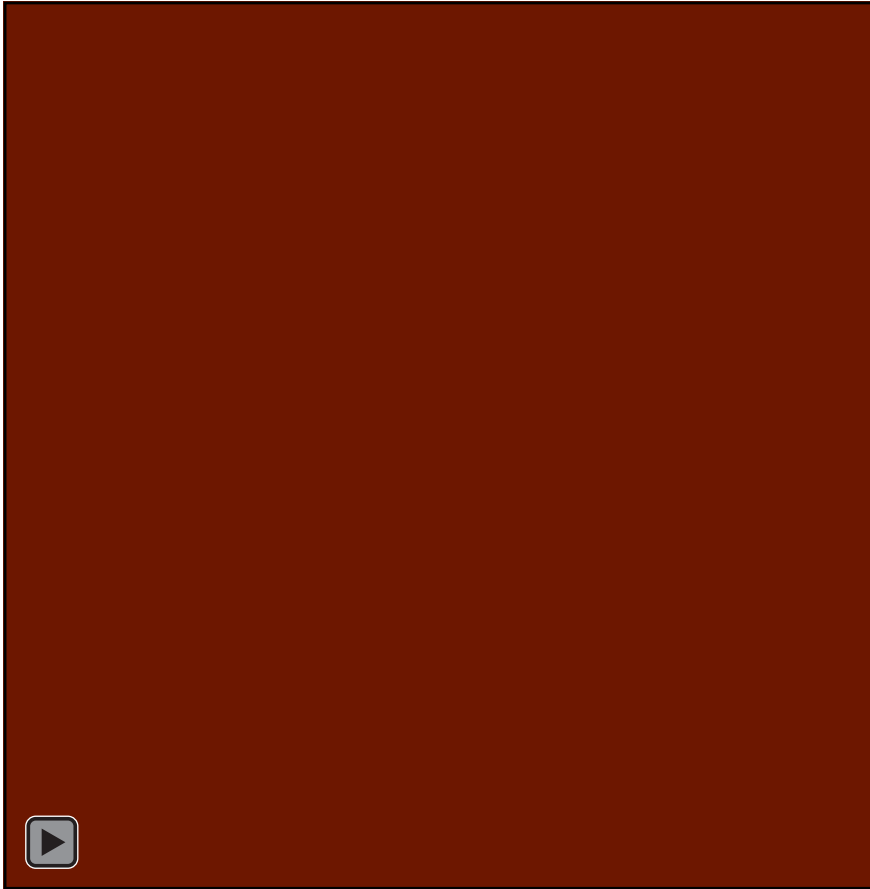


# Example: European Spallation Source

[M.Lindroos, ESS]



# time resolved neutron scattering – making coffee

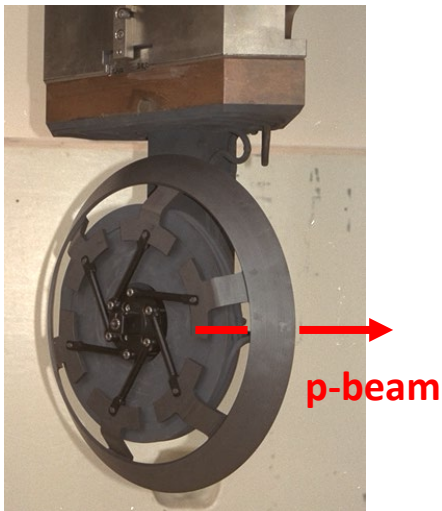
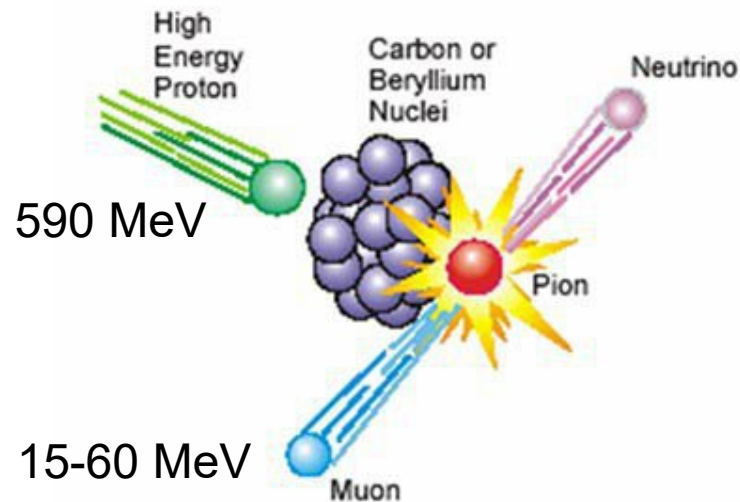


**neutron  
radiography**

**[courtesy: PSI]**

# Muon production using targets

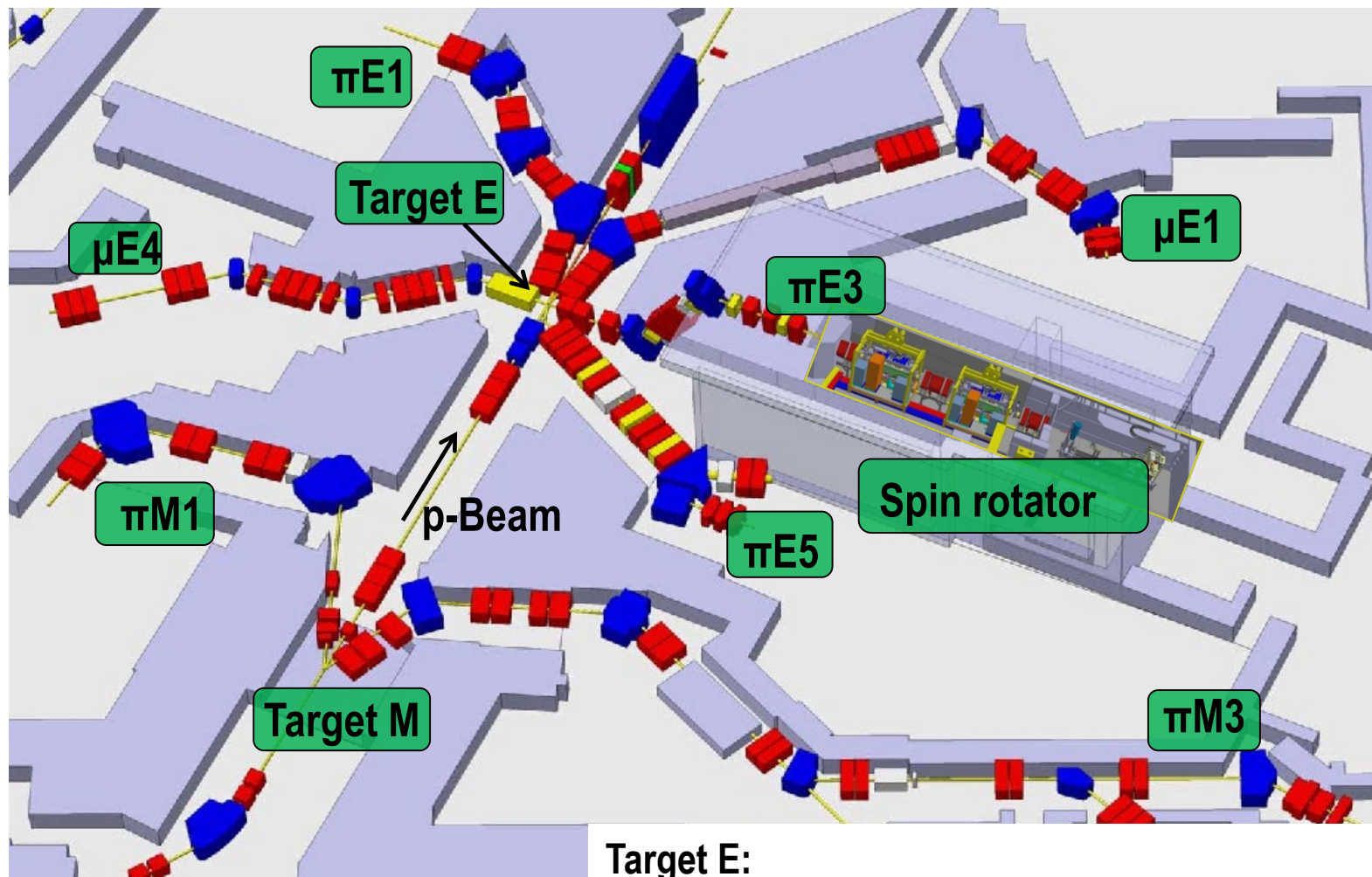
## Production of muons:



### Graphite TARGET CONE

Mean diameter:	450 mm
Operating Temperature:	1700 K
Rotational Speed:	1 Turn/s
Target thickness:	40 mm
Power deposition:	50 kW

# PSI $\mu/\pi$ Secondary Beam Lines



**Target M:**

$\pi M1$ : 100-500 MeV/c Pions

$\pi M3$ : 28 MeV/c Surface Muons

**Target E:**

$\pi E1$ : 10 - 500 MeV/c High Intensity Pions und Muons

$\mu E1$ : Polarized Muon Beam

$\pi E3$ : 28MeV/c Surface polarized Muons

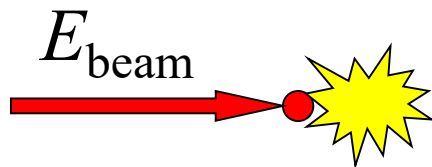
$\mu E4$ : 30 - 100 MeV/c High Intensity Polarized Muons

$\pi E5$ : 10 - 120 MeV/c High Intensity Muons

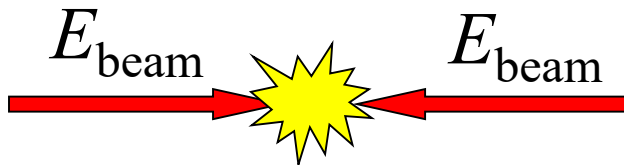
# Next: Accelerators for Particle Physics Research

# Fixed target vs collisions

What energy is available in collisions (center-of-mass energy)?

fixed target geometry: 

$$E_{\text{cm}} = \sqrt{2mc^2} \cdot \sqrt{E_{\text{beam}}}$$

Colliding beams: 

$$E_{\text{cm}} = 2 \cdot E_{\text{beam}}$$

The energy gain with collisions is enormous!  
LHC: 14TeV vs. 0.15TeV

the first collider facility AdA was  
built 1961 in Frascati by  
Bruno Touschek



# LHC – the largest accelerator today

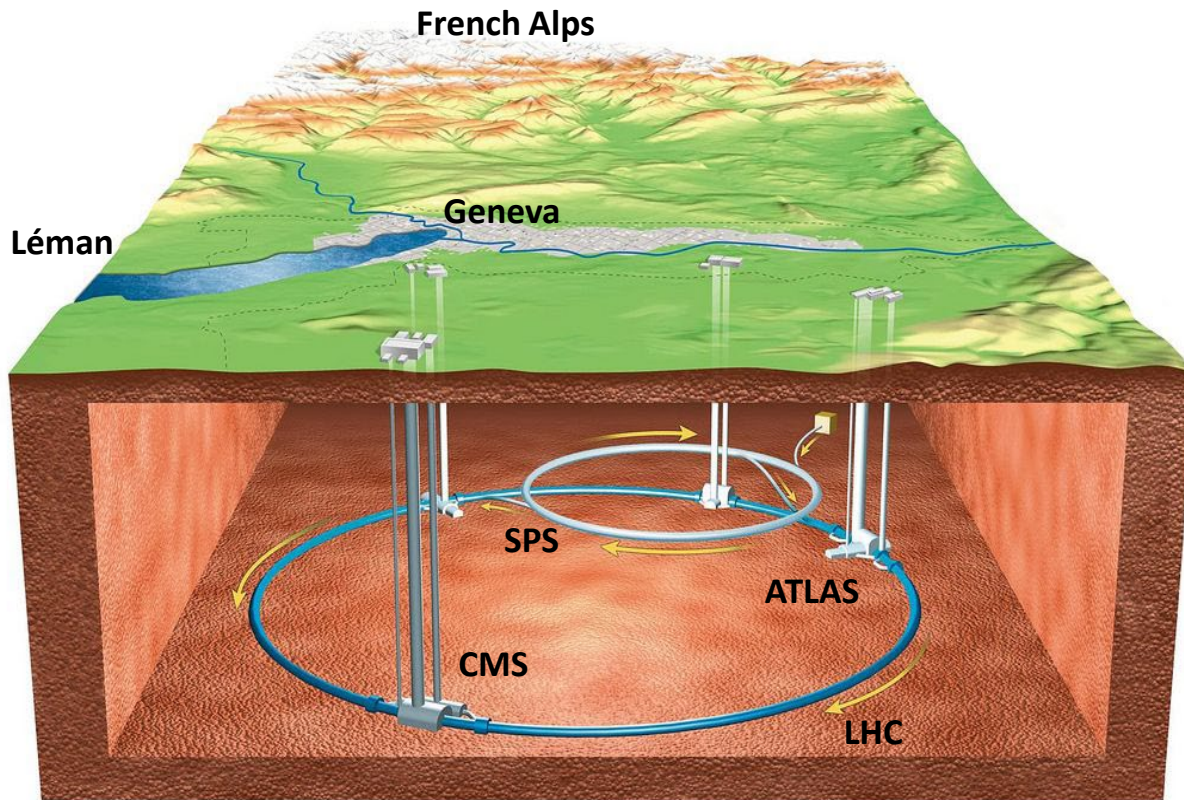
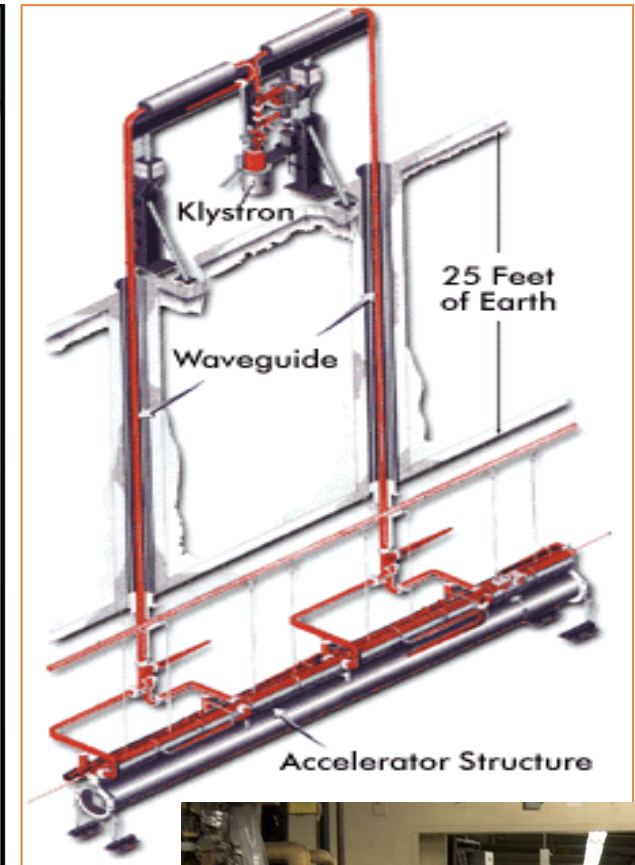
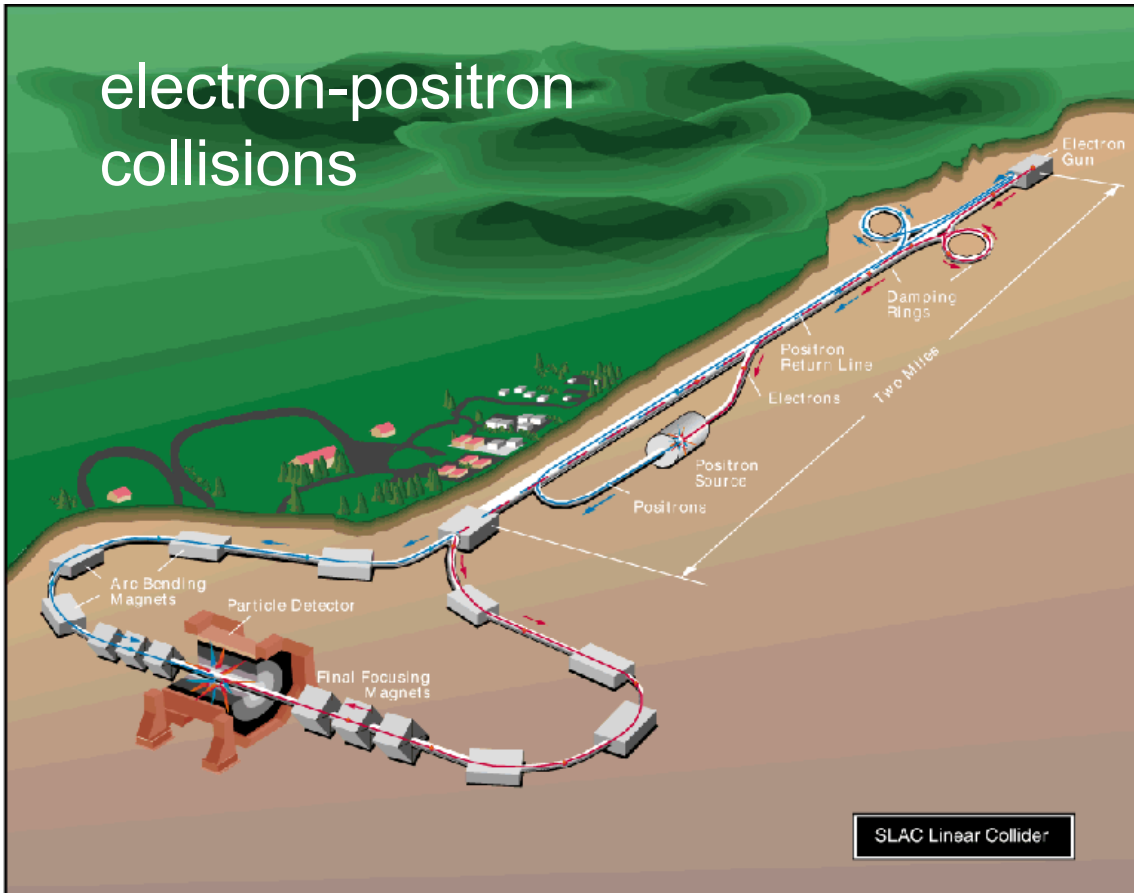


Image: Philippe Mouche

- 27 km circumference
- 100 m below ground
- 7 TeV per beam
- 99,9999991% of  $c$
- **CMS/ATLAS**: local generation of Big Bang conditions, Higgs etc.
- **LHCb**: CP violation in B-Meson-Systems
- **ALICE**: Heavy Ions, Quark-Gluon-Plasma

# Stanford Linear Collider (SLC, till 1997)

electron-positron  
collisions



only linear collider in world, Menlo Park/California/USA  
Linac: 3km length  
2x50GeV  $e^+/e^-$  collisions, polarised!



Klystron-Gallery –  
“view to infinity”

# 24 Nobel Prizes in Physics with direct contributions from accelerators

Year	Name	Accelerator-Science Contribution to Nobel Prize-Winning Research
1939	<b>Ernest O. Lawrence</b>	Lawrence invented the cyclotron at the University of Californian at Berkeley in 1929 [12].
1951	<b>John D. Cockcroft and Ernest T.S. Walton</b>	Cockcroft and Walton invented their eponymous linear positive-ion accelerator at the Cavendish Laboratory in Cambridge, England, in 1932 [13].
1952	Felix Bloch	Bloch used a cyclotron at the Crocker Radiation Laboratory at the University of California at Berkeley in his discovery of the magnetic moment of the neutron in 1940 [14].
1957	Tsung-Dao Lee and Chen Ning Yang	Lee and Yang analyzed data on K mesons ( $\theta$ and $\tau$ ) from Bevatron experiments at the Lawrence Radiation Laboratory in 1955 [15], which supported their idea in 1956 that parity is not conserved in weak interactions [16].
1959	Emilio G. Segrè and Owen Chamberlain	Segrè and Chamberlain discovered the antiproton in 1955 using the Bevatron at the Lawrence Radiation Laboratory [17].
1960	Donald A. Glaser	Glaser tested his first experimental six-inch bubble chamber in 1955 with high-energy protons produced by the Brookhaven Cosmotron [18].
1961	Robert Hofstadter	Hofstadter carried out electron-scattering experiments on carbon-12 and oxygen-16 in 1959 using the SLAC linac and thereby made discoveries on the structure of nucleons [19].
1963	Maria Goeppert Mayer	Goeppert Mayer analyzed experiments using neutron beams produced by the University of Chicago cyclotron in 1947 to measure the nuclear binding energies of krypton and xenon [20], which led to her discoveries on high magic numbers in 1948 [21].
1967	Hans A. Bethe	Bethe analyzed nuclear reactions involving accelerated protons and other nuclei whereby he discovered in 1939 how energy is produced in stars [22].
1968	Luis W. Alvarez	Alvarez discovered a large number of resonance states using his fifteen-inch hydrogen bubble chamber and high-energy proton beams from the Bevatron at the Lawrence Radiation Laboratory [23].
1976	Burton Richter and Samuel C.C. Ting	Richter discovered the $J/\psi$ particle in 1974 using the SPEAR collider at Stanford [24], and Ting discovered the $J/\psi$ particle independently in 1974 using the Brookhaven Alternating Gradient Synchrotron [25].
1979	Sheldon L. Glashow, Abdus Salam, and Steven Weinberg	Glashow, Salam, and Weinberg cited experiments on the bombardment of nuclei with neutrinos at CERN in 1973 [26] as confirmation of their prediction of weak neutral currents [27].

1980	James W. Cronin and Val L. Fitch	Cronin and Fitch concluded in 1964 that CP (charge-parity) symmetry is violated in the decay of neutral K mesons based upon their experiments using the Brookhaven Alternating Gradient Synchrotron [28].
1981	Kai M. Siegbahn	Siegbahn invented a weak-focusing principle for betatrons in 1944 with which he made significant improvements in high-resolution electron spectroscopy [29].
1983	William A. Fowler	Fowler collaborated on and analyzed accelerator-based experiments in 1958 [30], which he used to support his hypothesis on stellar-fusion processes in 1957 [31].
1984	Carlo Rubbia and <b>Simon van der Meer</b>	Rubbia led a team of physicists who observed the intermediate vector bosons W and Z in 1983 using CERN's proton-antiproton collider [32], and van der Meer developed much of the instrumentation needed for these experiments [33].
1986	Ernst Ruska	Ruska built the first electron microscope in 1933 based upon a magnetic optical system that provided large magnification [34].
1988	Leon M. Lederman, Melvin Schwartz, and Jack Steinberger	Lederman, Schwartz, and Steinberger discovered the muon neutrino in 1962 using Brookhaven's Alternating Gradient Synchrotron [35].
1989	Wolfgang Paul	Paul's idea in the early 1950s of building ion traps grew out of accelerator physics [36].
1990	Jerome I. Friedman, Henry W. Kendall, and Richard E. Taylor	Friedman, Kendall, and Taylor's experiments in 1974 on deep inelastic scattering of electrons on protons and bound neutrons used the SLAC linac [37].
1992	Georges Charpak	Charpak's development of multiwire proportional chambers in 1970 were made possible by accelerator-based testing at CERN [38].
1995	Martin L. Perl	Perl discovered the tau lepton in 1975 using Stanford's SPEAR collider [39].
2004	David J. Gross, Frank Wilczek, and H. David Politzer	Gross, Wilczek, and Politzer discovered asymptotic freedom in the theory of strong interactions in 1973 based upon results from the SLAC linac on electron-proton scattering [40].
2008	Makoto Kobayashi and Toshihide Maskawa	Kobayashi and Maskawa's theory of quark mixing in 1973 was confirmed by results from the KEKB accelerator at KEK (High Energy Accelerator Research Organization) in Tsukuba, Ibaraki Prefecture, Japan, and the PEP II (Positron Electron Project II) at SLAC [41], which showed that quark mixing in the six-quark model is the dominant source of broken symmetry [42].



**2013:** François Englert and Peter W. Higgs "for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at **CERN's Large Hadron Collider**"

# Particle Accelerators

Next: about this course ...

# References and accessible Reading Material

## available on the internet:

P. Schmüser & J. Rossbach, Basic course on accelerator optics:

<https://cds.cern.ch/record/247501/files/p17.pdf>

F.Tecker, Longitudinal Dynamics:

<https://arxiv.org/pdf/1601.04901.pdf>

L.Rivkin, Electron dynamics in rings in the presence of radiation :

<https://cds.cern.ch/record/375974/files/p45.pdf>

**Book, H.Wiedemann, Particle Accelerators, download pdf !:**

<https://link.springer.com/book/10.1007%2F978-3-319-18317-6>

CERN Accelerator School (CAS) proceedings homepage (huge!)

<https://cas.web.cern.ch/previous-schools>

CERN Accelerator School on Medical Applications:

<https://cds.cern.ch/record/2271793/files/33-8-PB.pdf>

## books, papers:

S.Peggs, T.Satogata, *Introduction to Accelerator Dynamics*, Cambridge University Press, 2017

A. Wolski, *Beam Dynamics in high energy particle accelerators*, Imperial College Press, 2014

A. W. Chao, M. Tigner, *Handbook of Accelerator Physics and Engineering*, World Scientific 1999

E. D. Courant and H. S. Snyder, *Annals of Physics*: **3**, 1-48 (1958)

M. Sands, SLAC-121, 1969

**Physics of Electron Storage Rings: An Introduction.**

<https://digital.library.unt.edu/ark:/67531/metadc865991/>

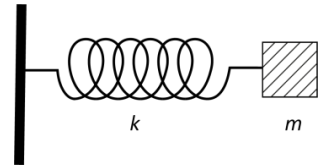
# Mathematical Prerequisites

linear differential equations:

$$\ddot{x} + \omega^2 x = 0, \quad x(t) = A \sin \omega t$$

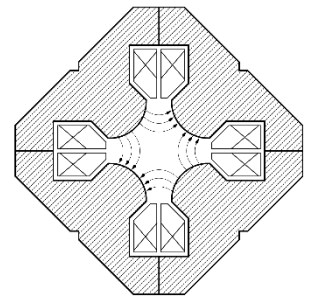
$$x'' + K(s)x = 0$$

$$\frac{\Delta x}{\Delta t} \propto x \longrightarrow \dot{x} = \alpha x, \quad x(t) = A \exp \alpha t$$



vector differential equations:

$$\nabla \times \vec{B} = 0, B_z = 0, \quad \rightarrow \quad \frac{\partial}{\partial x} B_y - \frac{\partial}{\partial y} B_x = 0$$



# More Mathematical Prerequisites

linearization and Taylor expansion:

$$B_z(R + x) \approx B_z(R) + \frac{dB_z}{dR}x$$
$$\frac{1}{R + x} \approx \frac{1}{R} \cdot \left(1 - \frac{x}{R}\right)$$

R = position of beam center  
x = particle, distance to center

matrix calculations:

$$\vec{x}_2 = \mathbf{M} \cdot \vec{x}_1$$

$$\det \mathbf{M}_1 = 1, \det \mathbf{M}_2 = 1 \longrightarrow \det (\mathbf{M}_1 \cdot \mathbf{M}_2) = 1$$

matrix eigenvalues, eigenvectors:

$$\mathbf{M}\vec{v} = \lambda\vec{v}, \quad \det (\mathbf{M} - \lambda\mathbf{I}) = 0 \quad \text{eigenvalue equation, characteristic polynomial}$$

$$\mathbf{M}^n = \mathbf{v}\boldsymbol{\lambda}^n\mathbf{v}^{-1}$$

$\boldsymbol{\lambda}$  = diagonal matrix of eigenvalues  
 $\mathbf{v}$  = matrix with eigenvectors in cols.

# Outline of LPAP Accelerator Physics Course

## Lectures

## Dates

Accelerator Concepts, their History and their Applications	12.09.2024
An Overview on Physics and Technology of Accelerators	19.09.2024
Transverse Dynamics :: Equation of Motion and Solutions	26.09.2024
Transverse Dynamics :: Lattice Design and Beam Properties	03.10.2024
Transverse Dynamics :: Lattice Imperfections and Hamilton Formalism	10.10.2024
Longitudinal Beam Dynamics	17.10.2024
Synchrotron Radiation and Electron Dynamics	31.11.2024
Synchrotron Light Sources	07.11.2024
Medical Application (Marco Schippers, PSI)	14.11.2024
Particle Colliders	21.11.2024
CERN Visit	28.11.2024
Collective Effects (Tatiana Pieloni, EPFL)	05.12.2024
Accelerator Case Studies	12.12.2024
Recap and exam example	19.12.2024

special tutorial  
mid-term test-exam

special tutorial  
exam preparation

# Possibilities for TPs, Master thesis

CERN (postdocs, doctoral students)

- LHC and its upgrades (high luminosity, energy)
- Future Circular Collider study
- Muon Collider Study

PSI (postdocs, graduate students)

- SwissFEL: X-Ray Free Electron Laser
- Swiss Light Source (SLS)
- neutron, muon beams
- medical applications: cancer proton therapy

Other accelerator labs around the world

# Why learn about accelerators?

Accelerator is a challenging **part** of experimental setups.

*It is important to understand*

**Its limitations**

**Its potential**

**You** can be user or designer of future machines

***Help to specify, design them***

**It's fun!**

# Appendix: Magnetic Rigidity (proton)

Lorentz force  $\vec{F}_B = e \cdot \vec{v} \times \vec{B}$

$B, v$  perpendicular  $F_B = evB$

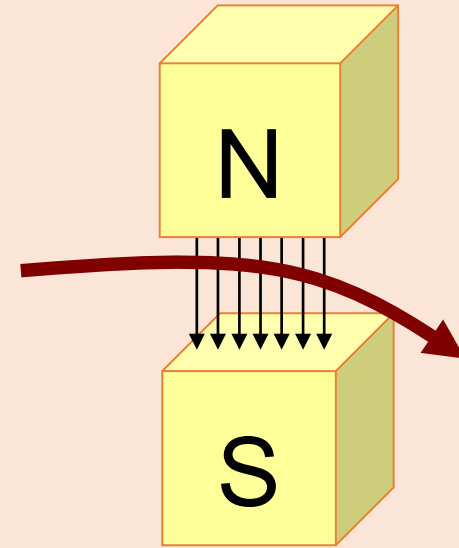
centrifugal force  $F_c = -m \frac{v^2}{\rho}$

$$F_B + F_c = 0 \longrightarrow evB = m \frac{v^2}{\rho}$$

$$B\rho = \frac{mv}{e}$$

Magnetic rigidity

$$B\rho = \frac{p}{e}$$



$B$  = magnetic field

$\rho$  = local bending radius

$p$  = momentum

$e$  = elementary charge

# Appendix: Magnetic Rigidity in Practical Units

$$B\rho = \frac{p}{e} = \frac{mv}{e} = \beta\gamma \frac{m_0 c}{e}$$

$$= \beta\gamma \frac{m_0 c^2}{ce}$$

$$= \beta \frac{E_{\text{tot}}}{ce}$$

$$= \beta \frac{10^9}{c} E_{\text{tot}} [\text{GeV}]$$

↓

$$B\rho [\text{Tm}] \approx 3.3356 \cdot E_k [\text{GeV}]$$

$$B\rho [\text{Tm}] = 3.3356 \cdot p [\text{GeV}/c]$$

$B$  = magnetic field

$\rho$  = local bending radius

$p$  = momentum

$e$  = elementary charge

$E_k$  = kinetic energy

total energy:

$$E_{\text{tot}} = E_k + m_0 c^2$$

approximations:

$$\beta \approx 1, cp \approx E_k$$

$$\text{for } E_k \gg m_0 c^2$$

see also Wiedemann, p.101, eq.5.6