Neutronics Exercises

14 Depletion

14.1 *Fuel evolution*

Exercise description:

For a reactor fuelled with low enriched uranium, show that the ^{239}Pu concentration (relative to that of ^{235}U), after an irradiation corresponding to a fluence of θ n/cm^2 may be expressed as: $\frac{N_9(\theta)}{N_5(\theta)} = \frac{N_{8,0}}{N_{5,0}}\frac{\sigma_{a,8}}{\sigma_{a,9}}\frac{1-e^{-\sigma_{a,9}\theta}}{e^{-\sigma_{a,5}\theta}}$

$N_{5,0}$ being the initial ^{235}U concentration and $N_{8,0}$ being that of ^{238}U. It is assumed that the ^{238}U concentration remains constant (it indeed does not vary significantly). $\sigma_{a,5}$, $\sigma_{a,8}$, and $\sigma_{a,9}$ are the microscopic (1-group) absorption cross-sections of ^{235}U , ^{238}U and ^{239}Pu, respectively.

(a) Calculate the $^{239}Pu/^{235}U$ ratio for a reactor operating during 2yrs with a constant flux of 4.10^{13}n/cm^2.s, given the following data:

σ_{a5} = 344 b σ_{a8} = 1.5 b σ_{a9} = 820 b

$N_{50} = 1.7 \cdot 10^{20}$ cm^{-3}, $N_{80} = 6.7 \cdot 10^{21}$ cm^{-3}

(b) What is the ratio of the fission rate of ^{239}Pu to that of ^{235}U at the end of the 2-year period? Take $\sigma_{f,5}$= 293b and $\sigma_{f,9}$= 607b.

Knowledge to be applied: $\theta = \int_t \Phi(t)\mathrm{d}t$, $\frac{N_9(\theta)}{N_5(\theta)} = \frac{N_{8,0}}{N_{5,0}}\frac{\sigma_{a,8}}{\sigma_{a,9}}\frac{1-e^{-\sigma_{a,9}\theta}}{e^{-\sigma_{a,5}\theta}}$, $R = \Phi N\sigma$

Expected results: (a) $\frac{N_9(\theta)}{N_5(\theta)} = 0.15$ (b) $\frac{F_9}{F_5} = 0.31$

Exercise solution:

(a) $\theta = 4 \times 10^{13} \times 2 \times 365 \times 24 \times 3600 = 2.52 \times 10^{21}\ n/\mathrm{cm}^2$

$$\frac{N_9(\theta)}{N_5(\theta)} = \frac{N_{8,0}}{N_{5,0}}\frac{\sigma_{a,8}}{\sigma_{a,9}}\frac{1-e^{-\sigma_{a,9}\theta}}{e^{-\sigma_{a,5}\theta}} = \frac{6.7\times10^{21}}{1.7\times10^{20}}\frac{1.5}{820}\frac{1-e^{-2.07}}{e^{-0.87}} = 0.07\frac{0.874}{0.419} = 0.15$$

(b) $\frac{F_9}{F_5} = \frac{N_9}{N_5}\frac{\sigma_{f,9}}{\sigma_{f,5}} = 0.15\frac{607}{293} = 0.31$

14.2 *Average burn-up*

Exercise description:

(a) Let's consider a 100MW$_{th}$ reactor made of 120 fuel assemblies. Knowing that each fuel element contains 10 kg of fuel, what is the average burn-up in MWd/kg after a 1-year operational period?

(b) The initial percentage of fissile atoms in the fuel is 15.2%. What is the fraction of fissile material which, on average, (i) has undergone fission, (ii) has been destroyed?

(Consider that 10^6 MWd correspond to the fissioning of 1t of fissile material and use σ_f= 582 b, σ_a= 681 b for the fissile nuclide.)

Expected results: (a) $Burnup = 30.4\frac{MWd}{kg}$ (b) (i) $F_{fissioned}$=20% (ii) $F_{destroyed}$=23.4%

Exercise solution:

(a) $Burnup = \frac{E_{year}}{m_{fuel}} = \frac{100MW \cdot 365d}{120 \cdot 10kg} = 30.4\,MWd/kg$

(b) (i) The above average burn-up implies that ~ 304 MWd have been produced by the 10kg fuel element. This amount corresponds to the fissioning of 0.304 kg of fissile material. Since, of the 10 kg, 1.52 kg is initially fissile, fraction of fissile material fissioned = 0.304 / 1.52 = **20%**

(ii)

$$F_{fissioned} = \frac{\Delta N_{fuel}}{N_{fuel}} = N_{fuel}\sigma_f\Phi = 0.2$$

$$F_{destroyed} = \frac{\Delta N_d}{N_{fuel}} = N_{fuel}\sigma_a\Phi = \frac{\sigma_a}{\sigma_f}F_{fissioned} = 0.234$$

14.3 *Xe poisoning I*

Exercise description:

(a) ^{135}Xe is the fission product with the largest poisoning effect in a thermal reactor. The direct production rate from fission is relatively low (~ 0.3%) and it is primarily formed by the β^--decay of the much more abundant fission product, ^{135}I (production rate ~ 6.1%): $I\overrightarrow{\beta^-, T_{1/2} = 6.6h}Xe\overrightarrow{\beta^-, T_{1/2} = 9.1h}Cs(\text{stable})$

For a reactor at steady-state with a flux Φ, show that the equilibrium xenon concentration (relative to the number density of fissile atoms) is given by: $\frac{N_x}{N_f} = \frac{(\gamma_i+\gamma_x)(\sigma_f/\sigma_x)}{1+\lambda_x/(\Phi\sigma_x)}$, where γ_i is the fission yield for ^{135}I, γ_x that for ^{135}Xe, σ_f the microscopic cross-section for thermal fission, σ_x that for Xe absorption, and λ_x is the disintegration constant for ^{135}Xe. (N.B.: One may neglect absorptions in ^{135}I.)

(b) Show that the reactivity effect of ^{135}Xe at equilibrium in a large thermal reactor fuelled with ^{235}U, is given approximately by: $\rho = -\frac{\gamma_i+\gamma_x}{\bar{\nu}p\varepsilon(1+\lambda_x/(\Phi\sigma_x))}$, where $\bar{\nu}$, p, ε have their usual meaning. Calculate the reactivity effects for a reactor operating with a thermal neutron flux of (i) 10^{10} , (ii) 10^{12} , (iii) 10^{14} n/cm^2.s , using the data from table.

(c) What is theoretically the maximal reactivity effect?

$\bar{\nu} = 2.42$,	$p = 0.925$,	$\varepsilon = 1.040$	
$\gamma_1 = 0.061$,	$\gamma_x = 0.003$,	$\sigma_x = 2.6 \cdot 10^6$ b ,	$\lambda_x = 2.1 \cdot 10^{-5} s^{-1}$

Knowledge to be applied: $\rho = \frac{k-k_0}{k} \simeq \frac{f-f_0}{f}$, $f = \frac{\Sigma_{a,c}}{\Sigma_{a,c}+\Sigma_{a,m}+\Sigma_{a,Xe}}$ $k_0 = k_\infty = 1 = \eta fp\varepsilon$,

Expected results: (b) ρ= (i) -3.4.10^{-5}, (ii) -3.0.10^{-3}, (iii) -2.5% (c) ρ_{max}=-2.75%

Exercise solution:

(a)) At equilibrium, for both ^{135}I and ^{135}Xe, Production = Destruction.

$$\frac{dN_i(t)}{dt} = -\lambda_i N_i(t) + \gamma_i\sigma_f N_f\Phi = 0 \Rightarrow N_i = \frac{\gamma_i}{\lambda_i}\sigma_f N_f\Phi$$

$\frac{dN_x(t)}{dt} = -\lambda_x N_x(t) - \sigma_x N_x\Phi + \lambda_i N_i(t) + \gamma_x\sigma_f N_f\Phi = 0 \Rightarrow \lambda_i N_i + \gamma_x\sigma_f N_f\Phi = \lambda_x N_x + \sigma_x N_x\Phi$Substituting for N_i:

$$\gamma_i\sigma_f N_f\Phi + \gamma_x\sigma_f N_f\Phi = \lambda_x N_x + \sigma_x N_x\Phi \Rightarrow \frac{N_x}{N_f} = \frac{(\gamma_i+\gamma_x)\sigma_f\Phi}{\sigma_x\Phi+\lambda_x} = \frac{(\gamma_i+\gamma_x)(\sigma_f/\sigma_x)}{1+\lambda_x/(\Phi\sigma_x)}$$

(b) The ^{135}Xe affects f, the thermal utilization factor, such that $\rho = \frac{k-k_0}{k} \simeq \frac{f-f_0}{f}$, where k is the k_∞ value with Xe poisoning and k_0 is that without Xe. Neglecting the neutron absorption in the cladding, f and f_0 are given by: $f = \frac{\Sigma_{a,c}}{\Sigma_{a,c}+\Sigma_{a,m}+\Sigma_{a,Xe}}$ and $f_0 = \frac{\Sigma_{a,c}}{\Sigma_{a,c}+\Sigma_{a,m}}$, where the indices m, c, Xe denote moderator, fuel and xenon, respectively. Substituting it into reactivity equation $\rho = -\frac{\Sigma_{a,Xe}}{\Sigma_{a,c}+\Sigma_{a,m}}$. For a reactor of large dimensions, one may neglect the leakage and the criticality condition without xenon is simply:

$k_0 = k_\infty = 1 = \eta fp\varepsilon = \bar{\nu}\frac{\Sigma_f}{\Sigma_{a,c}}\frac{\Sigma_{a,c}}{\Sigma_{a,c}+\Sigma_{a,m}}p\varepsilon$ so that $\Sigma_{a,c} + \Sigma_{a,m} = \bar{\nu}\Sigma_f p\varepsilon$. From the two equation above: $\rho = -\frac{1}{p\varepsilon\bar{\nu}}\frac{\Sigma_{a,Xe}}{\Sigma_f}$. Finally, substituting for N_x/N_f : $\rho = -\frac{\gamma_i+\gamma_x}{p\varepsilon\bar{\nu}(1+\lambda_x/(\Phi\sigma_x))}$. Substituting the numerical values,

$$\rho = -\frac{0.061 + 0.003}{2.42 \times 0.925 \times 1.040(1 + 2.1 \times 10^{-5}/(2.6 \times 10^6 \times 10^{-24}\Phi))}$$
$$= -\frac{0.0275}{1 + 8.08 \times 10^{12}/\Phi}$$

Thus, for the 3 cases, the ρ-values are: (i) -3.4.10^{-5}, (ii) -3.0.10^{-3}, (iii) -2.5.10^{-2}=-2.5% .

(c) Theoretically, the reactivity effect is maximal for $\Phi \to \infty$, i.e. $\rho_{max} = -0.0275$

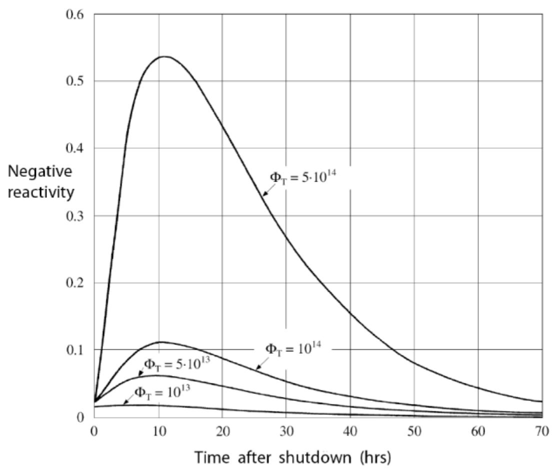
14.4 *Xe poisoning II*

Exercise description:

A reactor is in steady-state operation with equilibrium concentrations $N_{i,0}$ and $N_{x,0}$ of ^{135}I and ^{135}Xe, respectively. The reactor is shutdown.

(a) Show that the ^{135}Xe concentration varies as: $N_x(t) = N_{x,0}e^{-\lambda_x t} + \frac{\lambda_i}{\lambda_i - \lambda_x} N_{i,0}\left(e^{-\lambda_x t} - e^{-\lambda_i t}\right)$

Considering that $N_{i,0}$ and $N_{x,0}$ depend on the flux level, it is clear that the increase in the Xe concentration following the shutdown can prevent restarting the reactor during a certain time, in the case of a high-flux reactor. The figure alongside indicates the evolution of the Xe-concentration (through the negative reactivity in the system due to the presence of Xe-135) after reactor shutdown for different operational flux values. Consider now the case of a nominal flux value of 10^{14} n/cm^2.s.



(b) Estimate the waiting time necessary in order to be able to restart the reactor, if the maximal reactivity reserve held by the control system is: (i) 10%, (ii) 5%, (iii) 2%. (Assume that it is not possible to restart before the Xe concentration has passed its peak value.)

Knowledge to be applied: After t=0, $\frac{dN_x(t)}{dt} = -\lambda_x N_x(t) + \lambda_i N_i(t)$,

Expected results: (a) $N_x(t) = N_{x,0}e^{-\lambda_x t} + \frac{\lambda_i}{\lambda_i - \lambda_x} N_{i,0}\left(e^{-\lambda_x t} - e^{-\lambda_i t}\right)$ (b) reading off from the curve Φ=10^{14}, the waiting time is: (i) $t \approx 10h$, (ii) $t \approx 31h$, (iii) $t \approx 44h$,

Exercise solution:

After t=0, $\frac{dN_x(t)}{dt} = -\lambda_x N_x(t) + \lambda_i N_i(t) = -\lambda_x N_x(t) + \lambda_i N_{i,0} e^{-\lambda_i t}$

With $e^{-\lambda_x t}$ as integration factor, $N_x(t) = -\frac{\lambda_i}{\lambda_i - \lambda_x} N_{i,0} e^{-\lambda_i t} + \mathrm{C}e^{-\lambda_x t}$

At t=0, $N_x(0) = N_{x,0} \Rightarrow C = N_{x,0} + \frac{\lambda_i}{\lambda_i - \lambda_x} N_{i,0}$

Thus, $N_x(t) = N_{x,0}e^{-\lambda_x t} + \frac{\lambda_i}{\lambda_i - \lambda_x} N_{i,0}\left(e^{-\lambda_x t} - e^{-\lambda_i t}\right)$

For the 3 "restart" cases, reading off from the curve Φ=10^{14}, the waiting time is:

(i) $t \approx 10h$, (ii) $t \approx 31h$, (iii) $t \approx 44h$,