Neutronics Exercises

16 Reactivity feedbacks

16.1 *Doppler effect reactivity*

Exercise description:

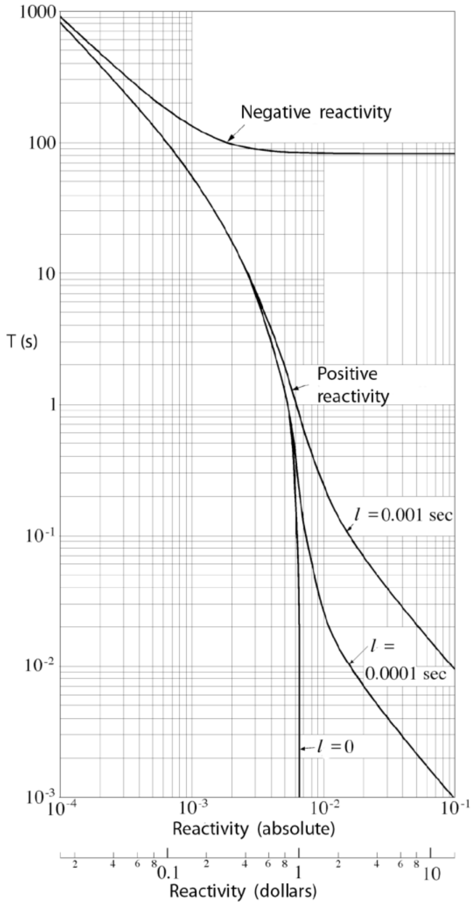
Consider the following situation:

In a reactor fuelled with ^{235}U and with $\Lambda=10^{-3}$ s, the average fuel temperature (T_c) changes suddenly from 400°C to 1050°C.

(a) What is the step change in reactivity (in $), if the fuel temperature (Doppler) coefficient is: (i) $+3.10^{-6}$/°C, (ii) -1.10^{-5}/°C? Assume that there are no other changes such as control rod movements, change in moderator temperature, etc.

(b) What is the stable period in the two cases?

(c) Estimate the corresponding values, assuming the complete absence of delayed neutrons.



Knowledge to be applied: $\rho_s = \Delta T_c \alpha_c$, figure, $T' = \frac{\Lambda}{|\rho|}$

Expected results: (a, i) $\rho_s = 30¢$

(a, ii) $\rho_s = -1\$$

(b, i) $T(30¢) = 20s$ (b, ii) $T(-1\$) = 85s$

(c, i) $T' = 0.51s$ (c, ii) $T' = 0.15s$

Exercise solution:

(a) The step change in reactivity

(i) $\rho_s = \Delta T_c \alpha_c = 650 \times 3 \times 10^{-6} = 1.95 \times 10^{-3} pcm(\simeq 0.3\$ = 30¢ for LWRs)$

(ii) $\rho_s = \Delta T_c \alpha_c = -650 \times 1 \times 10^{-5} = -6.5 \times 10^{-3} pcm\ (\simeq -1\$ for LWRs)$

(b) From the figure

(i) $T(30¢) = 20s$

(ii) $T(-1\$) = 85s$

(c) without delayed neutrons

(i) $T' = \frac{\Lambda}{|\rho|} = \frac{10^{-3}}{1.95 \times 10^{-3}} = 0.51s$

(ii) $T' = \frac{\Lambda}{|\rho|} = \frac{10^{-3}}{6.5 \times 10^{-3}} = 0.15s$

16.2 *Doppler coefficient*

Exercise description:

The temperature dependence of the resonance integral for fertile captures may be expressed by the following empirical relation: $I_{\text{eff}}(T_c) = I_{\text{eff}}(300°K)\left[1 + C\left(\sqrt{T_c} - \sqrt{300}\right)\right]$, where T_c is the fuel temperature in K, and C is a function of the fuel properties.

Constant C is given approximately by: $C = C_1 + (C_2/a\rho)$, where C_1, C_2 are constants for a given fuel type, a is the fuel radius in cm, and ρ is the fuel density in g/cm^3.

(a) Show that, for a thermal reactor, the resulting expression for the fuel temperature coefficient (due to broadening of the resonances) is: $\alpha_c = -\frac{C}{2\sqrt{T_c}} \ln\left[\frac{1}{p(300°K)}\right]$, where p is the resonance escape probability. Consider now a reactor fuelled with metallic uranium (ρ=19.1 g/cm^3), in which p=0.878 for T_c=300K. The fuel rods have a diameter of 2.8 cm.

(b) Calculate the Doppler coefficient corresponding to a fuel temperature of (i) 450°C , (ii) 1100°C, using appropriate data from the table.

Combustible	C_1 [10^{-4}]	C_2 [10^{-2}]
U^{238} (métal)	48	1.28
U^{238} (oxyde)	61	0.94
Th (métal)	85	2.68
ThO_2	97	2.40

N.B.: $\alpha_c \simeq \frac{1}{k}\frac{\text{dk}}{\text{dT}_c} \simeq \frac{1}{p}\frac{\text{dp}}{\text{dT}_c}$ with $p = \exp\left(-\frac{N_c V_c I_{\text{eff}}}{\xi_m N_m V_m}\right)$

Knowledge to be applied: $\alpha_c \simeq \frac{1}{p}\frac{\text{dp}}{\text{dT}_c}$, $p = \exp\left(-\frac{N_c V_c I_{\text{eff}}}{\xi_m N_m V_m}\right)$, $I_{\text{eff}}(T_c) = I_{\text{eff}}(300°K)\left[1 + C\left(\sqrt{T_c} - \sqrt{300}\right)\right]$

Expected results: (a) $\alpha_c = -\frac{C}{2\sqrt{T_c}} \ln\left[\frac{1}{p(300°K)}\right]$

(b, i) $\alpha_c(723°K) = -1.28 \times 10^{-5}/°C$ (b, ii) $\alpha_c(1373°K) = -9.3 \times 10^{-6}/°C$

Exercise solution:

(a) One has: $\alpha_c \simeq \frac{1}{p}\frac{\text{dp}}{\text{dT}_c}$ … (1) and $p = \exp(-\kappa I_{\text{eff}})$, where $\kappa = \frac{N_c V_c}{\xi_m N_m V_m}$ … (2)

Substituting (2) into (1), $\alpha_c \simeq \frac{1}{p}\left[-\kappa p \frac{\text{dI}_{\text{eff}}}{\text{dT}_c}\right]$ … (3)

From the expression for the resonance integral, $\frac{\text{dI}_{\text{eff}}}{\text{dT}_c} = I_{\text{eff}}(300°K)\frac{C}{2\sqrt{T_c}}$ … (4)

At 300K, $p(300°K) = \exp\left(-\kappa I_{\text{eff}}(300°K)\right)$ so that $I_{\text{eff}}(300°K) = \frac{1}{\kappa}\ln\left(\frac{1}{p(300°K)}\right)$… (5)

Finally, from (3), (4) and (5), $\alpha_c = -\frac{C}{2\sqrt{T_c}} \ln\left[\frac{1}{p(300°K)}\right]$

(b) For metallic ^{238}U: $C = C_1 + C_2/\text{al} = 48 \times 10^{-4} + 1.28 \times 10^{-2}/(1.4 \times 19.1) = 5.28 \times 10^{-3}$

$\Rightarrow \alpha_c = -\frac{5.28\times10^{-3}}{2\sqrt{T_c}} \ln\left[\frac{1}{0.878}\right] = -\frac{3.43\times10^{-4}}{\sqrt{T_c}}$. (i) $\alpha_c(723°K) = -1.28 \times 10^{-5}/°C$

(ii) $\alpha_c(1373°K) = -9.3 \times 10^{-6}/°C$

16.3 *Shut down margin*

Exercise description:

Consider a homogeneous thermal reactor, fuelled with highly enriched uranium (i.e. with p=ε=1), which is critical at a temperature of 230°C. The reactor is shut down and the temperature drops to the ambient value of 20°C. The change in temperature modifies the fuel and moderator cross-sections, such that the reactor parameters vary according to the values in the table. The delayed neutron fraction for U-235 is equal to 0.0065.

	230 °C	20 °C
η_c	2.055	2.060
f	0.592	0.596
M^2 (cm^2)	32.0	31.5

(a) What is the critical size of the reactor at 230°C, assuming spherical geometry?

(b) Estimate the minimal reactivity worth of control rods needed to ensure a safety margin of 2$ for the reactor's shutdown state at 20°C. You may neglect the changes in density and dimensions with temperature.

Knowledge to be applied: $B^2 = \left(\frac{\pi}{R_e}\right)^2 = B_m^2 = \frac{k_\infty - 1}{M^2}$, $k_{\text{eff}} = \frac{k_\infty}{1+B^2M^2}$, safety margin of 2$,

Expected results: (a) $R_e = 38.2\text{cm}$ (b) $\rho_{\min} = 3.8\$$

Exercise solution:

$$\text{(a)} B^2 = \left(\frac{\pi}{R_e}\right)^2 = B_m^2 = \frac{k_\infty - 1}{M^2} = \frac{\eta_c f - 1}{M^2} \underset{230°C}{=} \frac{2.055\times 0.592 - 1}{32.0} = 6.77\times 10^{-3}\text{cm}^{-2}$$

$$\Rightarrow R_e = \frac{\pi}{B} = 38.2\text{cm}$$

$$\text{(b) } k_{\text{eff}}(20°C) = \frac{k_\infty}{1+B^2M^2} = \frac{\eta_c f}{1+B^2M^2} \underset{20°C}{=} \frac{2.060\times 0.596 - 1}{1+6.77\times 10^{-3}\times 31.5} = \frac{1.2278}{1.2133} = 1.0120$$

$$\rho_{\min} = \frac{k-1}{k}\frac{1}{\beta}[\$] + 2[\$] = \frac{0.0119}{0.0065}[\$] + 2[\$] = 3.8\$$$

16.4 *Reactivity coefficients*

Exercise description:

In a certain PWR, the total reactivity associated with going from cold to hot power conditions for the moderator is -$ 6.00 and that associated with the fuel is -$ 4.14. In this reactor cold temperature is T_c=20 C; mean moderator operating temperature is T_H=320 C; and the mean fuel operating temperature is T_f=1700 C . Find the moderator and fuel temperature reactivity coefficients.

Expected results: $\alpha_M = -0.02\ \$/°C;\ \alpha_F = -0.00246\ \$/°C$

Exercise solution:

Moderator temperature coefficient:

$$\alpha_M = \frac{\Delta\rho}{\Delta T_M} = \frac{-6}{320 - 20} = -0.02\ \$/°C$$

Fuel temperature coefficient:

$$\alpha_F = \frac{\Delta\rho}{\Delta T_M} = \frac{-4.14}{1700 - 20} = -0.00246\ \$/°C$$