

11 mai 2022

## Series 10 : Mixture designs

### 1. Constraints

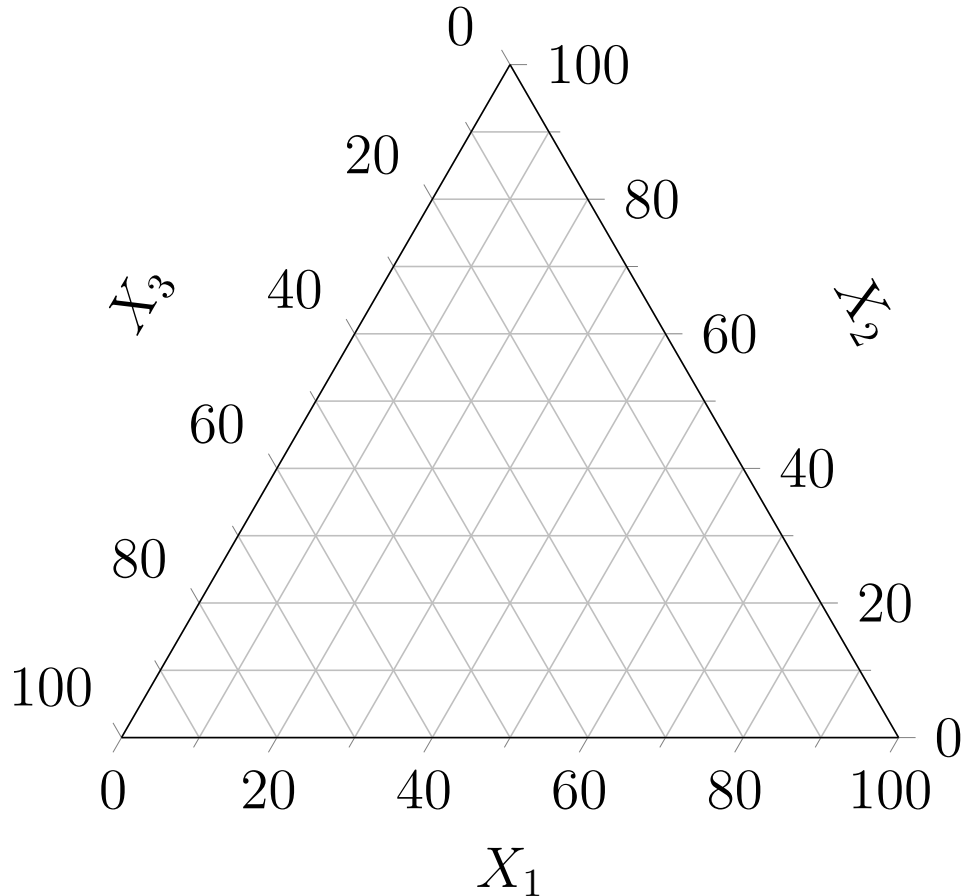
On a ternary diagram whose variables are  $x_1, x_2$  et  $x_3$ , let's determine the domains corresponding to the following constraints :

a)  $20\% \leq x_1 \leq 50\%$

b)  $\begin{cases} x_1 \geq 30\% \\ x_3 \leq 10\% \end{cases}$

c)  $\begin{cases} x_1 \leq 30\% \\ x_2 \leq 50\% \\ x_1 \leq x_3 \end{cases}$

d)  $x_1 + x_2 \leq x_3$



## 2. Simplex lattice design {q,3}

- a) Set up a simplex lattice design {3,3}
- b) Place the measurement points on a ternary diagram  
(*ternplot(A,B,C,linetype)* )
- c) Set up a simplex lattice design {4,3}
- d) Draw an axonometry of the design

## 3. Simplex lattice design {3,2}

The data  $Y_1$  and  $Y_2$  given in the table here below have been obtained with a *simplex lattice design* {3,2} for factors  $x_1, x_2$  and  $x_3$ .

$x_1$	$x_2$	$x_3$	$Y_1$	$Y_2$
0	0	1	1	1.1
0	1	0	1.5	1.4
1	0	0	1.3	1.4
0.5	0.5	0	3	3.2
0.5	0	0.5	3.4	3.3
0	0.5	0.5	3.6	3.7

- a) Using the least square fit method, determine the second degree Scheffé's model corresponding to the response  $Y_1$  of the table
- b) Consider responses  $Y_1$  and  $Y_2$  as replicates and infer an adequate model with the routine *fitlm(...)*
- c) Draw the response surface with the help of the functions *ternsurf(...)* and *terncontour(...)*

## 4. Analysis of the ternary functions

- a) Code a function *Scheffe(y, x1, x2, x3)* to draw in 3D the response surface of a Scheffé's model,
- b) With this function, draw the linear Scheffé's model  $y = 3x_1 + 2x_2$ . By varying the coefficients of the model, analyze and understand its geometry.
- c) Use the function to represent the quadratic Scheffé's model, for example  $y = x_1x_2 + x_1x_3 + x_2x_3$ . By varying the coefficients of the model, analyze its geometry.

- d) Use the function to represent the cubic Scheffé's model, for example  $y = x_1x_2(x_1 - x_2) + x_1x_3(x_1 - x_3) + x_2x_3(x_2 - x_3)$ . By varying the coefficients of the model, analyze its geometry.

Tips :

- Draw in a Cartesian plane using the *nan* value to draw only the ternary domain.
- Use symbolic calculation to manipulate functions and make transformations from the ternary coordinate system to the Cartesian coordinate system.

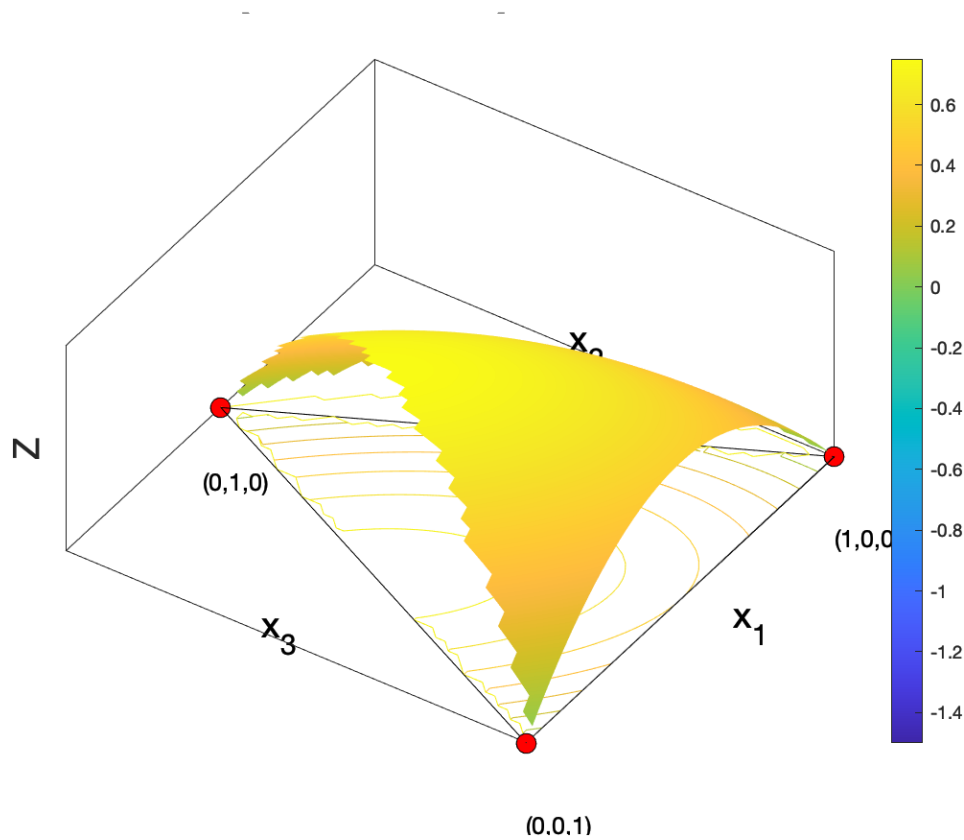


FIGURE 1 – Example of a quadratic Scheffé's model