Broad topic	Lecture title
Basic principles of NPP	Introduction / Review of nuclear physics
	Interaction of neutrons with matter
	Nuclear fission
	Fundamentals of nuclear reactors
	LWR plants
Modeling the beast	The diffusion of neutrons - Part 1
	The diffusion of neutrons - Part 2
	Neutron moderation without absorption
	Neutron moderation with absorption
	Multigroup theory
	Element of lattice physics
	Neutron kinetics
	Depletion
Reactor Concepts Zoo	Advanced LWR technology
	Breeding and LFR
	AGR, HTGR
	Channels, MSR and thorium fuel
Review session	

- Interaction of neutrons with matter
- Cross sections
- Mechanisms of neutron interactions
- Illustrations of cross sections
- The Doppler effect
- Anisotropic scattering
- On cross section libraries

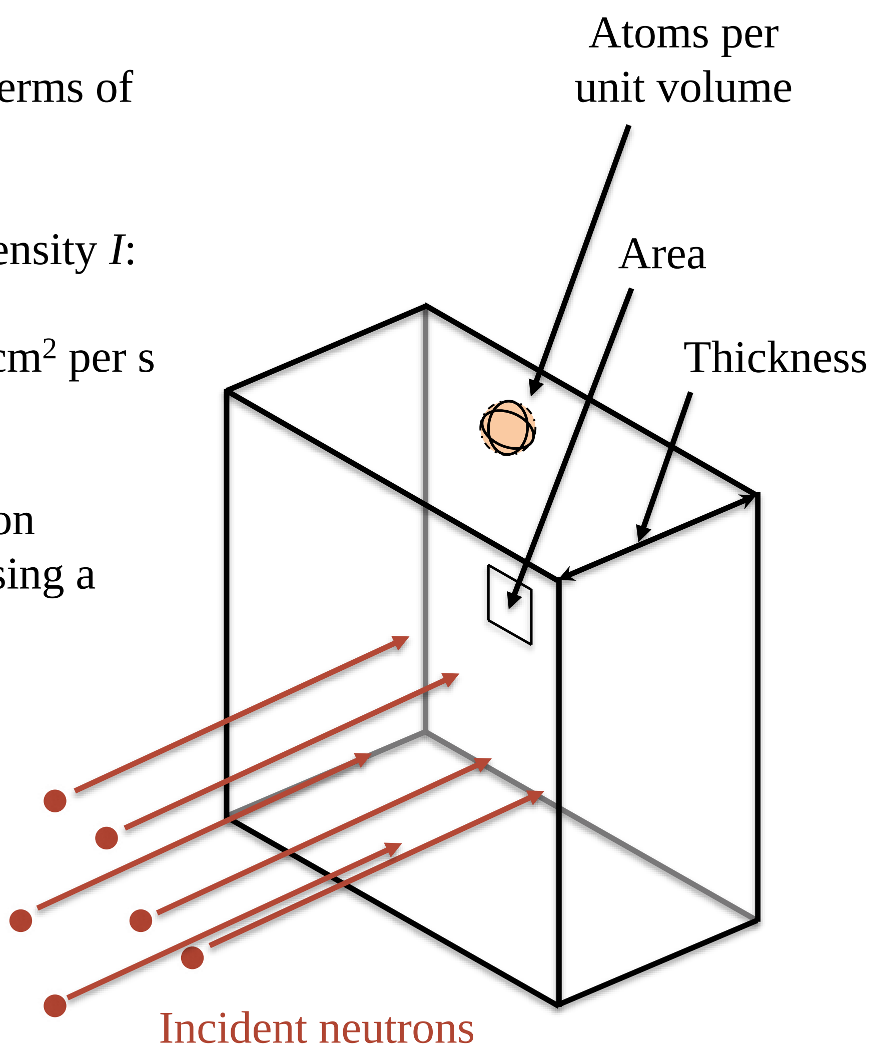
- The operation of a nuclear reactor depends fundamentally on the way in which neutrons interact with atomic nuclei.

Reaction	Is isotopic composition of nucleus changed ?	Is internal energy of nucleus changed ?	Neutron after	Particles after
(n,n)	No	No	1	-
(n,n’)	No	Yes	1	-
(n,γ)	Yes	Yes	0	γ
(n,p)	Yes	Yes	0	p
(n,α)	Yes	Yes	0	α
(n,2n)	Yes	Yes	2	-
(n,3n)	Yes	Yes	3	-
(n,np)	Yes	Yes	1	p
(n,fiss)	Yes	Yes	2-3	γ, …

Interactions of neutrons with matter are described in terms of *cross sections* (XS).

Considering monodirectional beam of neutrons of intensity *I*:

- I = number of neutrons which strike the target per cm^2 per s
  I = n × v = neutron density × neutron velocity
- In this case the beam intensity is equal to the neutron (scalar) flux Φ [n/cm^2s] – number of neutrons crossing a unit area per s



Neutrons incident on a target

- Intuitive concept: *Number of interactions* per unit time per unit volume (Interaction rate) [1/cm^{3}s] =

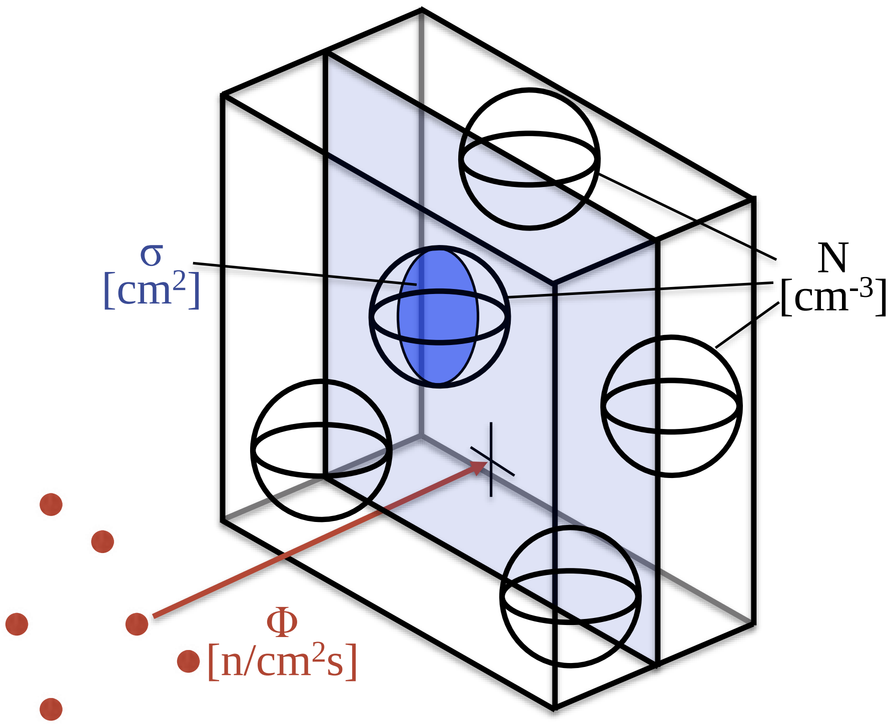
  Nucleus *cross section* [cm^2] X
  Number of *nuclei per unit volume* [1/cm^3] X
  Beam *intensity* (neutron flux) [n/cm^{2}s]

→ Interaction (or reaction) rate: $F = \sigma \times N \times \Phi$

- σ is the *microscopic* cross section [barn]
  → 1 barn (1b) is equal to 10^{-24} cm^2

- $\Sigma = \sigma \times N$ is the *macroscopic* cross section [1/cm]

→ Interaction (or reaction) rate: $F = \Sigma \times \Phi$



Neutrons incident on a target

Type of interactions

Neutrons interact with nuclei in a number of ways and each type of interaction is described by a characteristic cross section:

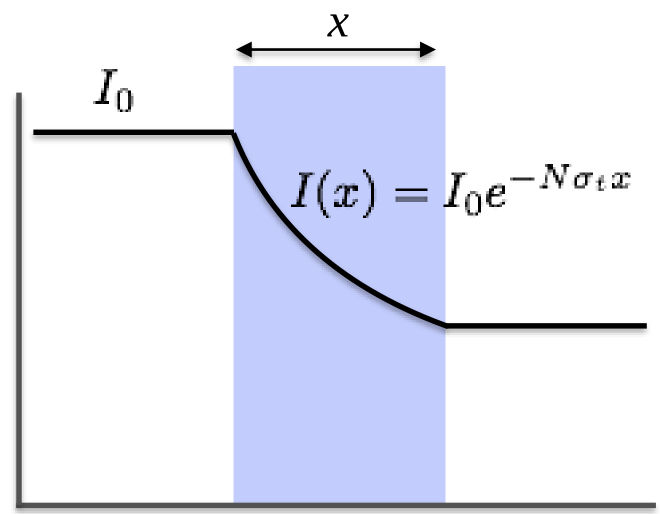
- Elastic scattering cross section σ_e
- Inelastic scattering cross section σ_i
- Radiative capture (n,γ) cross section σ_γ
- Fission cross section σ_f
- Cross section for (n,p) reaction σ_p
- Cross section for (n,α) reaction σ_α
- …

Scattering cross section σ_s (elastic + inelastic)

Absorption cross section σ_a (radiative capture, fission, (n,p), (n,α))

Total cross section σ_t (all of the above)

NEUTRON INTERACTIONS AND MACROSCOPIC CROSS SECTIONS



- Consider a target in monodirectional beam of intensity I_0.
- *I(x)* is the intensity of <u>non-interacted</u> neutrons.
- Decrease of *I* while traversing d*x* of the target:

$$-dI(x) = N\sigma_t I(x)dx$$

$$\frac{-dI(x)}{I(x)} = \underbrace{N\sigma_t}_{\Sigma_t = N\sigma_t} dx$$

- $\frac{dI(x)}{I(x)}$ is the fraction of neutrons which penetrated x without interaction and which interacted in dx
- Σ_t dx = Probability that a neutron interacts in dx
- Σ_t is the probability per unit path length that a neutron will undergo some sort of interaction

NEUTRON INTERACTIONS AND MACROSCOPIC CROSS SECTIONS

- Probability that a neutron has its first interaction in d*x* around *x* $p(x)\mathrm{d}x$

  =

  Probability that neutron survives up to *x* without interaction: $I(x)/I_0 = e^{-N\sigma_t x} = e^{-\Sigma_t x}$

  x

  Probability that neutron does interact in the next d*x* : Σ_t

$$p(x)\mathrm{d}x = \Sigma_t e^{-\Sigma_t x}\mathrm{d}x$$

- *p(x)* is a first interaction (or first collision) probability distribution function
- It represents the distribution of distances which neutron moves between interactions – *free path*.
- The average distance between two interactions – *mean free path*

$$\lambda = \frac{\int_0^\infty x p(x)\mathrm{d}x}{\int_0^\infty p(x)\mathrm{d}x} = \Sigma_t \int_0^\infty x e^{-\Sigma_t x}\mathrm{d}x = \frac{1}{\Sigma_t}$$

EPFL

- Homogeneous mixture of two nuclear species ***X*** and ***Y*** (N_X and N_Y atoms per cm^3)

  - $N_X \sigma_X$ - Probability per unit path that neutron interacts with a nucleus ***X***
  - $N_Y \sigma_Y$ - Probability per unit path that neutron interacts with a nucleus ***Y***

  - Probability per unit path that neutron interacts with *either* ***X*** or ***Y*** :

$$\Sigma = \Sigma_X + \Sigma_Y = N_X \sigma_X + N_Y \sigma_Y$$

  - For the molecule $X_m Y_n$

$$\Sigma = N_{X_m Y_n} \sigma_{X_m Y_n} = N_{X_m Y_n} (m\sigma_X + n\sigma_Y)$$

- These equations are based on the assumption that the nuclei X and Y act independently when they interact with neutrons.

  - for low-energy neutrons undergoing elastic scatterings on molecules, this assumption is not valid.

- **Neutron-electron** interactions are *negligible* (infinitesimal cross sections)
- **Neutron-neutron** interactions are *negligible* (probability to meet a nucleus is ~10^{14} times higher than to meet another neutron)
- Two fundamentally different mechanisms of neutron interaction with nucleus:

- *compound nucleus formation*:
neutron is absorbed, exciting nucleus which then relaxes by emission of:

. one neutron: elastic scattering
. one neutron and γ-ray: inelastic scattering
. γ-ray: (n,γ) reaction
. proton: (n,p) reaction
. α-particle: (n,α)
. in special case, fission (Chapter 3)

- *potential or shape scattering:*
neutron is not absorbed, but interacts with a nucleus as billiard balls do.

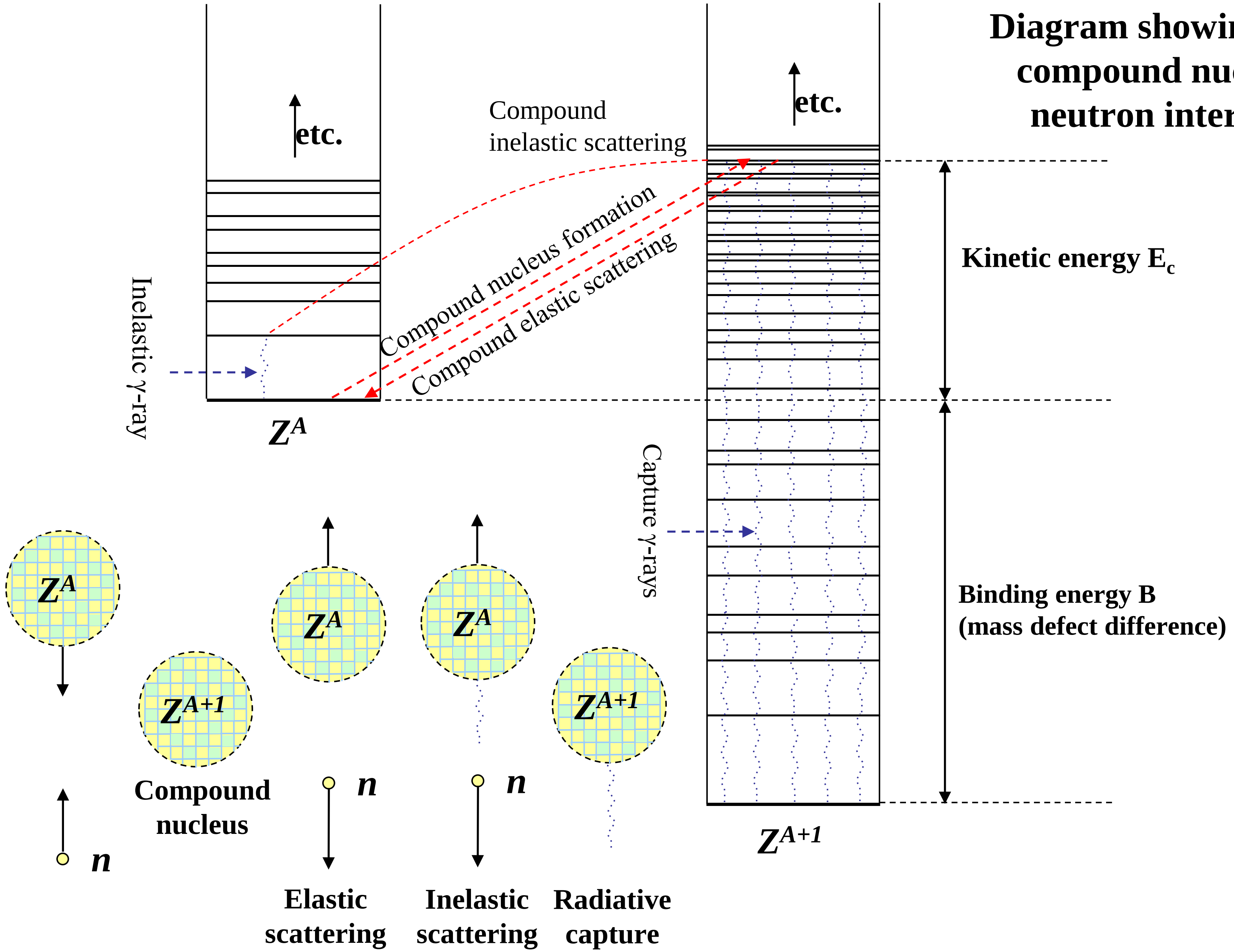
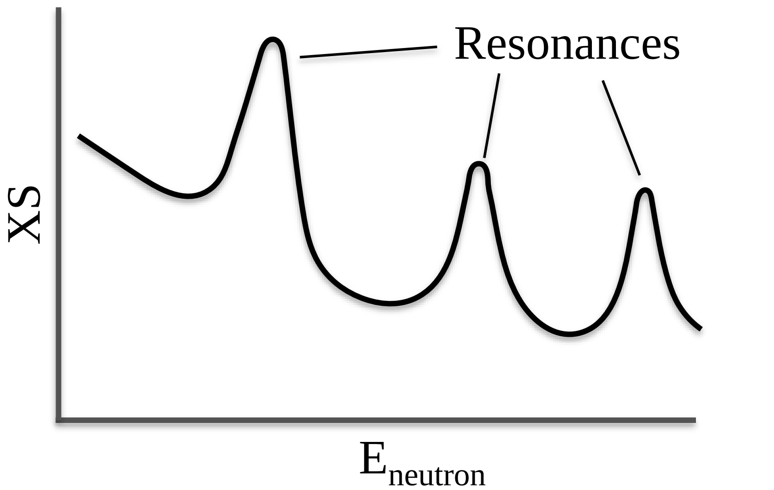


Diagram showing role of compound nucleus in neutron interaction

- Probability of formation of the compound nucleus is high if there is an excited state in the nucleus Z^{A+1} near E_c+B.



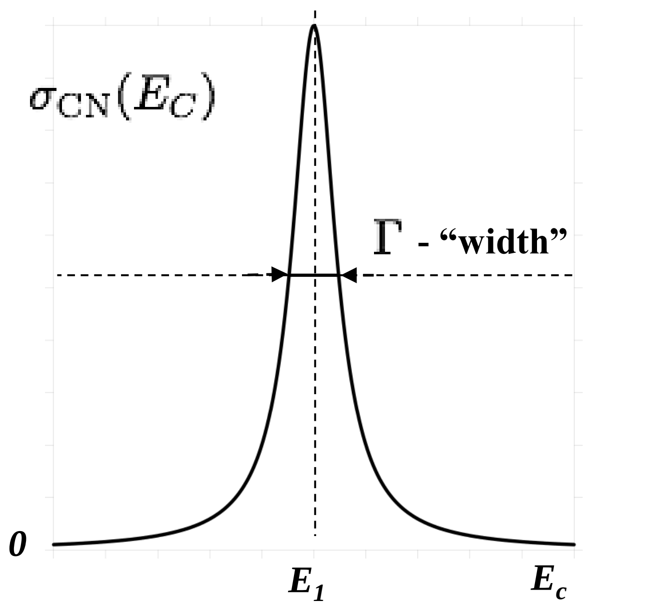
- The cross sections for neutron interactions (e.g. elastic scattering) through a compound nucleus formation can be written as:

$$\sigma_s(E_c) = \sigma_{CN}(E_c)\frac{\Gamma_n}{\Gamma}$$

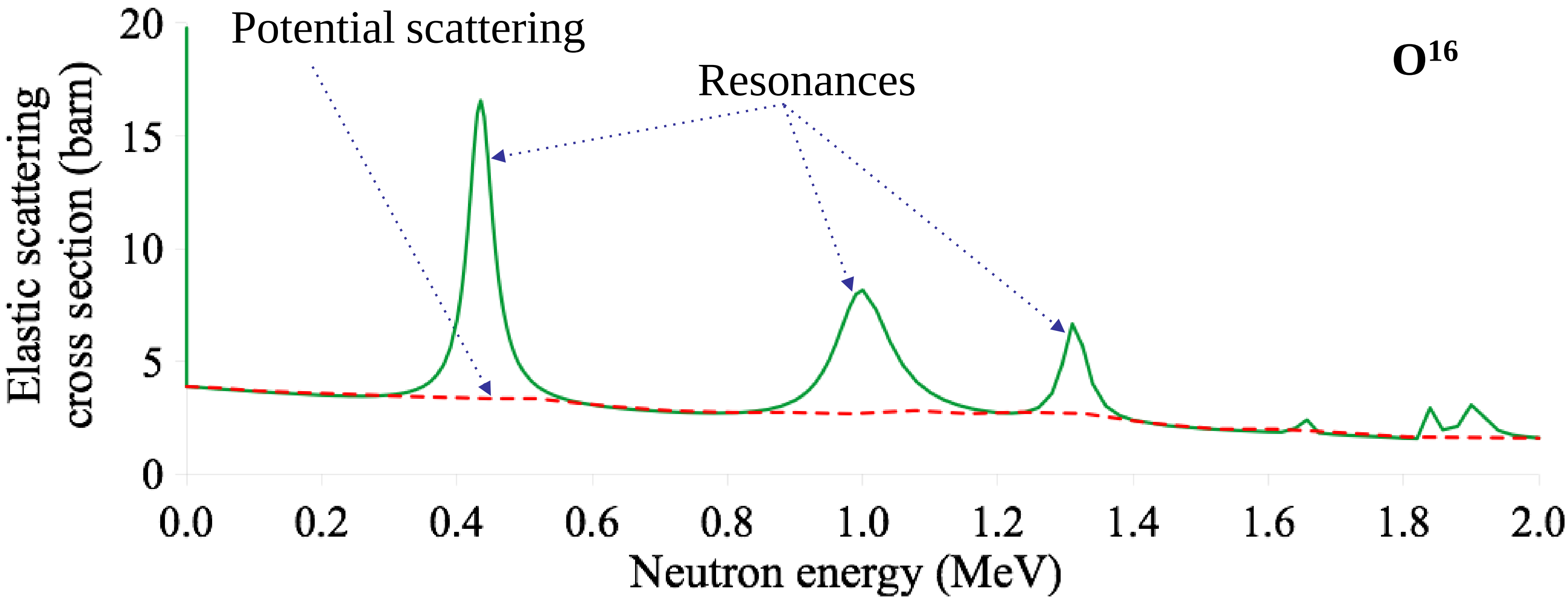
Total cross section for the formation of a compound nucleus × Probability that the compound nucleus decays by elastic neutron emission

- The energy dependence near an isolated resonance E_1 can be approximated by

$$\sigma_{CN}(E_C) = \frac{\text{constant}}{(E_c - E_1)^2 + \Gamma^2/4}$$

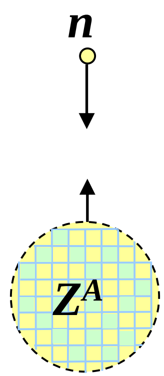


- The cross section for compound elastic scattering is significant only when neutron energy is close to the nucleus resonance.
- Potential elastic scattering on a nucleus takes place at *any* energy of incident neutrons.

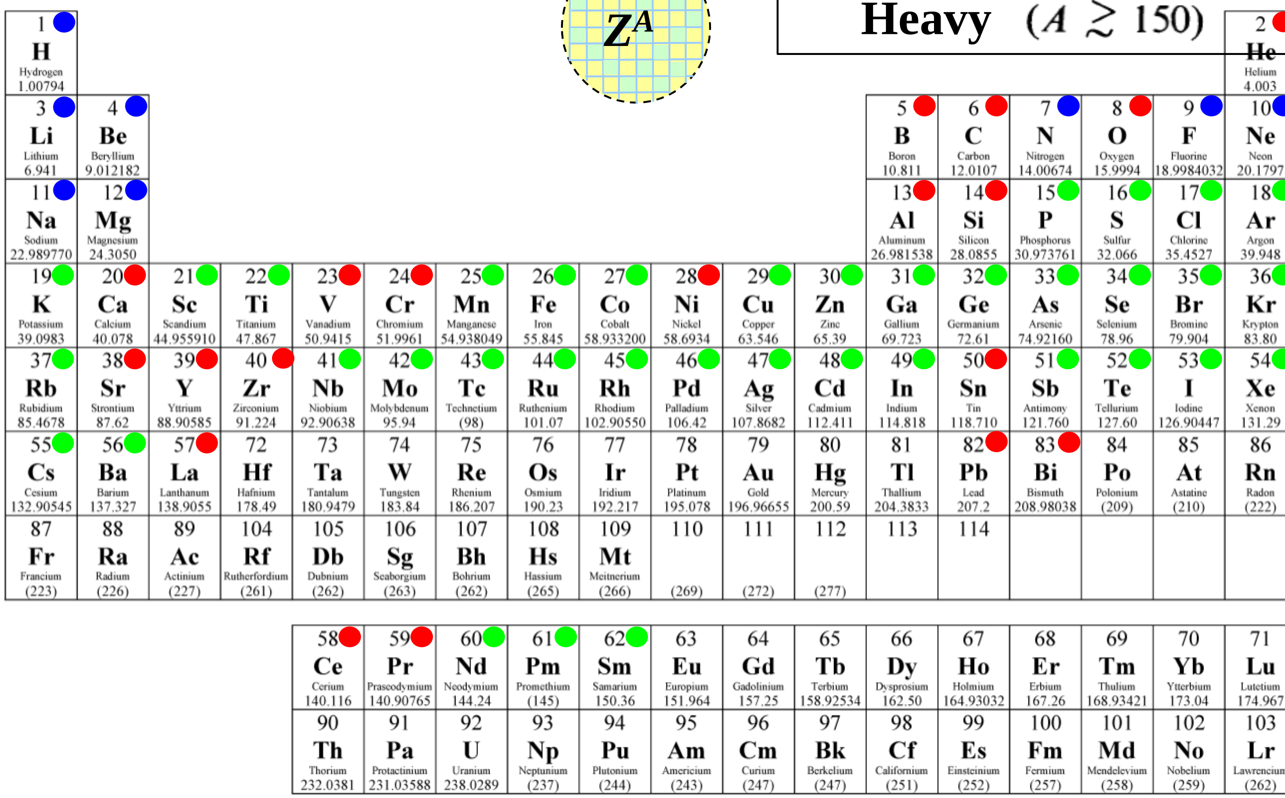


Let us consider now (in a limited way) actual experimental cross-section data. These data depend on:

- the energy of the incident neutron
- the nature of the target nucleus.



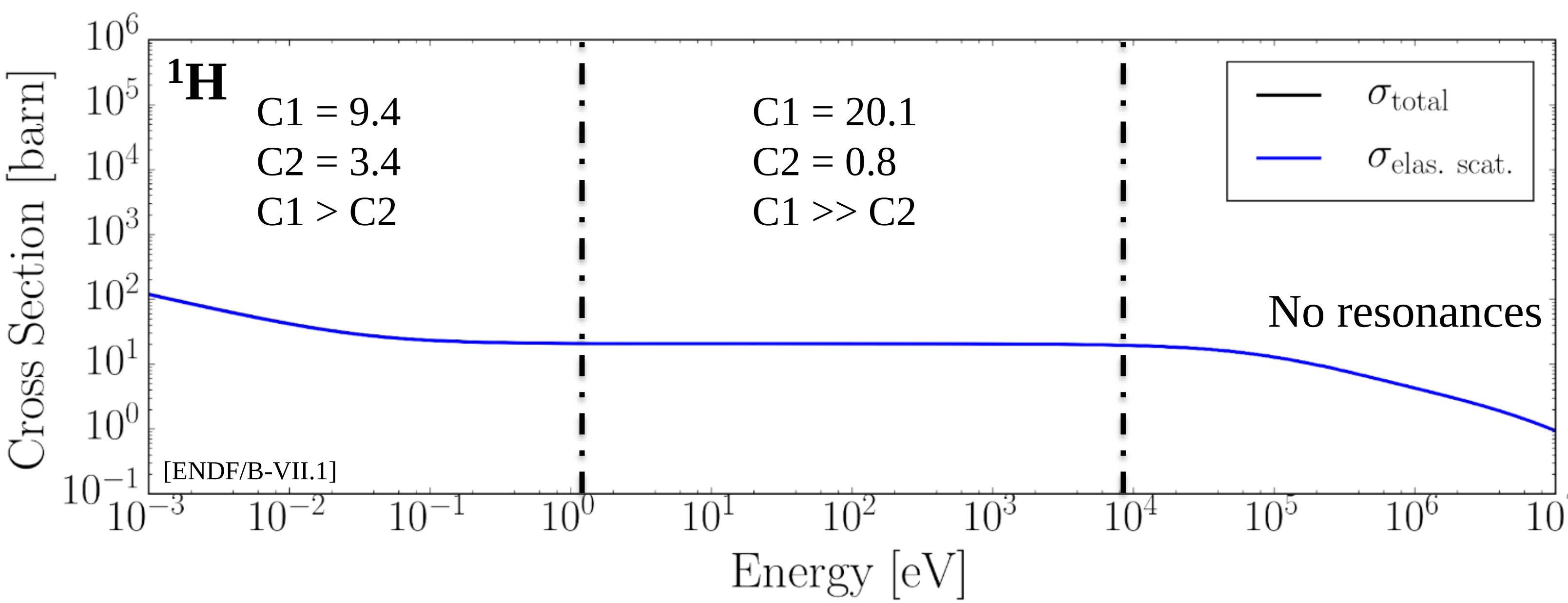
- **Light** $(A \lesssim 25)$
- **Magic** (*N* or *Z* = 2, 6, 8, 14, 20, 28, 50, 82, 126)
- **Intermediate** $(25 \lesssim A \lesssim 150)$
- **Heavy** $(A \gtrsim 150)$



Measured in transmission experiments

$$\sigma_t = C_1 + \frac{C_2}{\sqrt{E}} \qquad \text{or} \qquad \sigma_t = C_1 + \frac{C_2'}{v}$$

- C_1 is determined by the elastic scattering cross sections
- C_2 depends on (n,γ) or any other exothermic reaction



Light nuclides

Measured in transmission experiments

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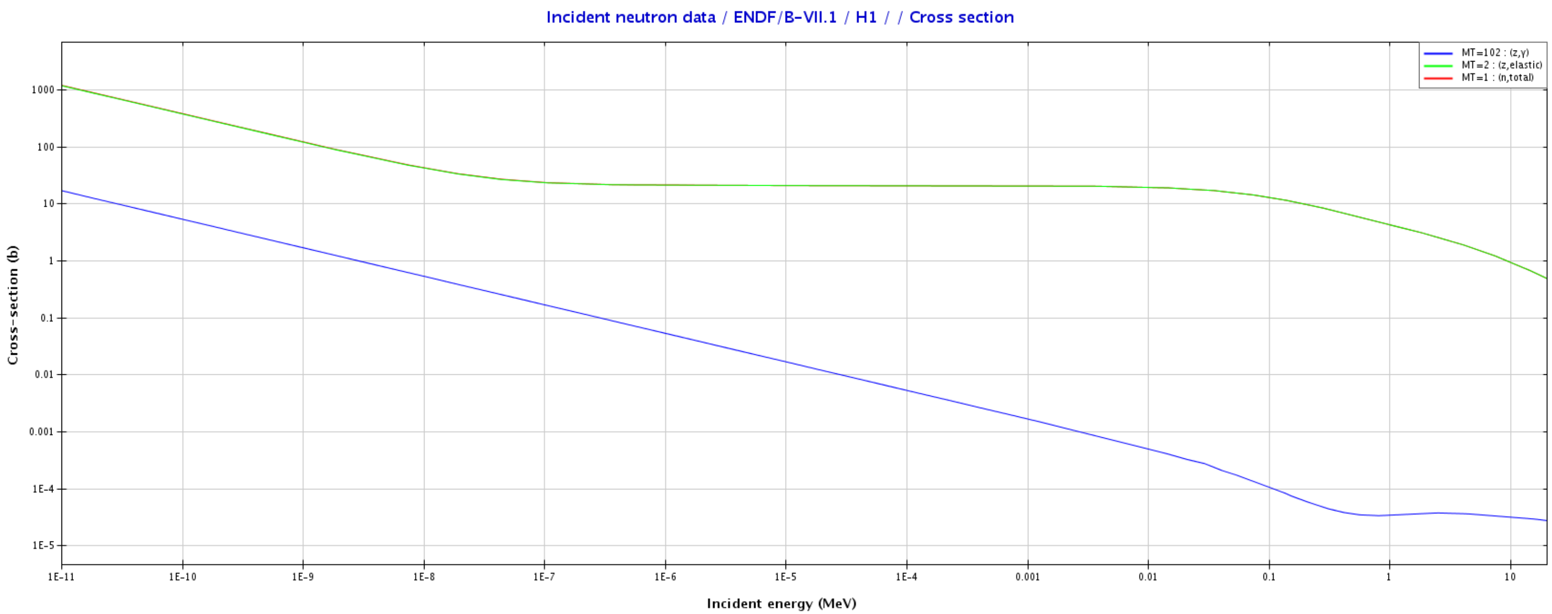
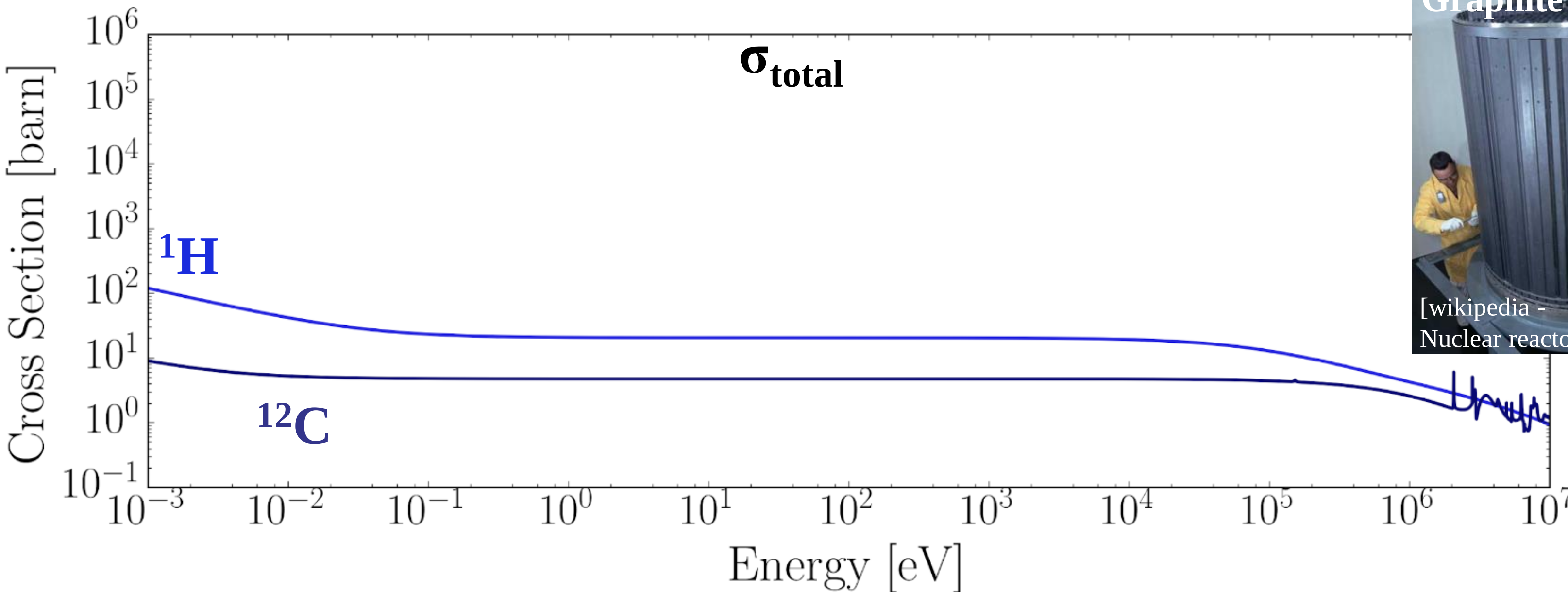
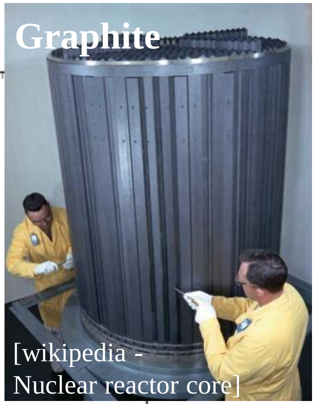
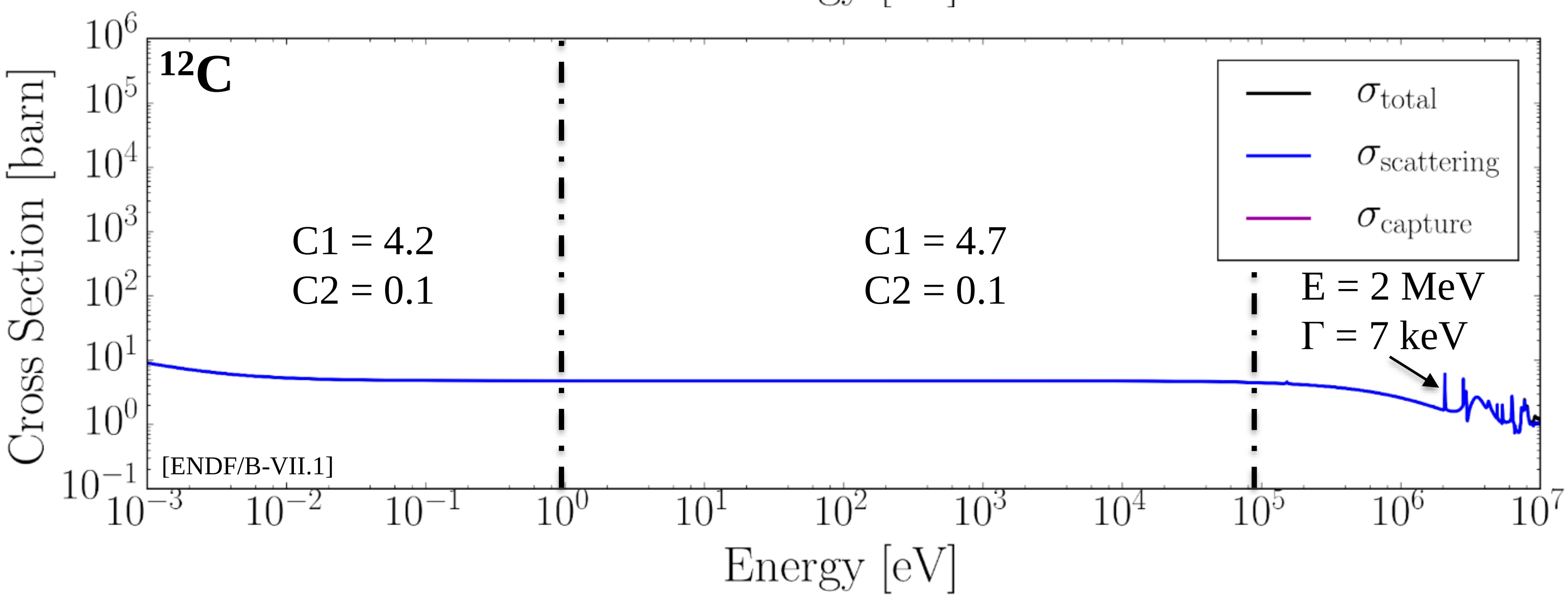


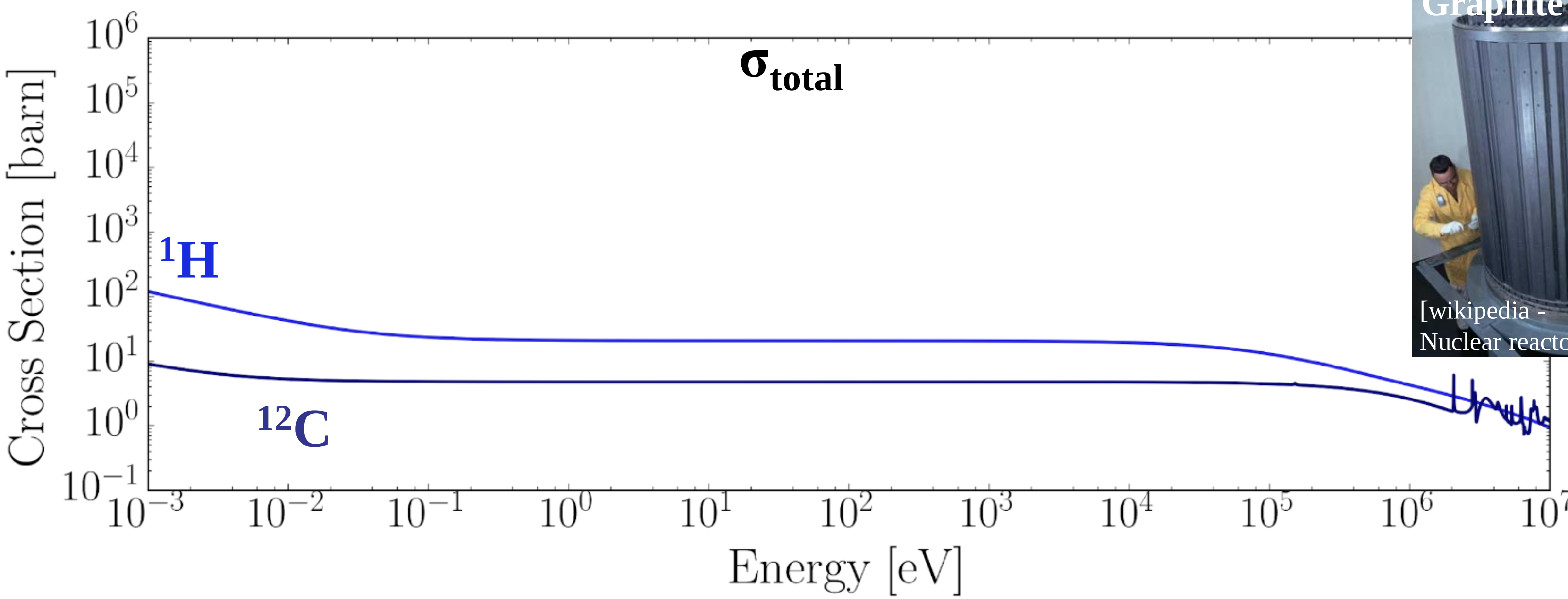
Light nuclides

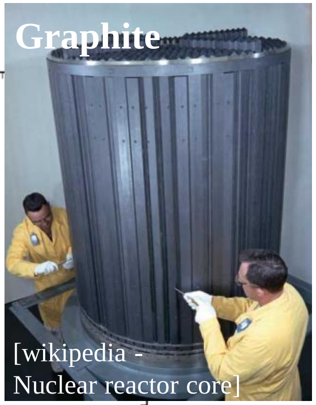


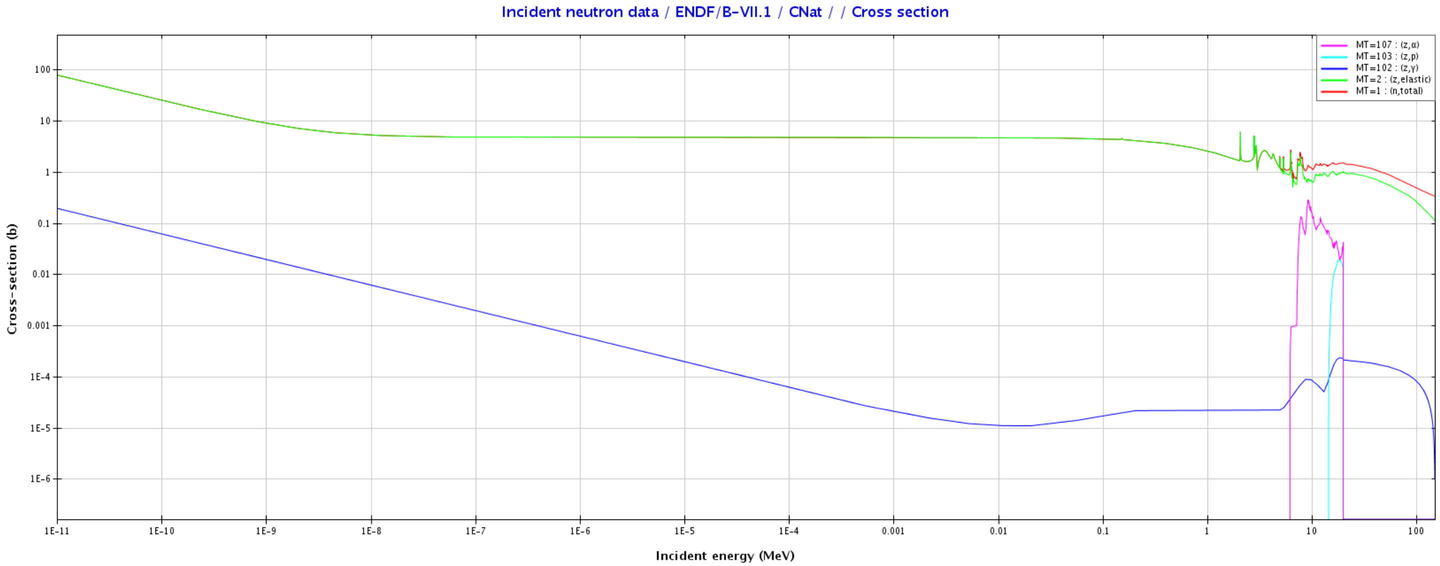




Light nuclides







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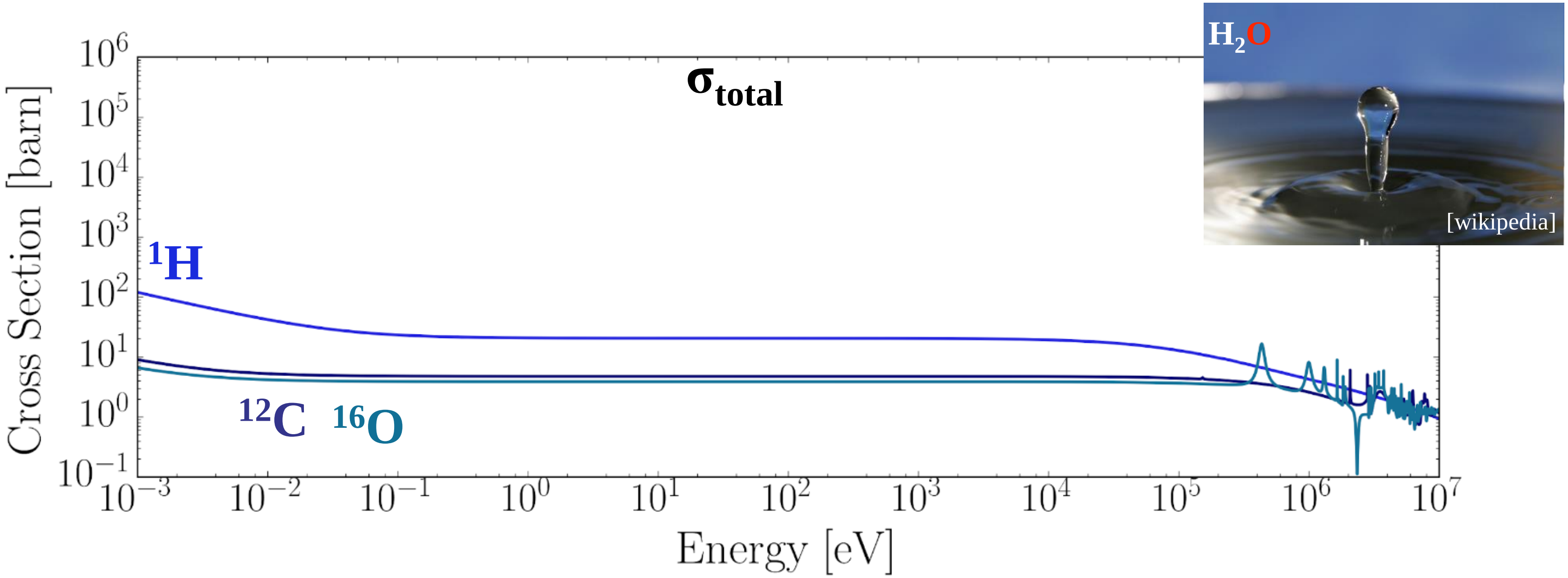

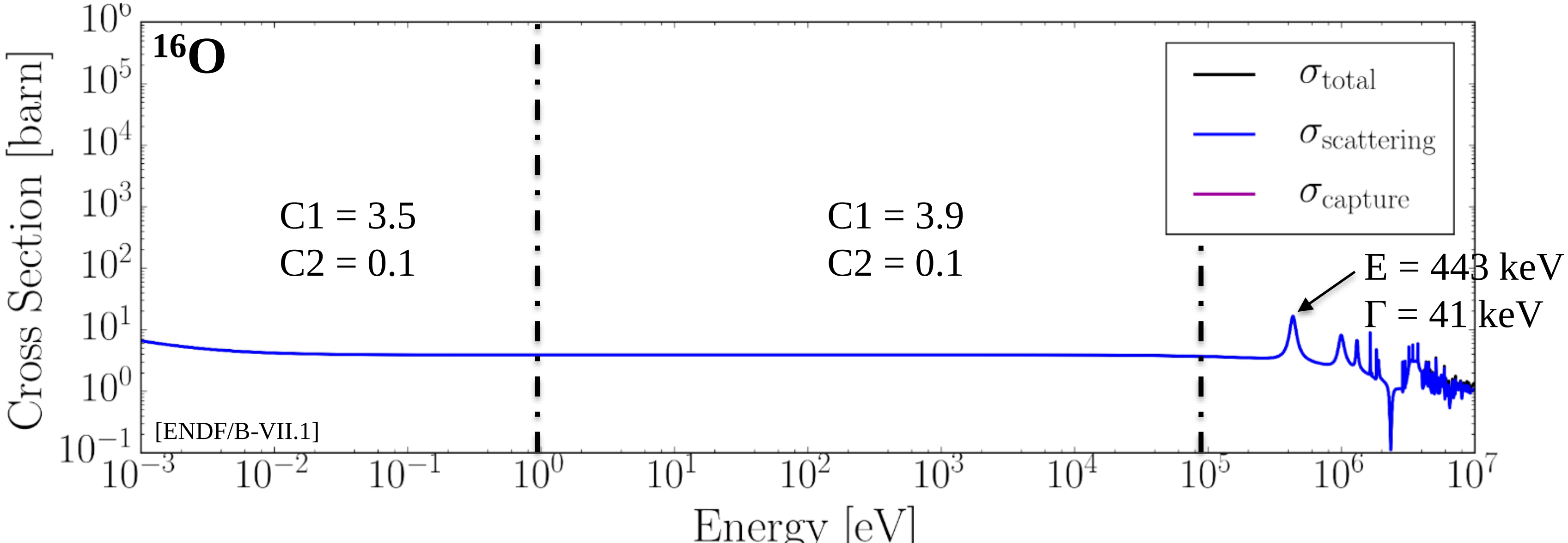

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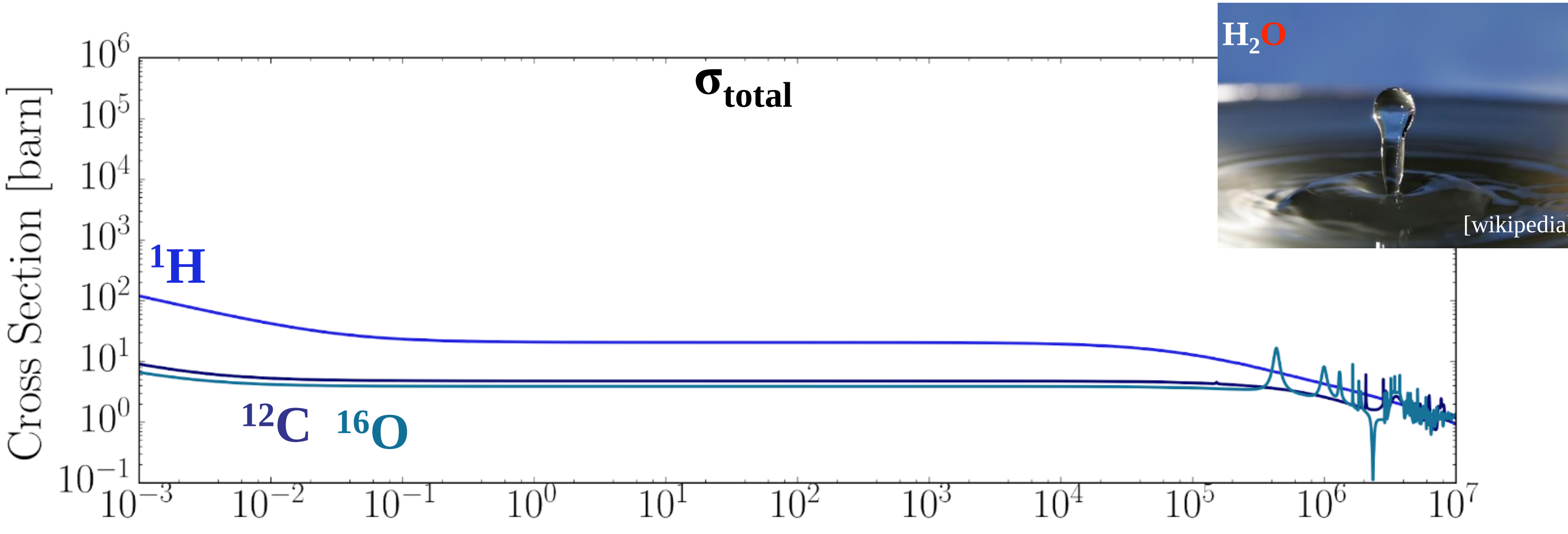
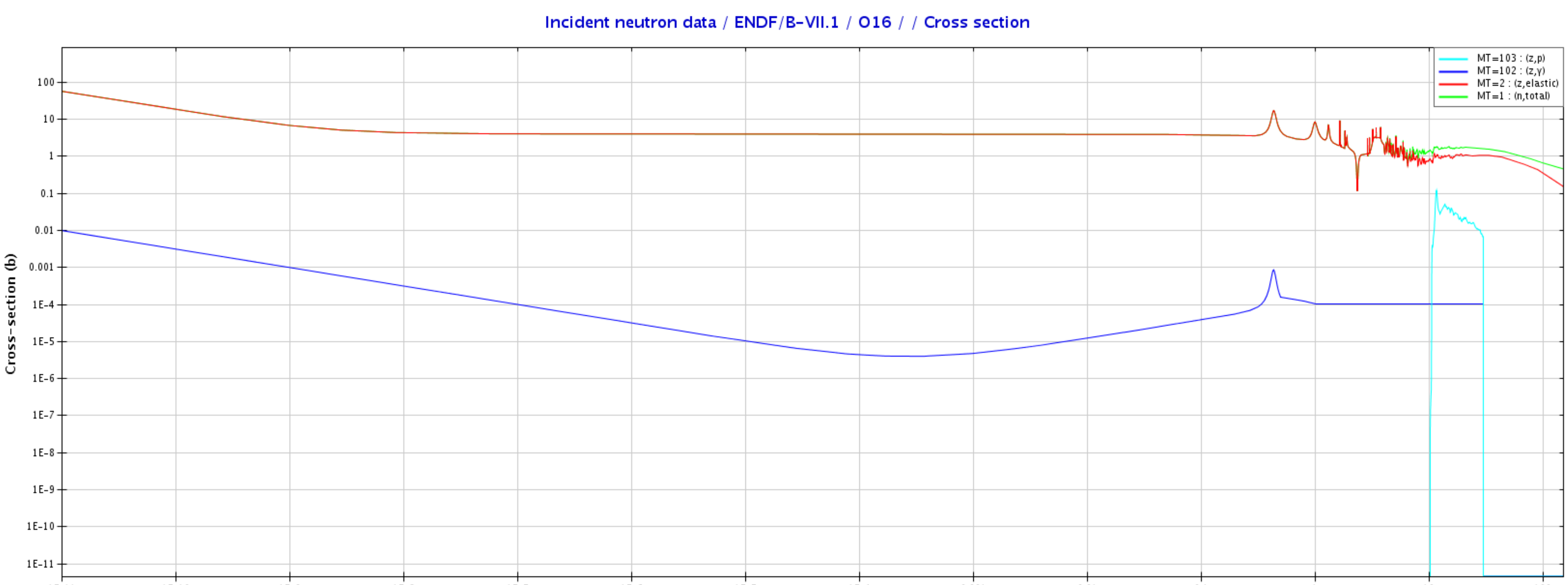
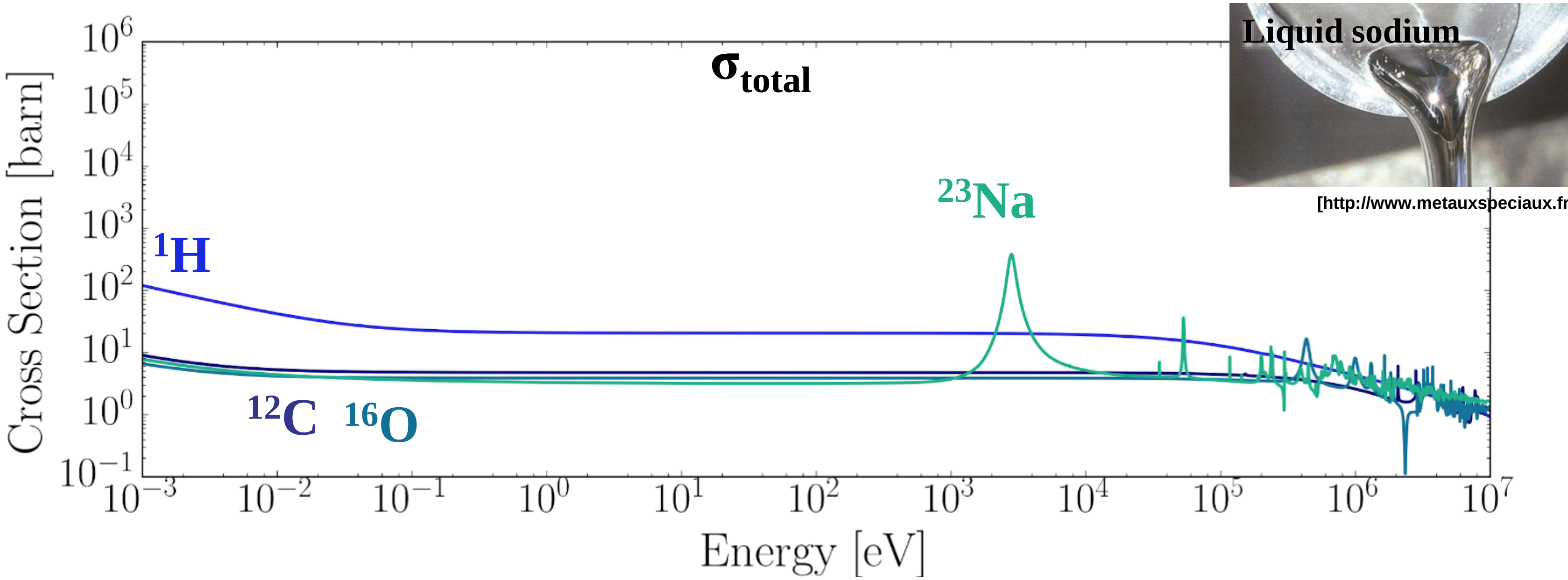
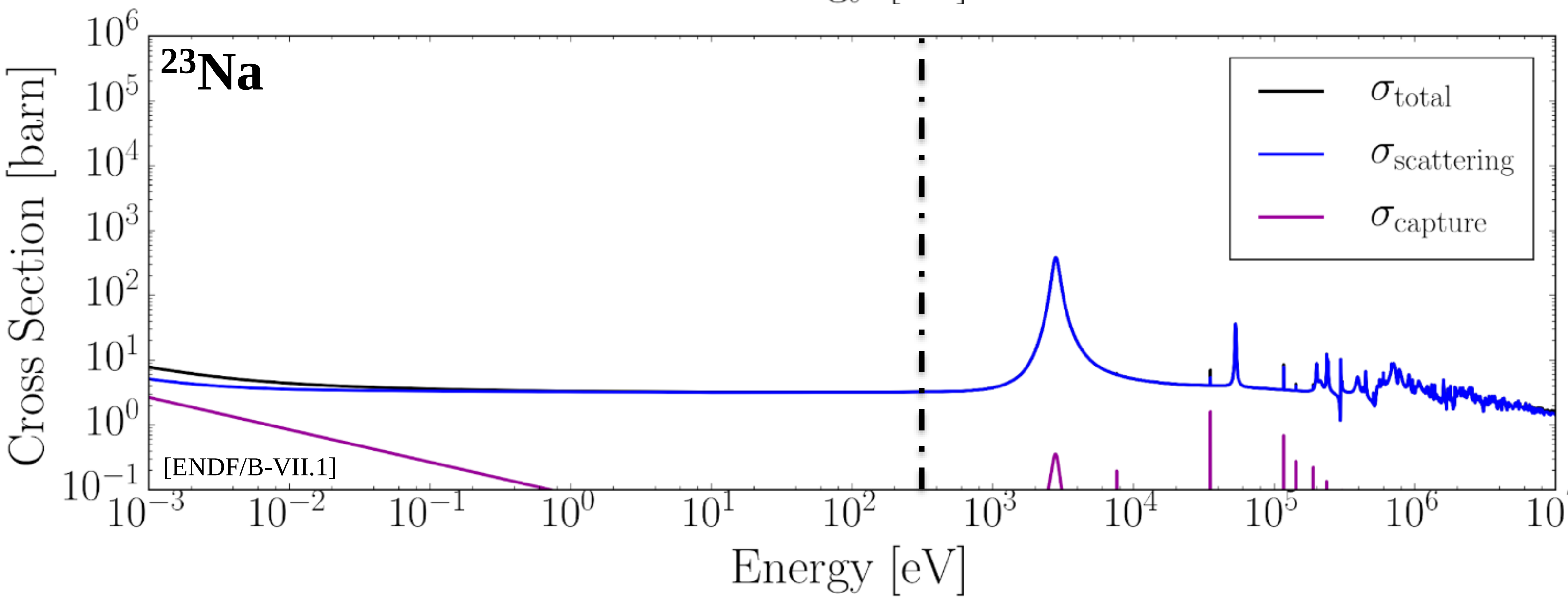




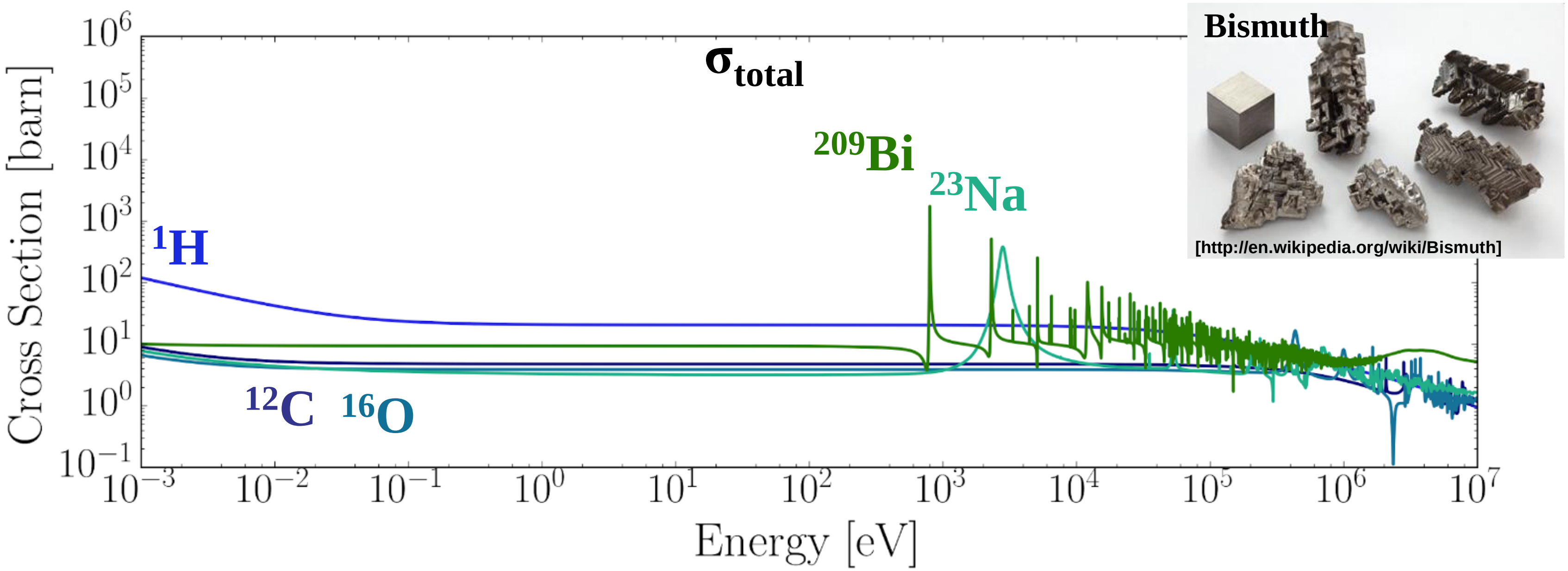
Light and magic nuclides

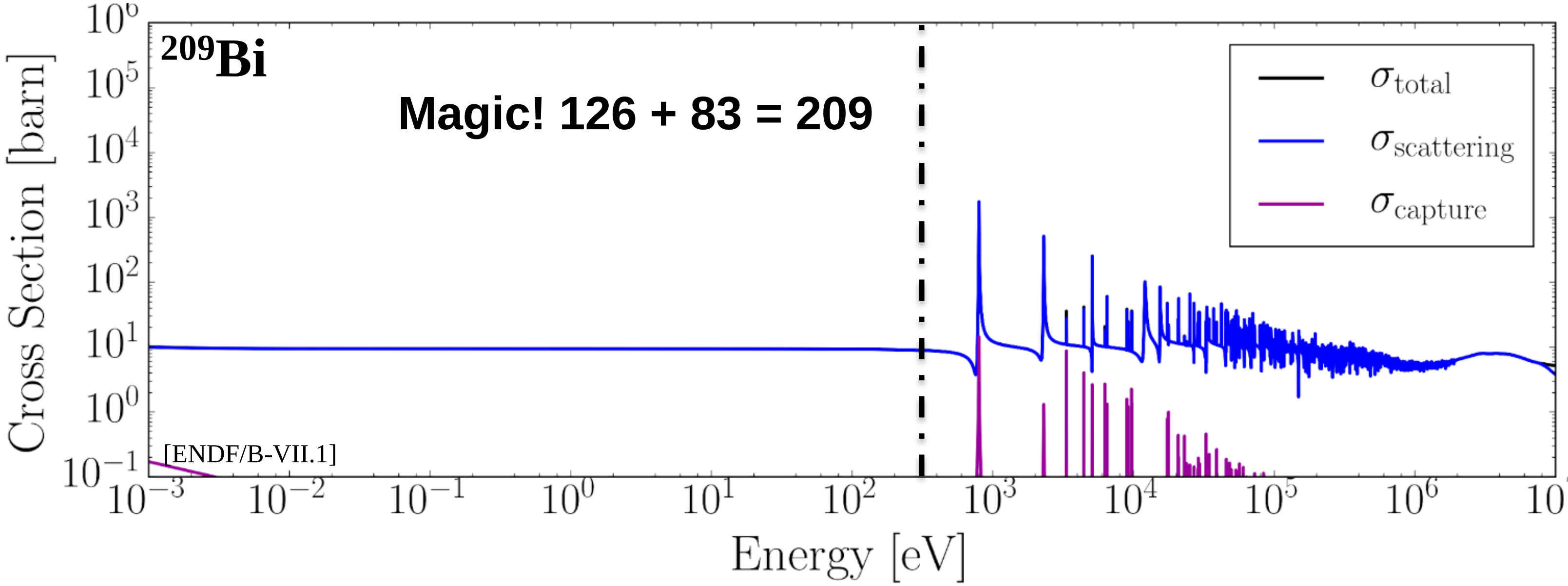


[http://www.metauxspeciaux.fr/]

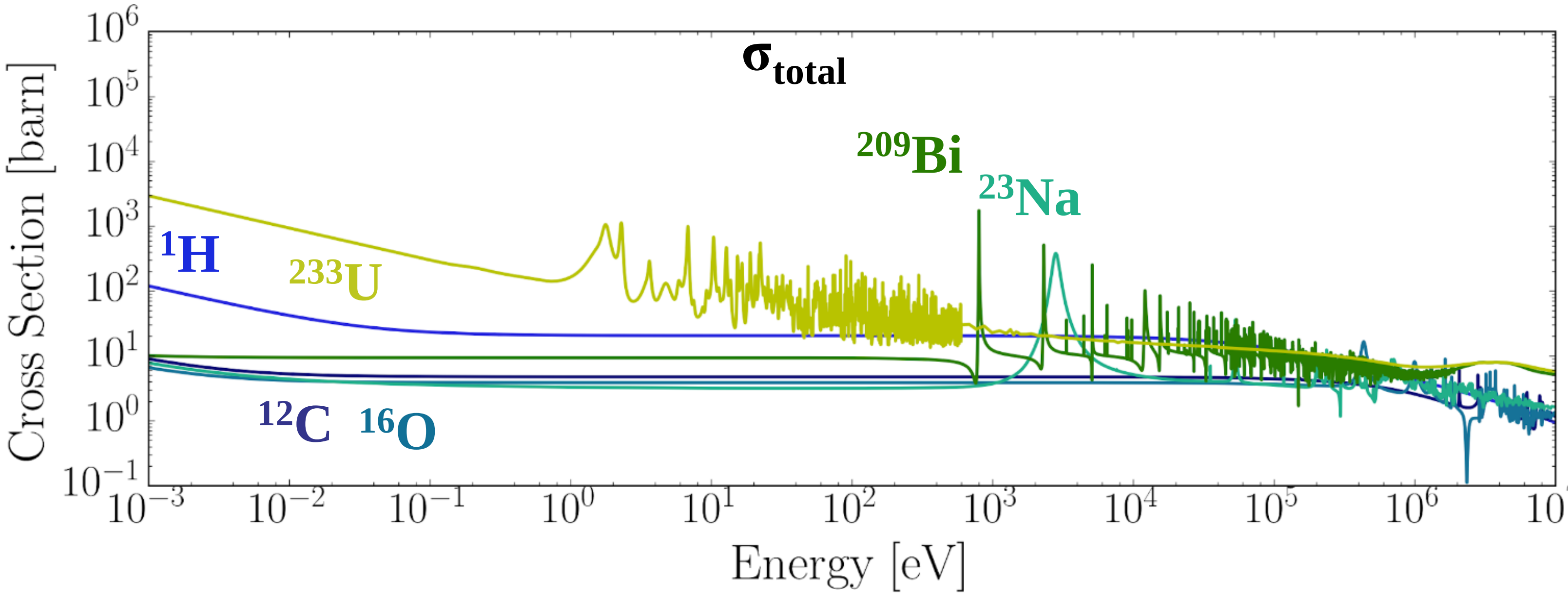


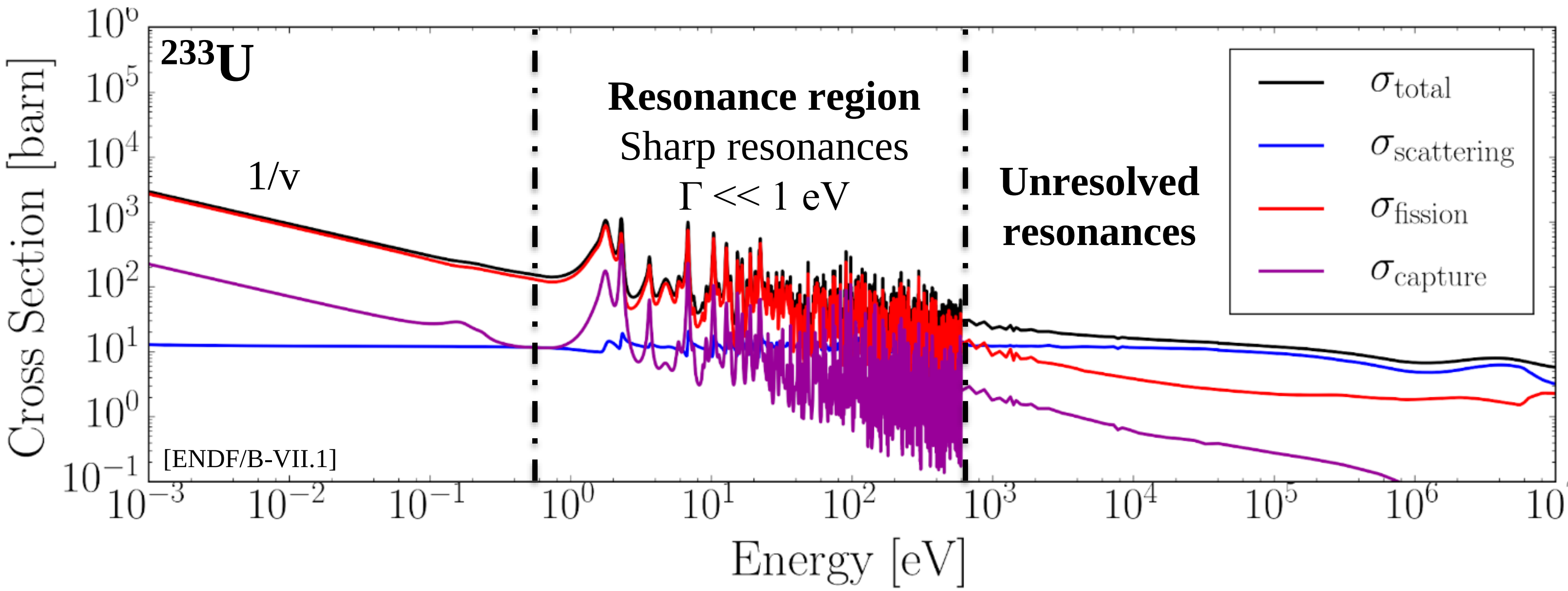
Light and magic nuclides





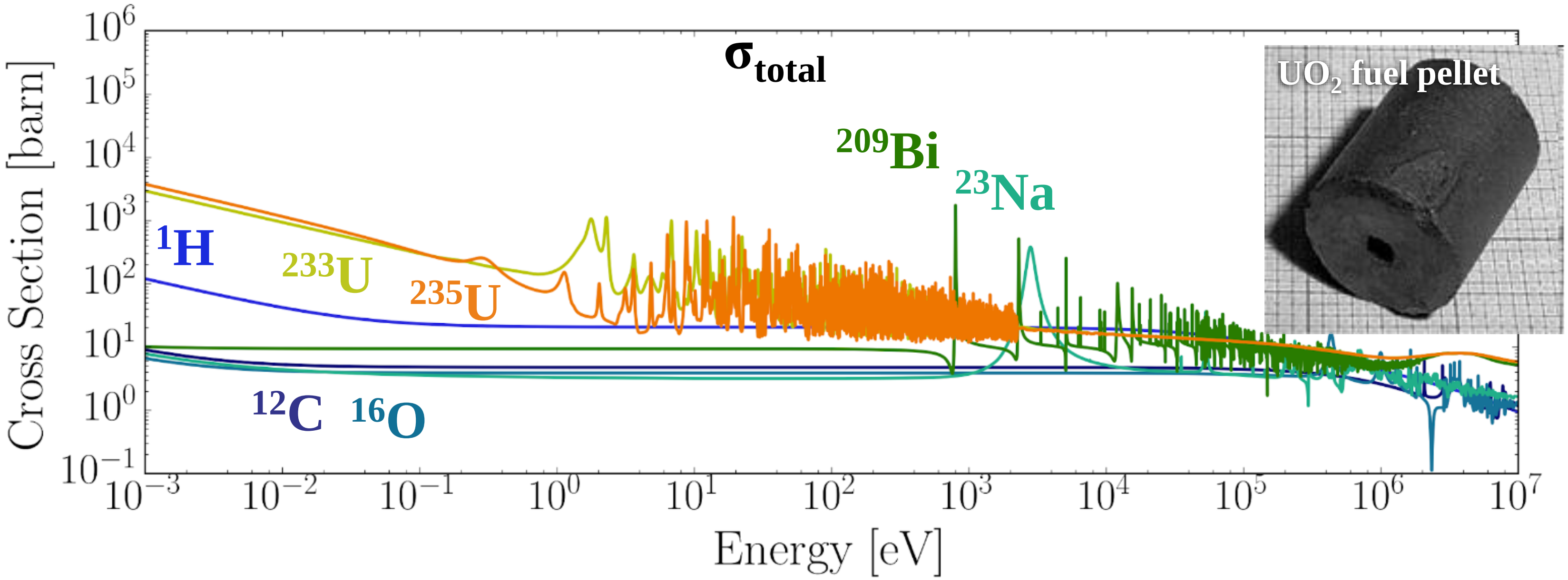
Heavy nuclides

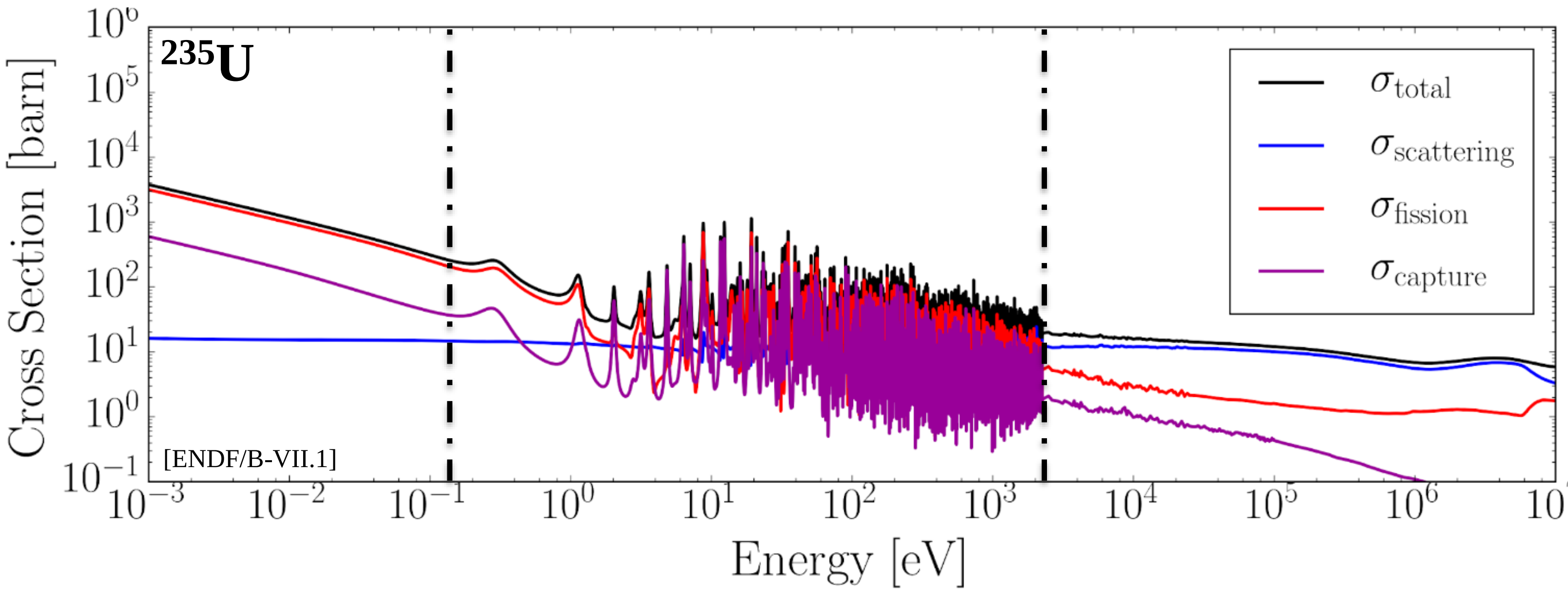




THE TOTAL CROSS SECTION

Heavy nuclides





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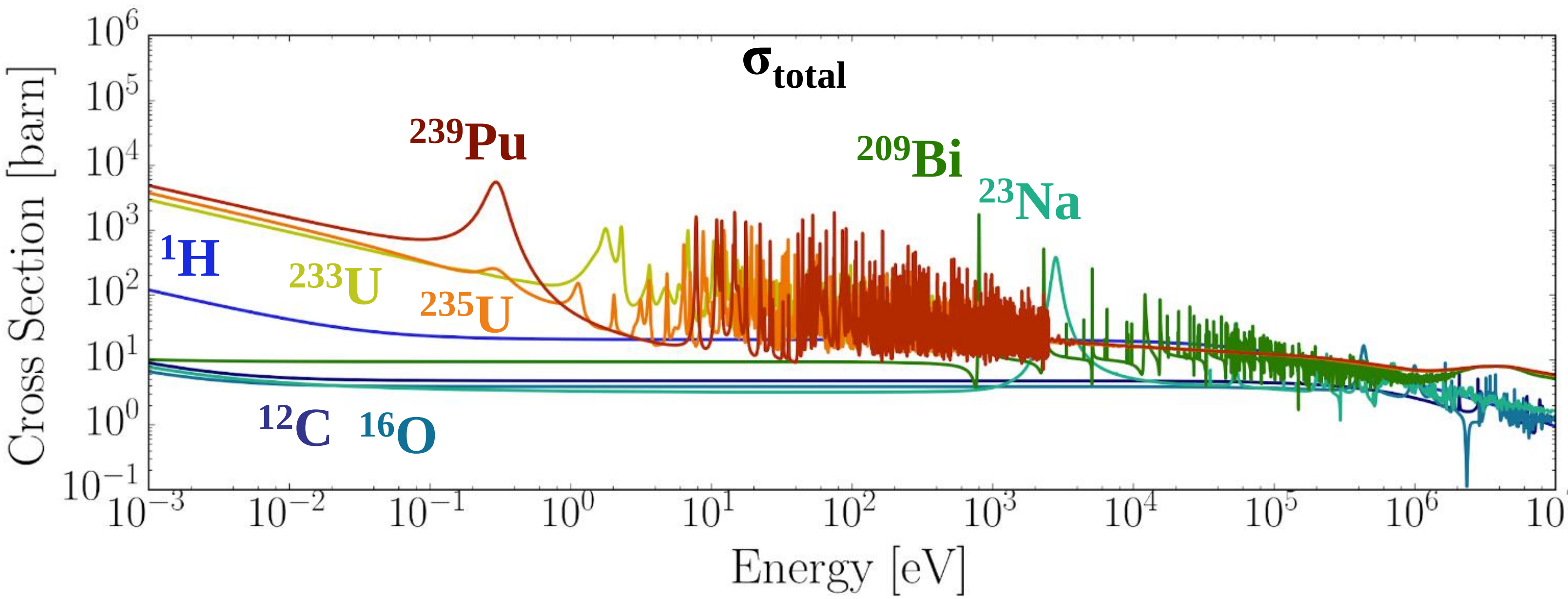

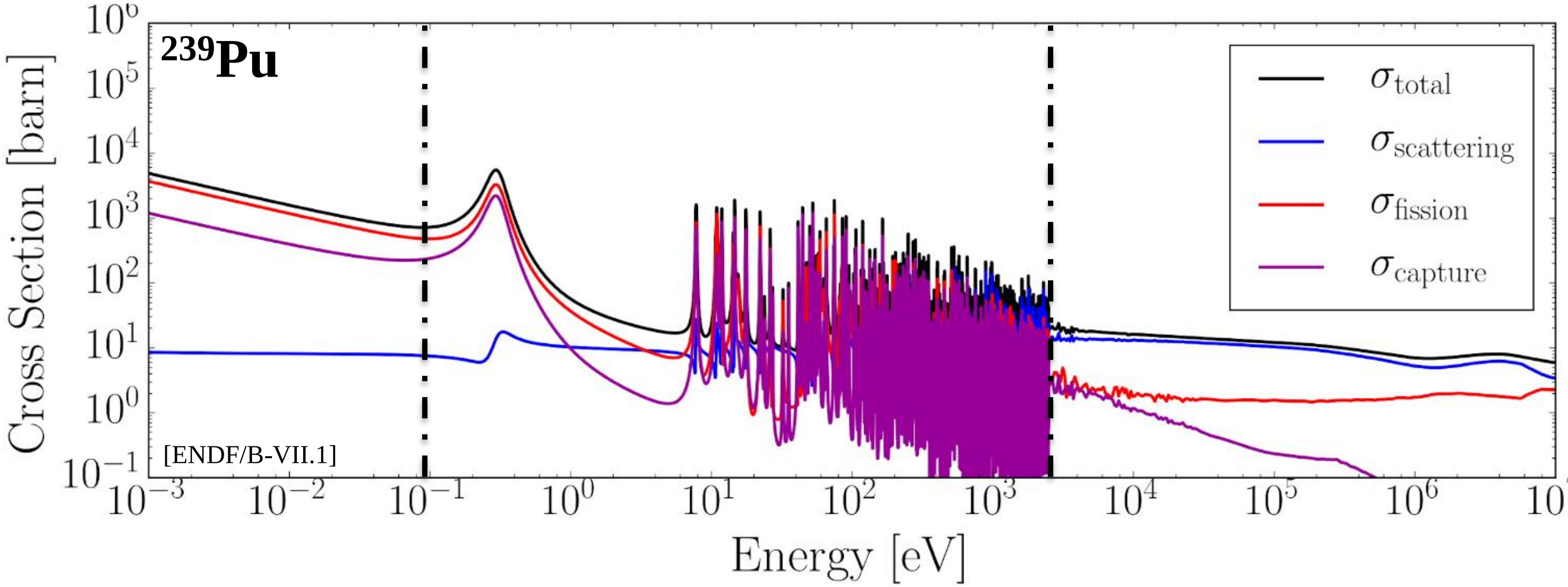

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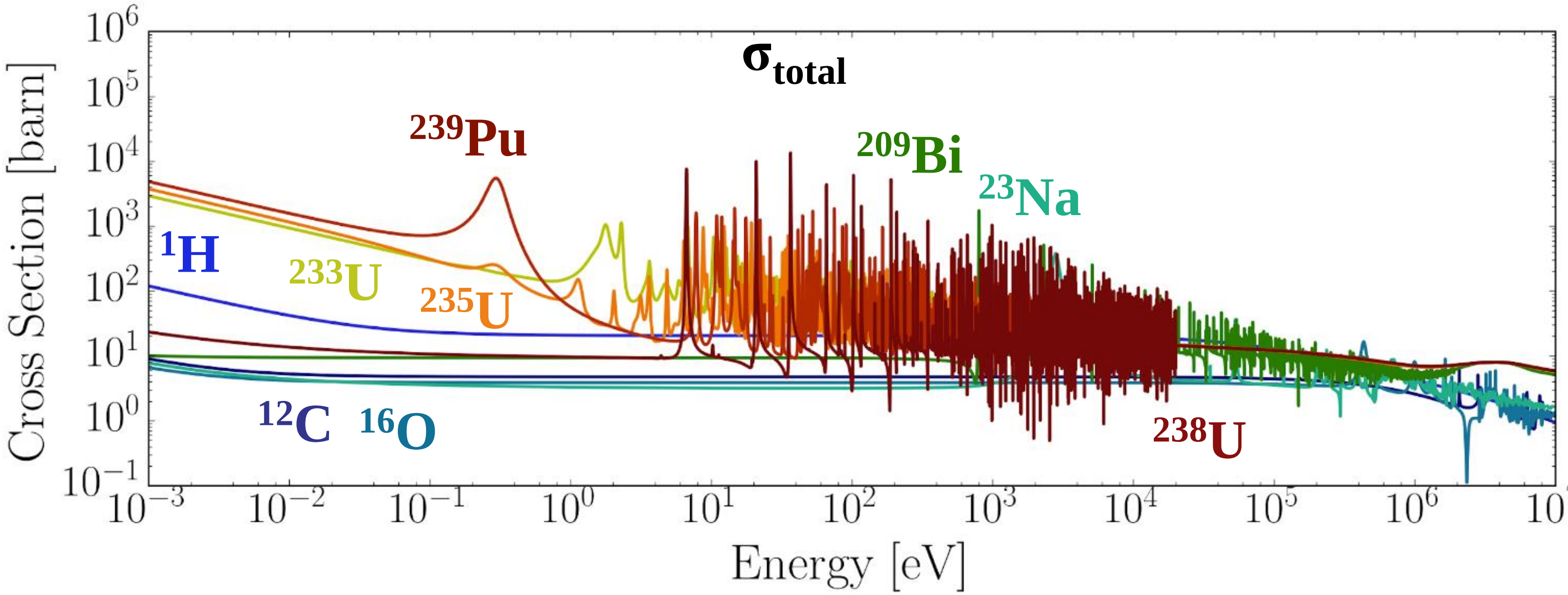

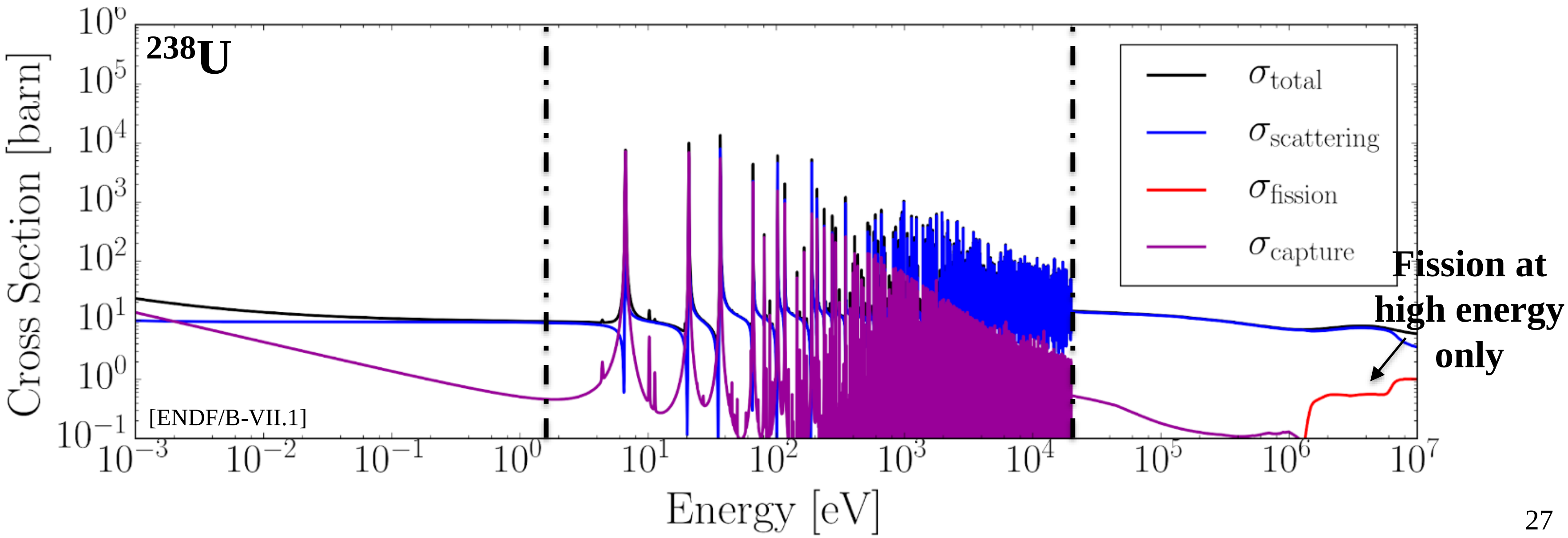

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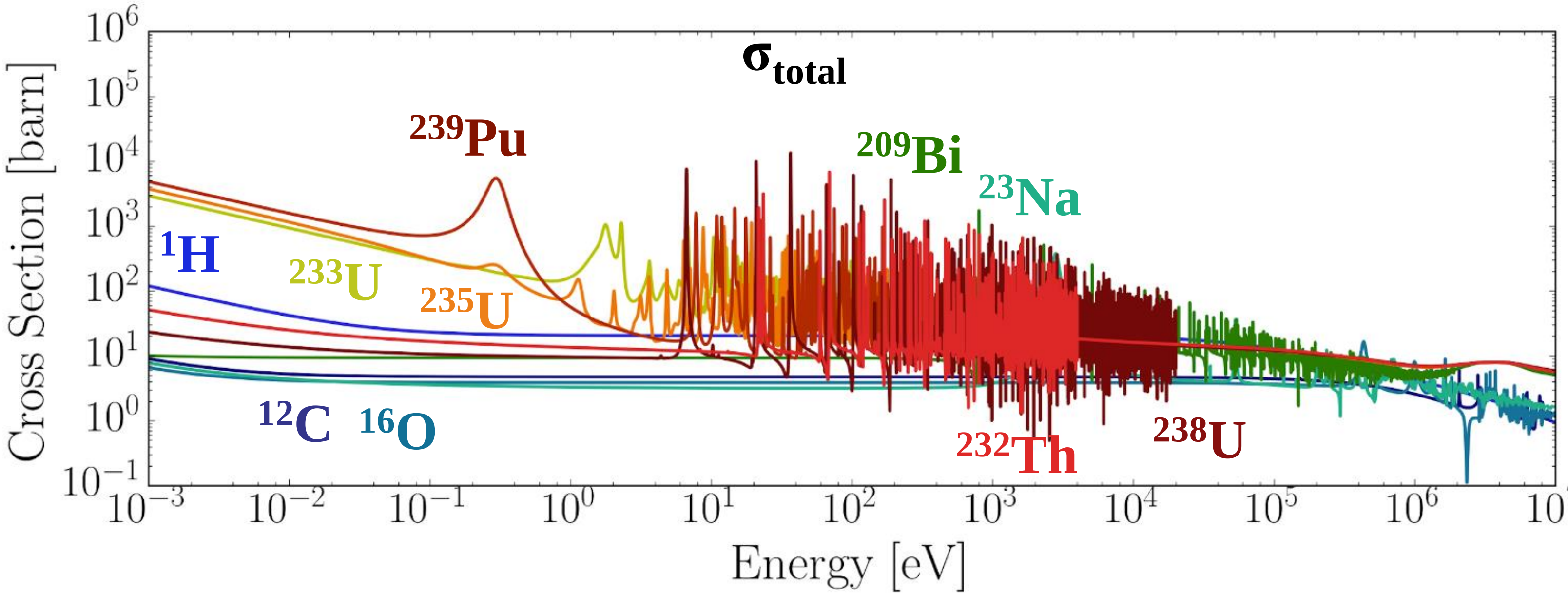
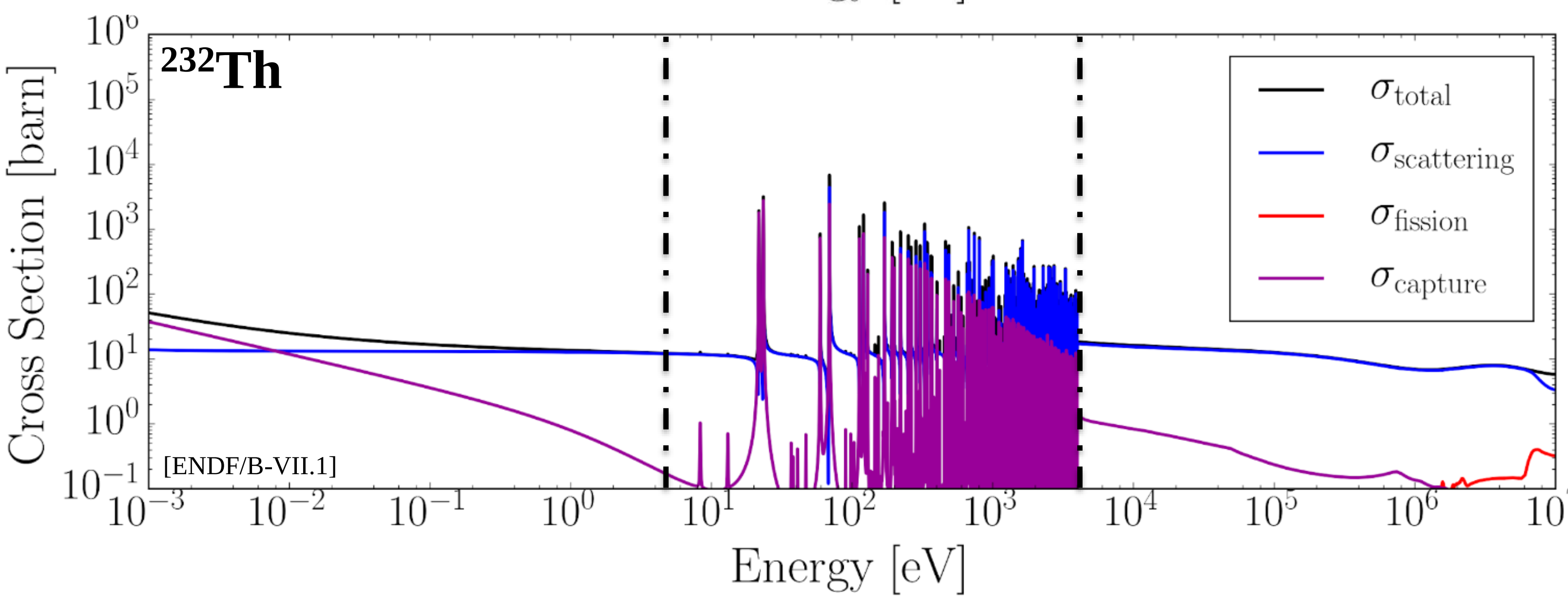
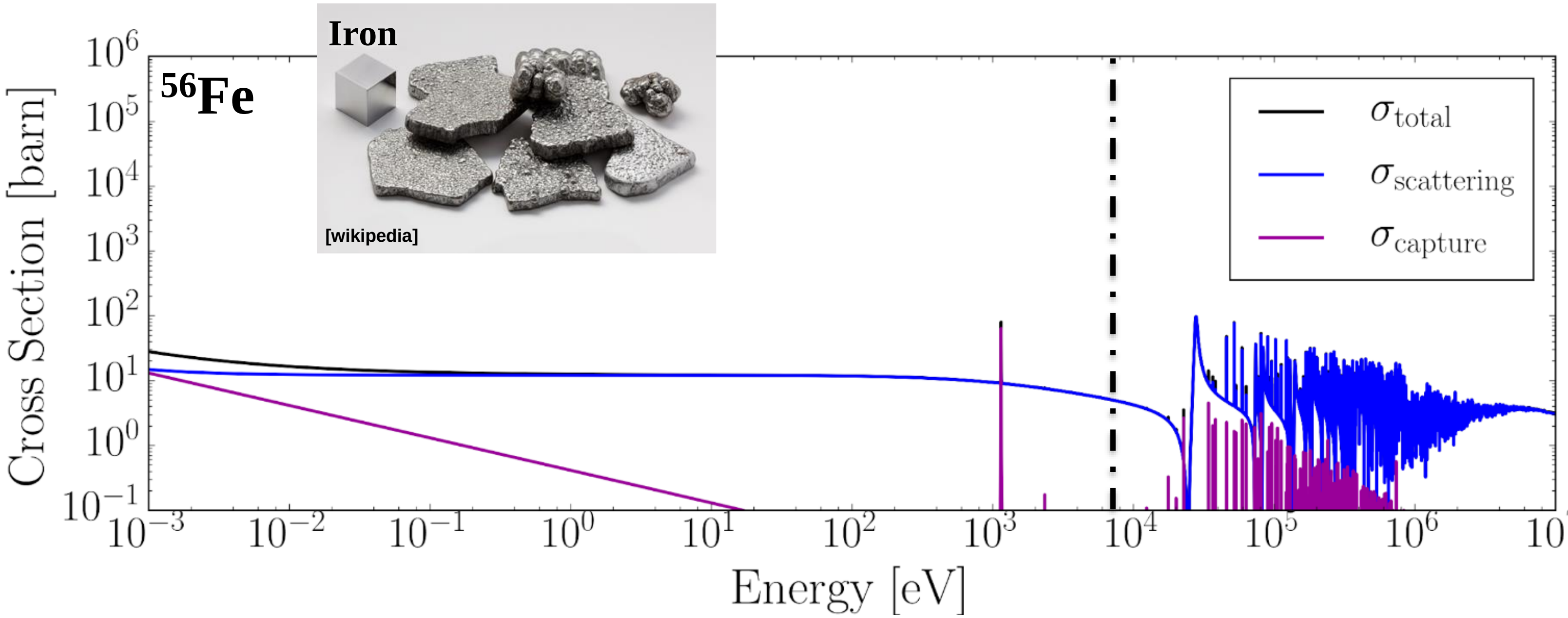
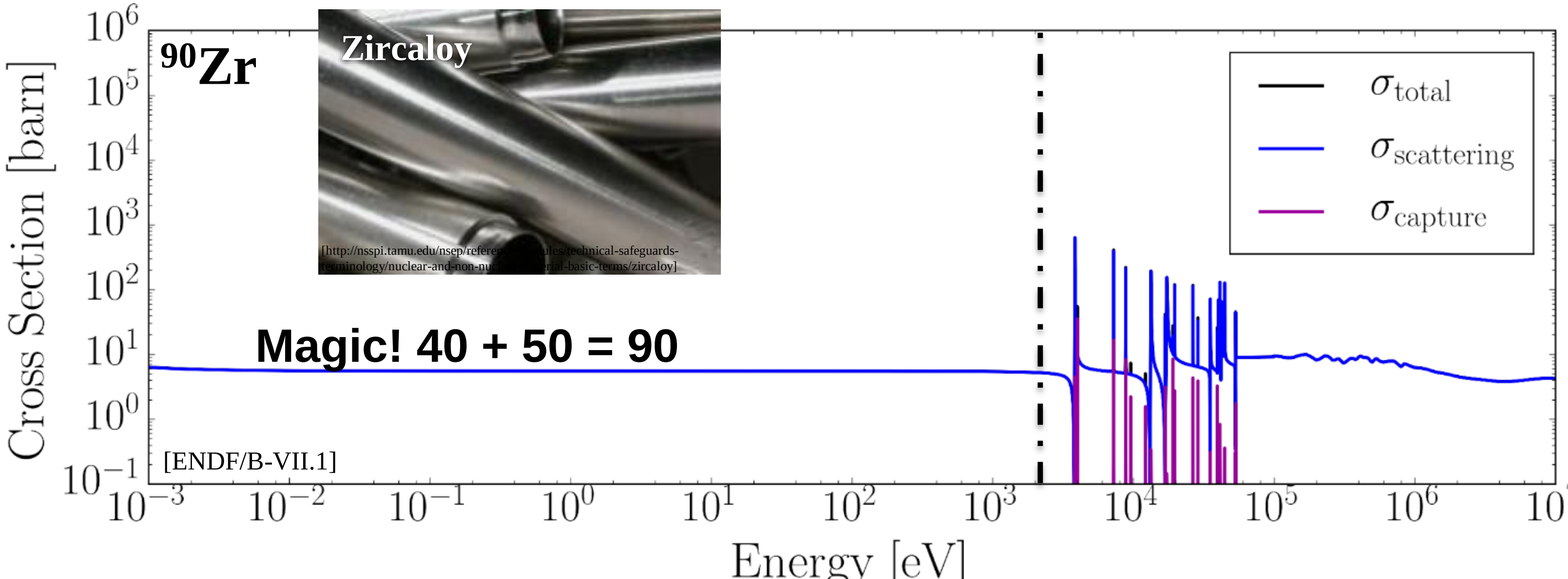


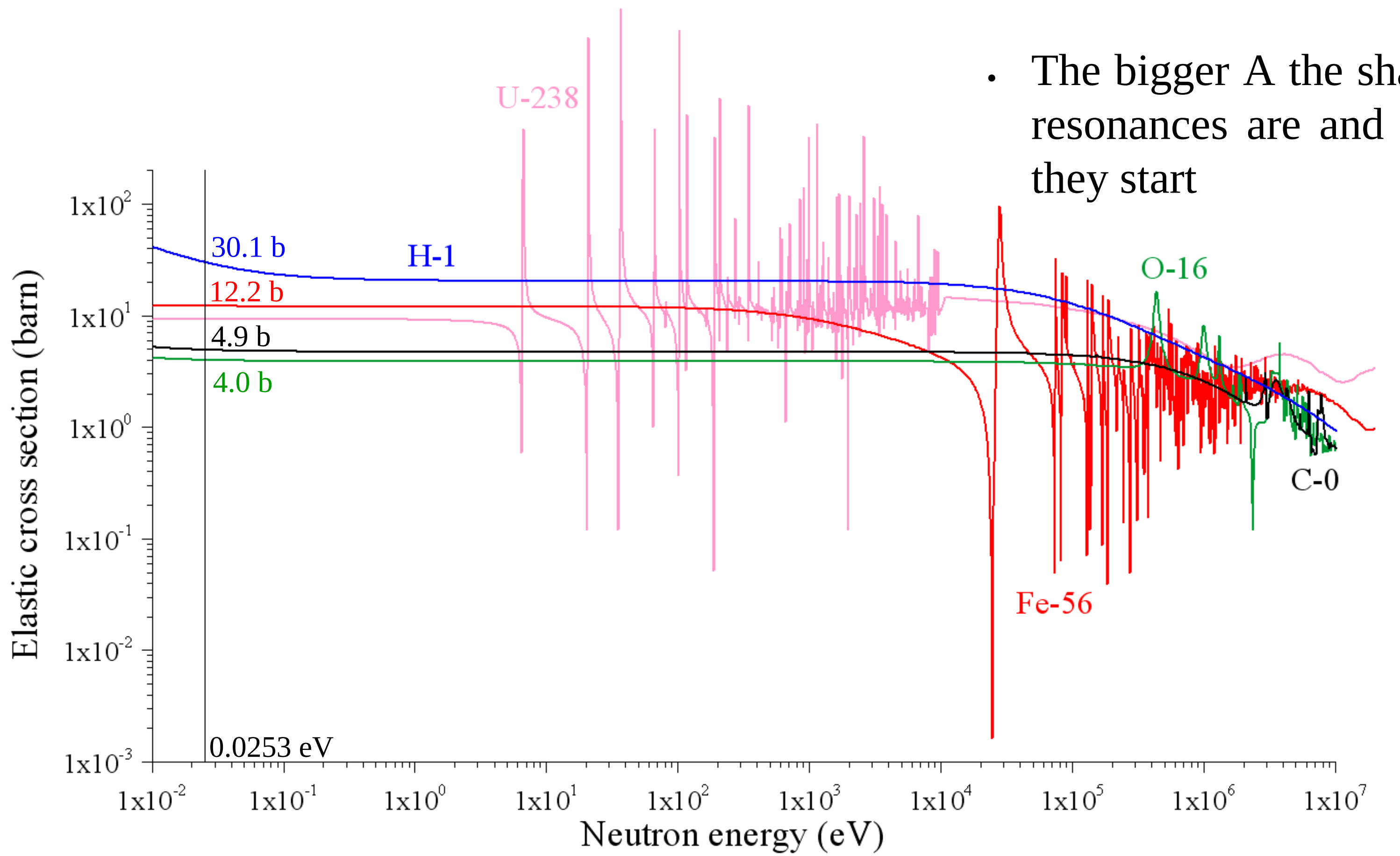


Intermediate nuclides





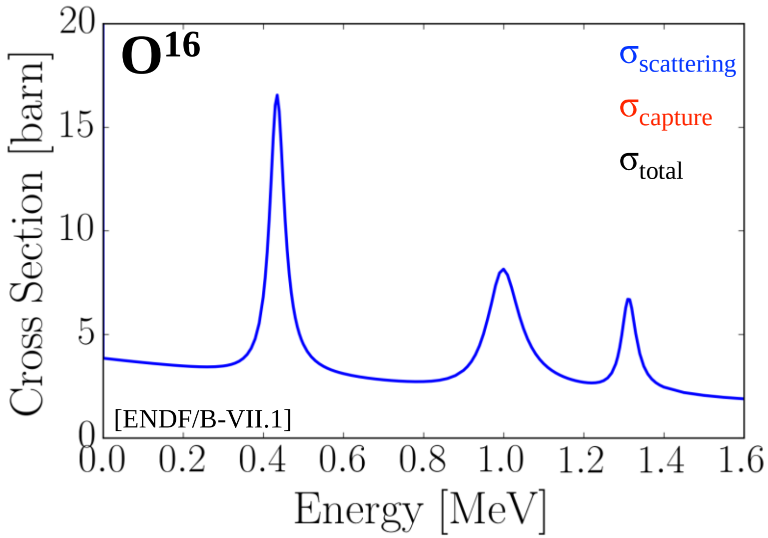
- Difficult to measure and analyze theoretically
- Low energy to ~ keV region: almost constant

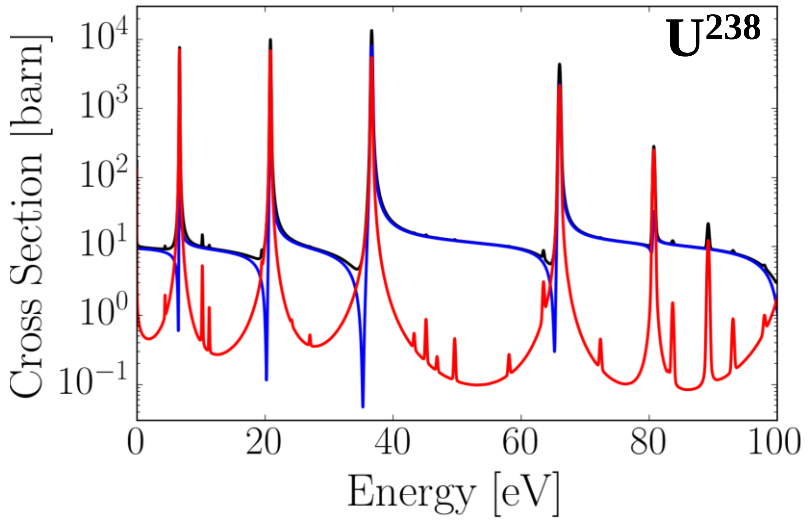


- High energy: reduction + resonances
- The bigger A the sharper and higher resonances are and at lower energy they start

Radiative capture

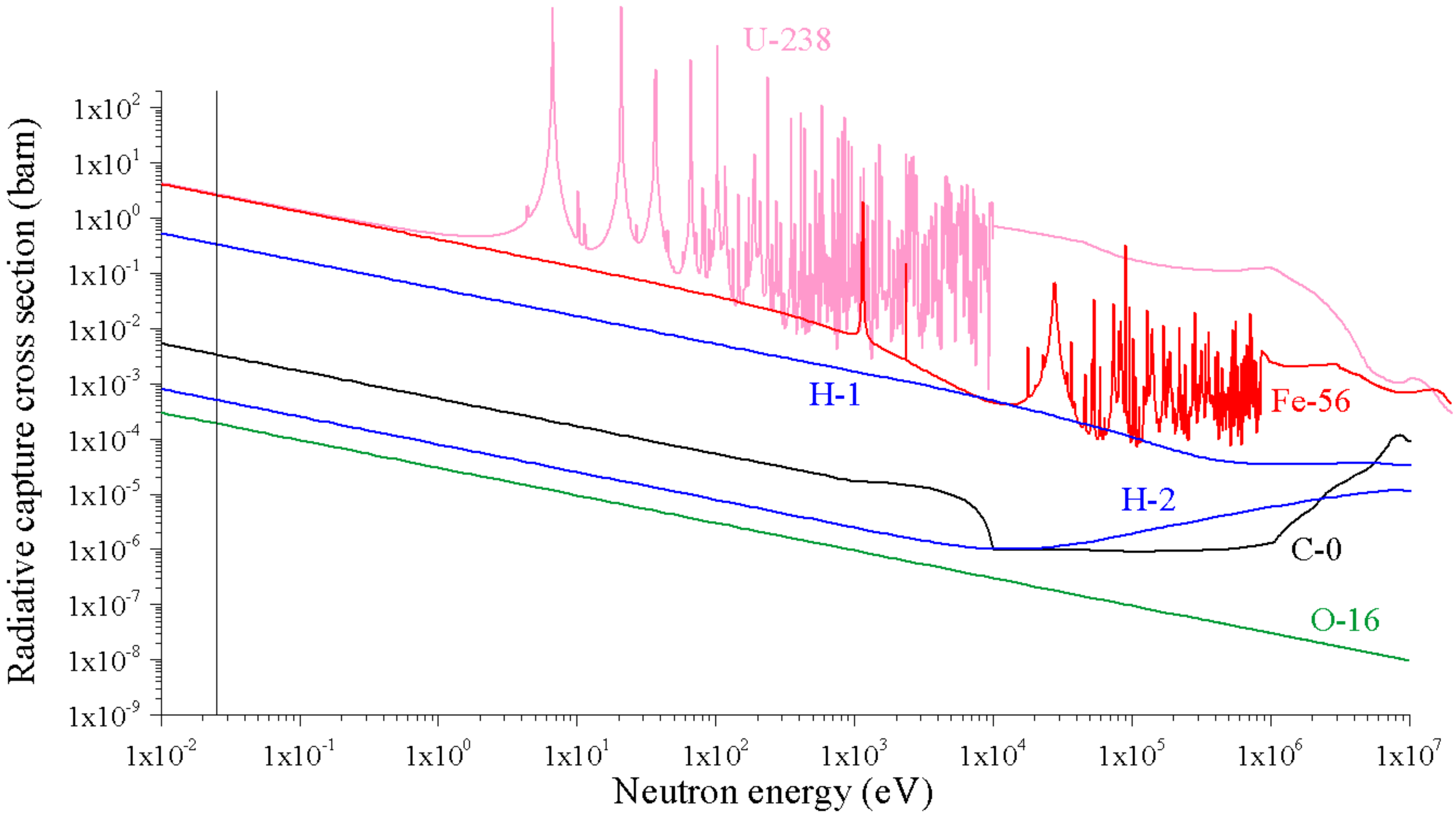
- Possible at all energies, but most probable at low energies
- For high A isotopes, capture and scattering resonances don't have the same shapes



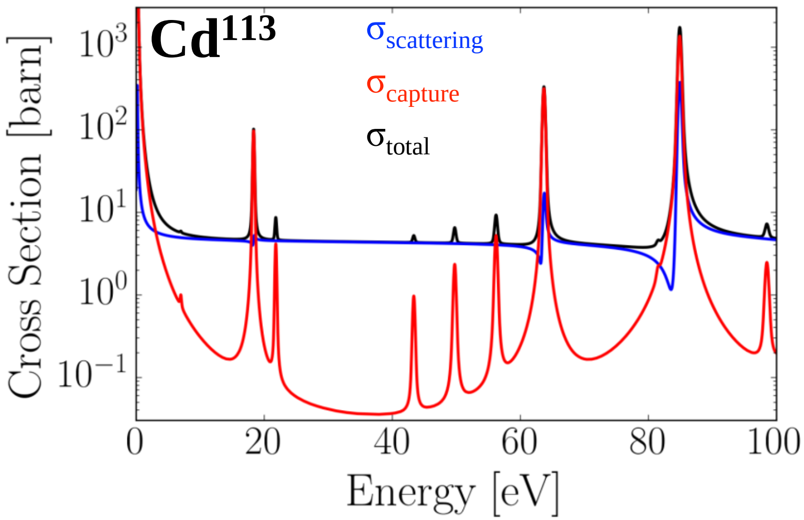


- Possible for all nuclei, but becomes increasingly important with increase of atomic number:
  - After a neutron absorption the excitation energy is above the virtual state (either nucleon or γ-ray can be emitted)
  - The excitation energy is divided between nucleons
  - The more nucleons the lower probability that one nucleon receives enough energy to leave the nucleus

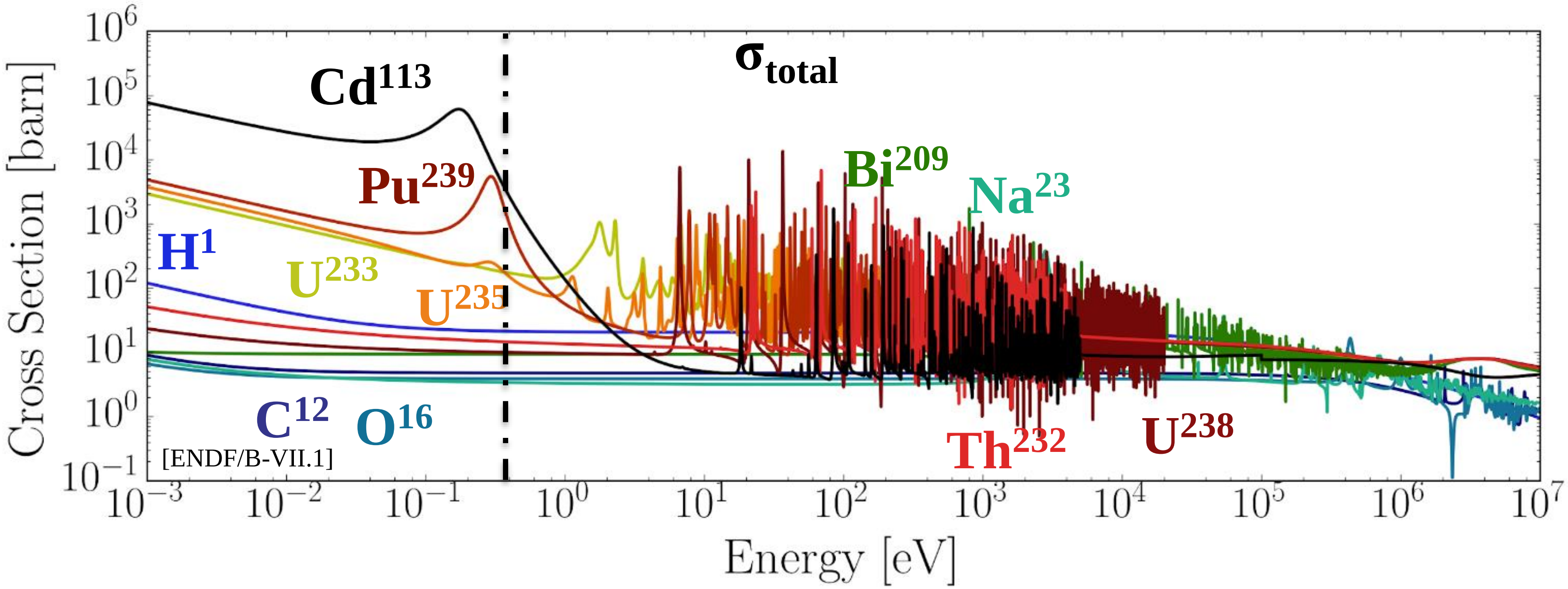
Radiative capture



Radiative capture - Cadmium



- (Very) high capture cross section at low energy
- Energy cut below 0.5 eV



Charged particle reactions

- Examples: (n,p), (n,α). Usually endothermic and threshold reactions with few exceptions.
- The most important exothermic charged particle reaction in nuclear reactor is:

$$B^{10}(n,\alpha)Li^{7}$$

B^{10} enriched B_4C is widely used as an absorber material in control and shutdown systems

- Another important reaction is:

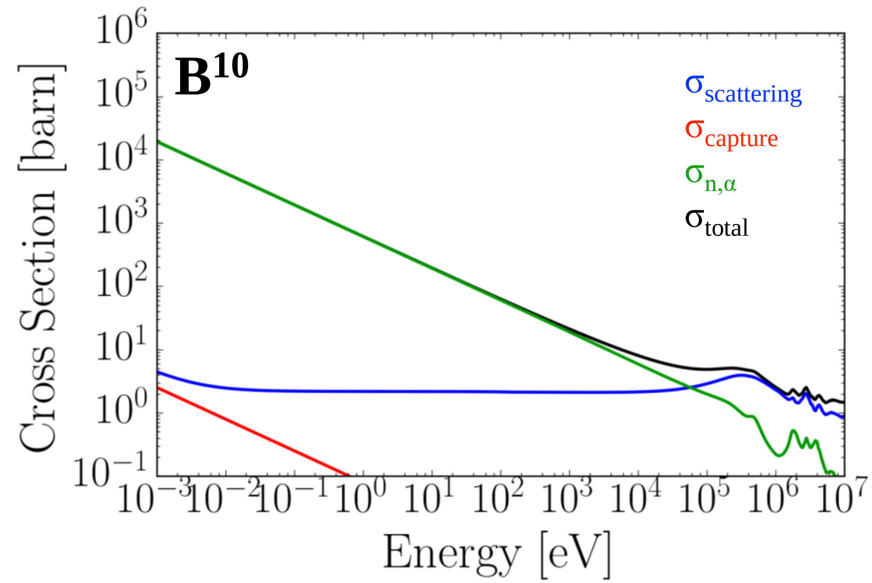
$$N^{14}(n,p)C^{14}$$

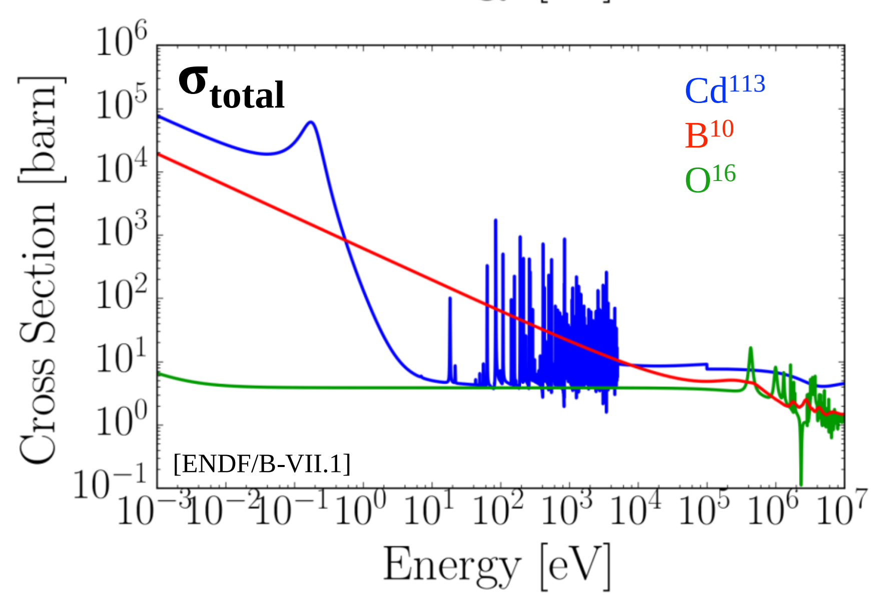
C^{14} ➞ long-lived potentially dangerous β^- emitter. Nuclear explosion: nitrogen in atmosphere.

- A source of water radioactivity :

$$O^{16}(n,p)N^{16}$$

N^{16} ➞ β^- decay with emission of 6-7 MeV γ-rays.



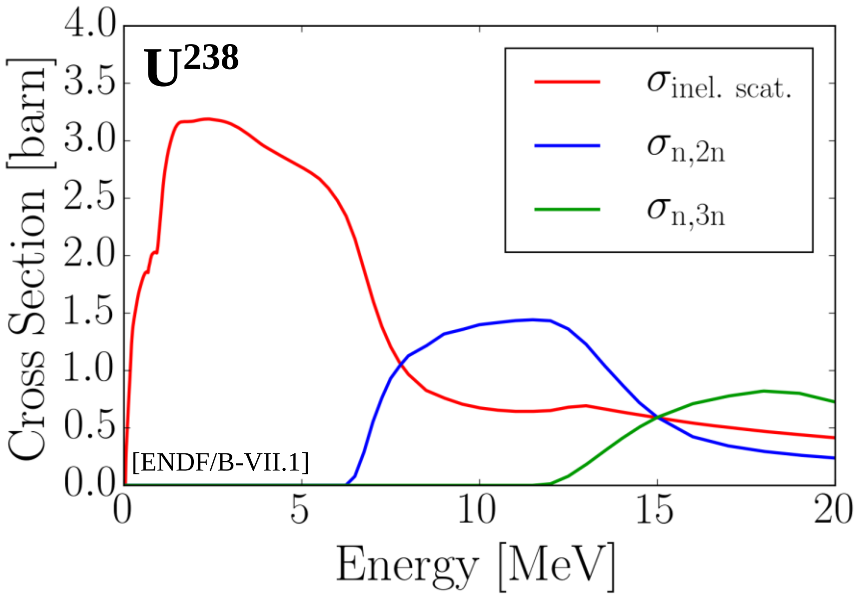


(n,2n) and (n,3n) reactions

- In an inelastic scattering most of the initial kinetic energy of an incident neutron remains in the residual nucleus (after emission of an inelastic neutron) and is released as γ-rays.

- When the initial energy becomes high enough, the emission of the inelastic neutron can be followed by the **emission of another neutron** (instead of a γ-ray).

- Most nuclei have an (n,2n) threshold in the range of 7-10 MeV. An important exception is ^{9}Be: 1.8 MeV.



(γ,n) reaction

- Energetic γ-rays are produced in a nuclear reactor as a result of:
  - fission;
  - neutron-nucleus interactions (radiative capture, inelastic scattering, etc.);
  - radioactive decay of fission products.

- An absorption of the most energetic γ-rays can result in the excited states of a nucleus which can decay with emission of a neutron: (γ,n) reaction

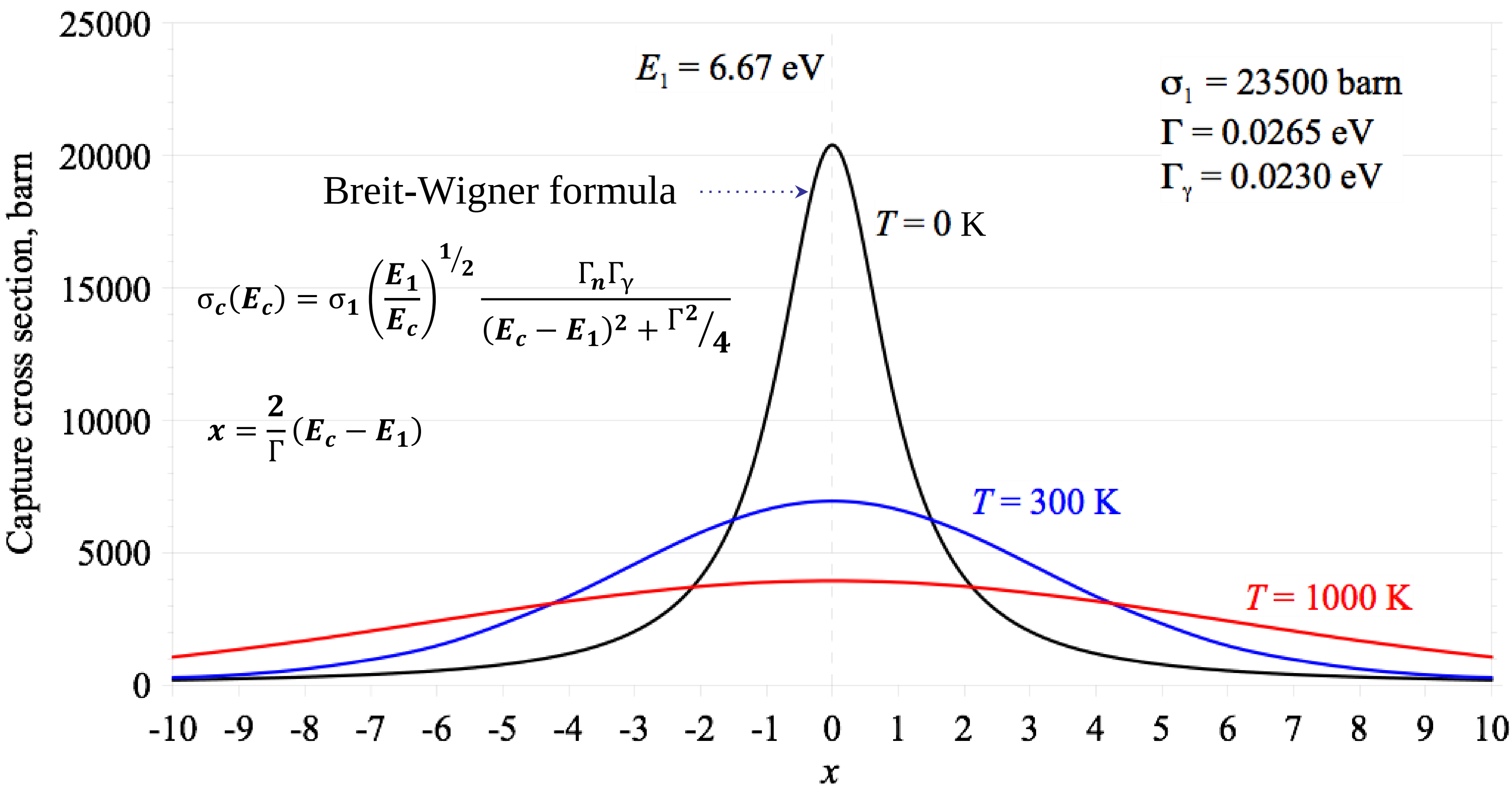
- The thresholds of (n,2n) and (γ,n) reactions are identical

- Unlike the (n,2n) reaction, (γ,n) reaction continues after the reactor shutdown.

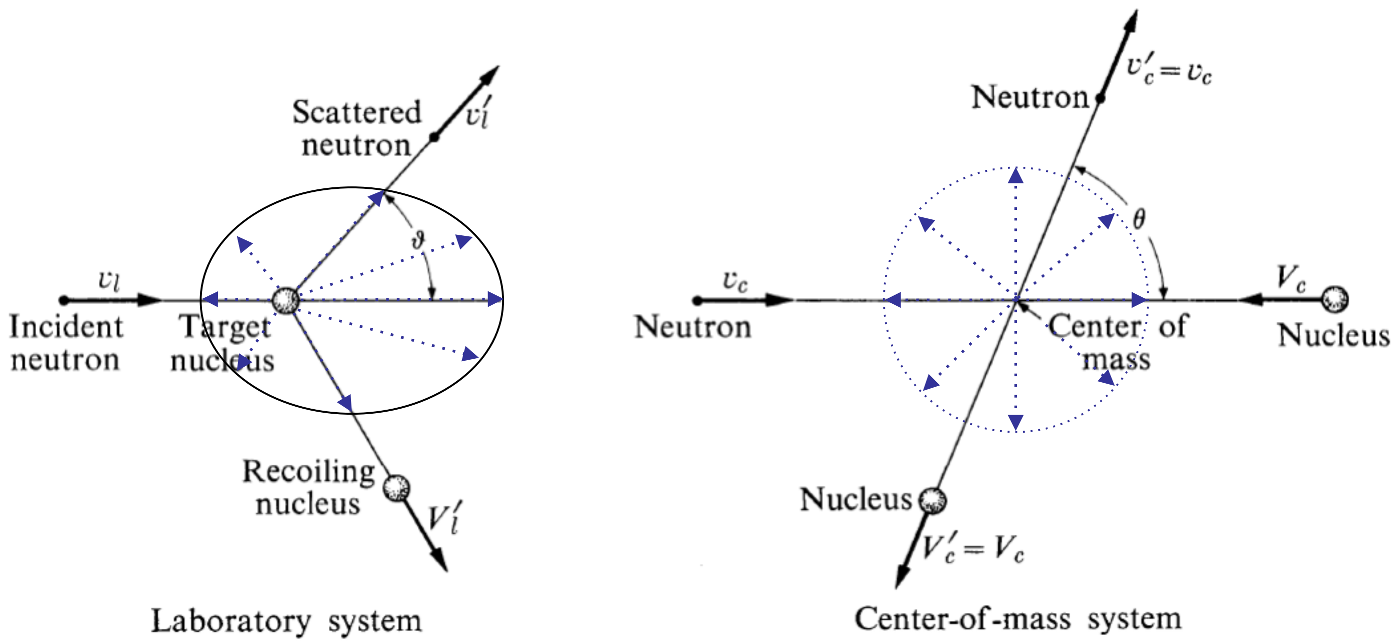
The Doppler effect.

- Up to now : before a neutron-nucleus interaction nucleus was supposed at rest in the laboratory (L) system.
- However, atoms are in continual motion due to their thermal energy.
- The monoenergetic (in L-system) beam of neutrons appears to have a smear of energies in center-of-mass (C) system, because of the thermal motion of the target nuclei.
- By analogy to similar phenomena in acoustics and optics this effect is known as *nuclear Doppler effect.*

Christian Doppler (1803 – 1853)



- Increase of the temperature results in widening of resonances (radiative capture, scattering, fission, …)
  → The neutron behaviour depends on the material temperature!



Elastic scattering of neutron by nucleus, as observed in laboratory and center-of-mass coordinates.

EPFL

To take into account the effect of scattering angular distribution on the neutron motion, a concept of the *transport cross section* is used. Consider scattering but non-absorbing medium.

- Before the first collision: $\overline{x}_0 = \lambda_s$
- After the first collision: $\overline{x}_1 = \overline{\lambda_s \cos\theta_1} = \lambda_s \overline{\mu}$
  projection to the axis of the original motion
- After the second collision: $\overline{x}_2 = \overline{\lambda_s \cos\alpha} = \overline{\lambda_s \cos\theta_1 \cos\theta_2} = \lambda_s \overline{\mu}^2$
  projection to the axis of the original motion
- After the nth collision: $\left.\begin{array}{l}\overline{x}_n = \lambda_s \overline{\mu}^n \\ \overline{\mu} < 1\end{array}\right\} \Longrightarrow \lim_{n\to\infty} \overline{x}_n = 0$

average value of the cosine of the scattering angle

One can define the *Transport mean free path:*

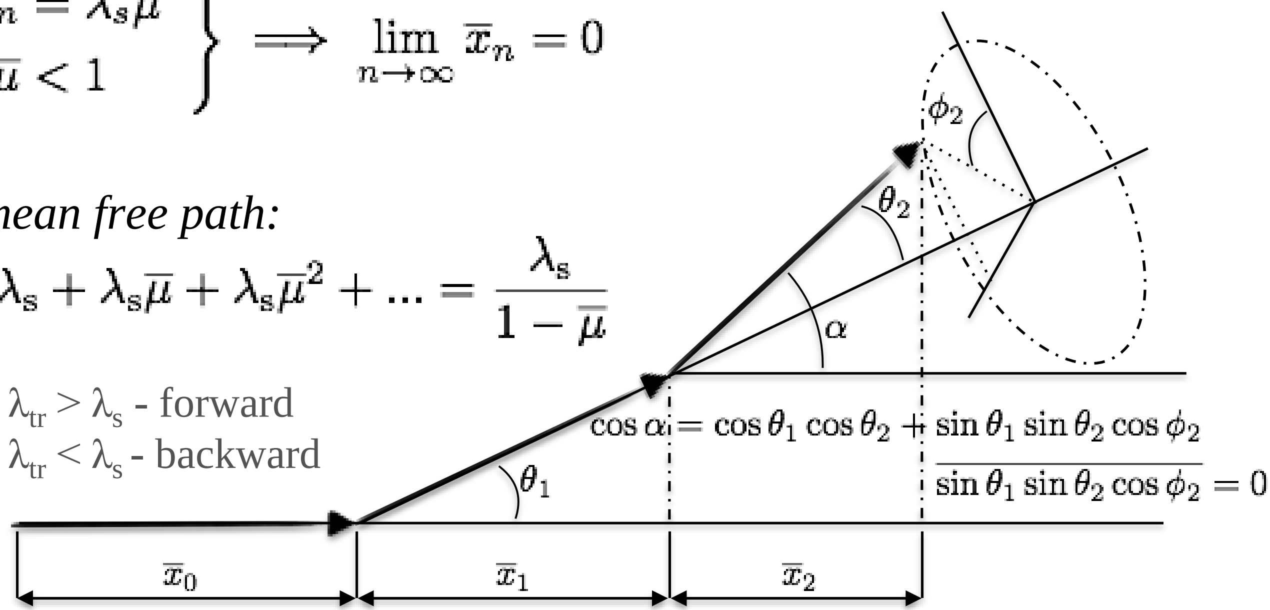
$$\lambda_{\mathrm{tr}} = \overline{x}_0 + \overline{x}_1 + \overline{x}_2 + \ldots = \lambda_{\mathrm{s}} + \lambda_{\mathrm{s}}\overline{\mu} + \lambda_{\mathrm{s}}\overline{\mu}^2 + \ldots = \frac{\lambda_{\mathrm{s}}}{1-\overline{\mu}}$$

Transport cross section:

$$\lambda_{\mathrm{tr}} = \frac{1}{N\sigma_{\mathrm{tr}}}$$

$$\sigma_{\mathrm{tr}} = \sigma_{\mathrm{s}}(1-\overline{\mu})$$

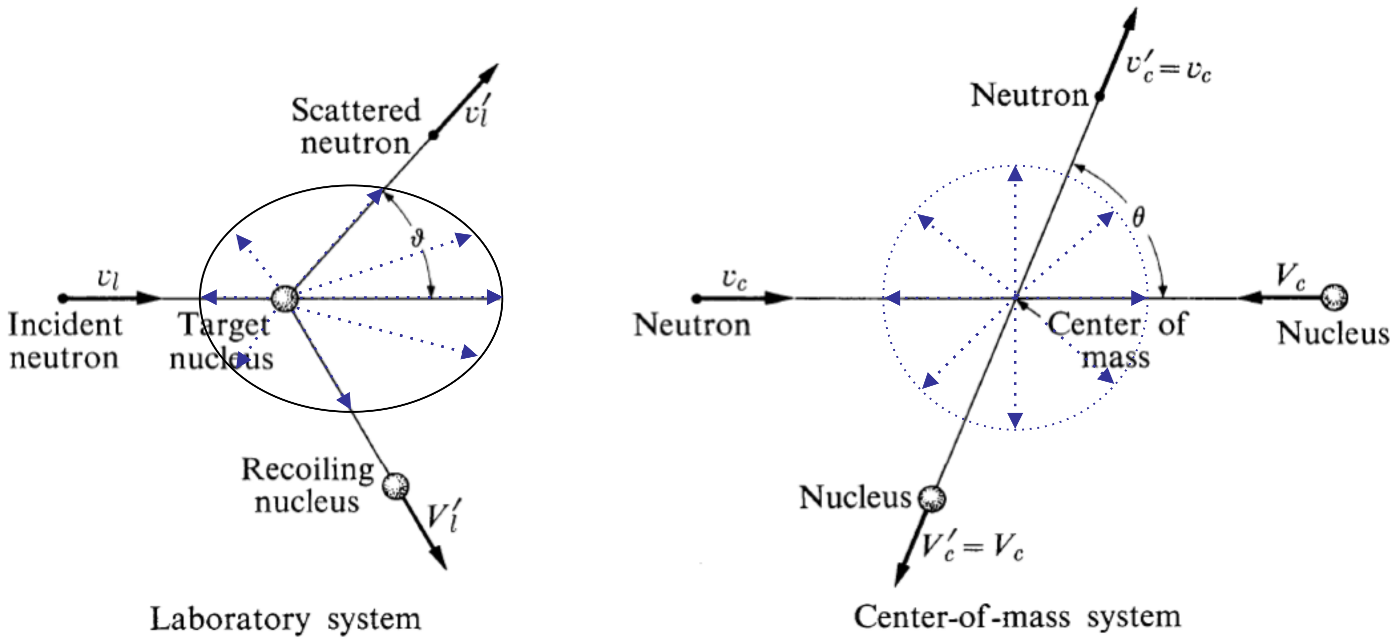
When $\lambda_{tr} > \lambda_s$ - forward
When $\lambda_{tr} < \lambda_s$ - backward



- In the case of isotropic scattering in the center-of-mass system:

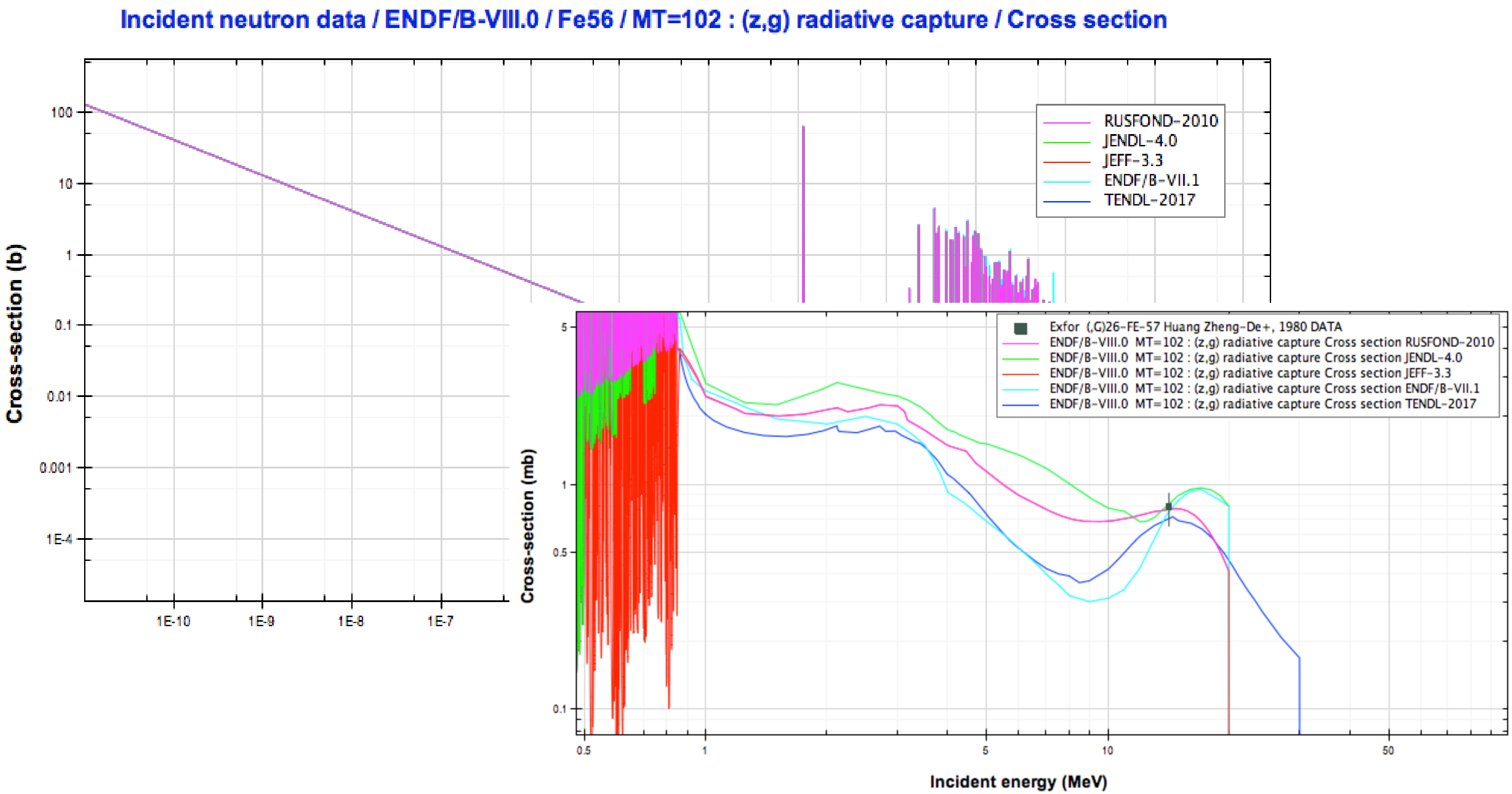
$$\bar{\mu} = \frac{2}{3A}$$

- If absorption is present: $\sigma_{tr} = \sigma_a + \sigma_s(1-\bar{\mu}) = \sigma_t - \sigma_s\bar{\mu}$ $\qquad \frac{1}{\lambda_{tr}} = \frac{1}{\lambda_t} - \frac{\bar{\mu}}{\lambda_s}$



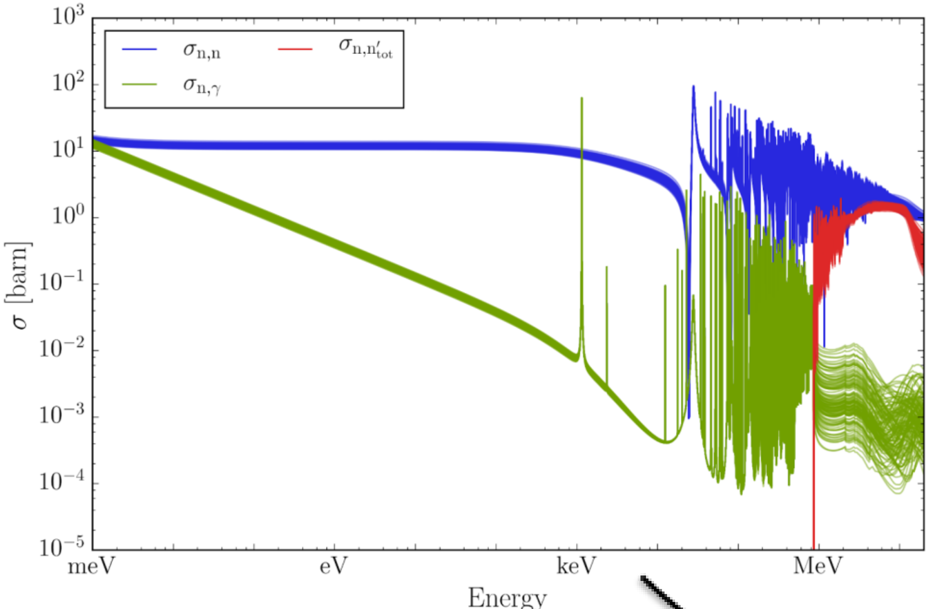
Elastic scattering of neutron by nucleus, as observed in laboratory and center-of-mass coordinates.

Exemple of the 56Fe (n,γ)

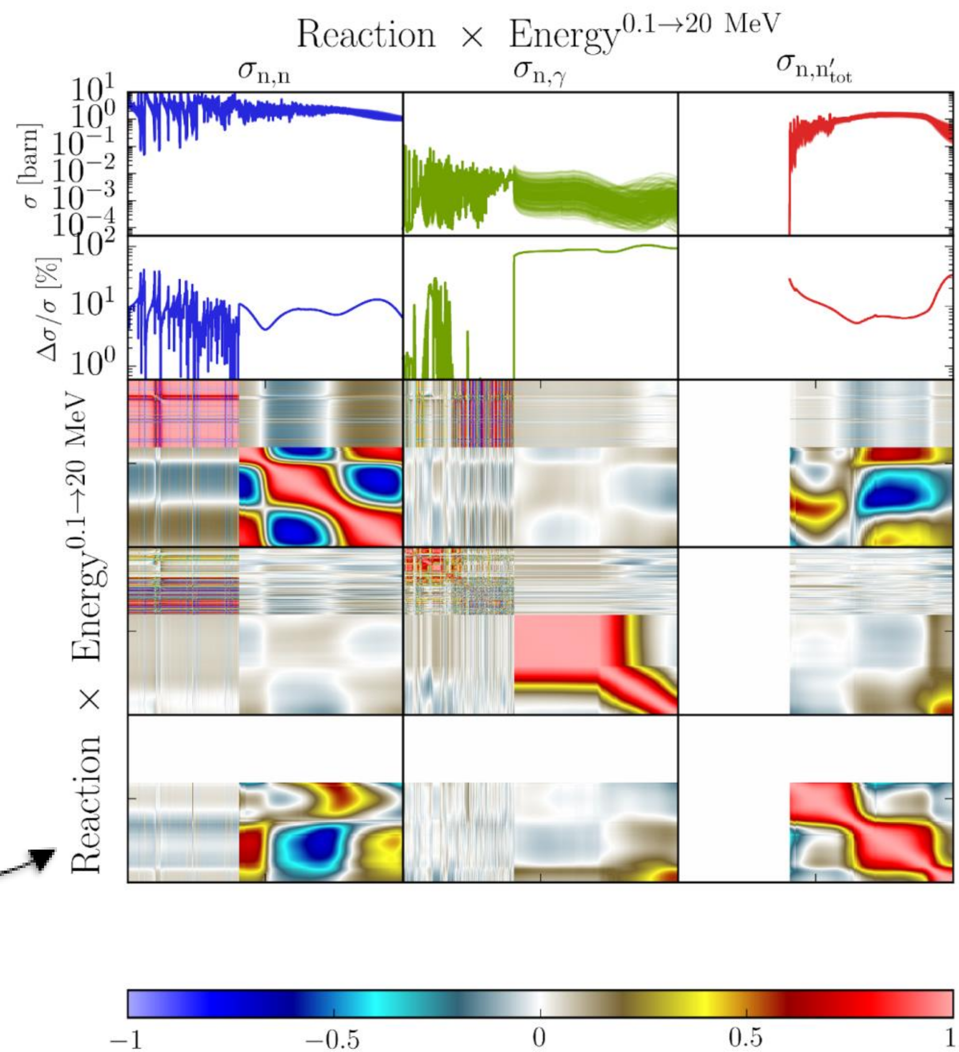


- The different libraries do not provide the same cross sections “values”…

Exemple of the 56Fe (n,γ)



same
“information”



Cross section dispersion

Uncertainty + correlation matrix

Summary

- Reaction rate = Flux x Cross-section (microscopic, macroscopic)

- Different types of reactions: absorption (fission, capture,…), scattering (elastic, inel.),…

- Energy dependence of cross-sections