Modelling and design of experiments

Chapitre 6: Qualitative factors

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Qualitative factors

6.1 Optimization of a workshop

Your company is producing mechanical pieces for the aeronautic industry. An analysis of last year results has shown that the workshop WA has a quality problem.

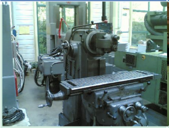
You are in charge of identifying the origin of the problem.

After discussing with the workshop supervisor, you have identified 3 possible factors that possibly affect the quality of the production :

1. The machines
2. The drills
3. The operators

You want now to evaluate their respective influence on the problem.

6.2 Optimisation of a workshop

Operators			
Machines			
Drills			

6.3 The mind map

Production quality in a workshop

- **Objective**: Identify the dominent factors to manage the production quality of a workshop
- **Type of investigation**: Experiment : factors can be manipulated
- **Factors**:
  1. Operators (3 levels)
  2. Machines(3 levels)
  3. Drills(3 levels)
- **Response(s)**: The quality could be mesured by different observations and measurements : the surface quality is chosen : the data is in μm the biggest defect detected on the surface.
- **Model(s)**: A constant coefficient model (CCM) with or without interactions
- **Strategies**:
  1. Factorial design (all the possible experiments)
  2. Latin square design (based on a magic square)

6.4 Factorial strategy

Table – Biggest defect detected on the surface in μm,

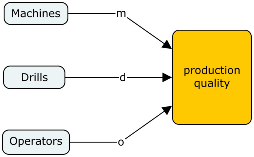
Operator	Drill	Machine		
		Deckel	Schaublin	Maho
Charlie	1mm	23.46	28.46	27.06
	5mm	17.66	24,07	23.79
	20mm	12.33	19.00	16.63
Pierre	1mm	24.22	28.56	28.06
	5mm	18.87	25.32	22.24
	20mm	10.85	19.21	17.75
Louis	1mm	22.55	29.69	28.50
	5mm	16.93	24.92	24.99
	20mm	12.25	17.24	16.72

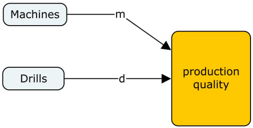
6.5 Constant coefficient model

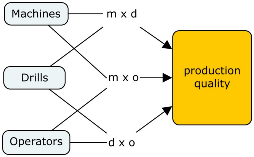
- Qualitative variables (or discreet variables)
- When a synthesis of observations is needed
  - Which ones are the important factors ?
  - Which one is the best machine ?
  - Which one is the best drill ?
  - Does the operator performance depend on the machine ?
- For those situations, a constant coefficient model is used

$$Y_{mhoi} = \mu + \alpha_m + \beta_h + \gamma_o + \epsilon_{mhoi}$$

6.5 The final causal model





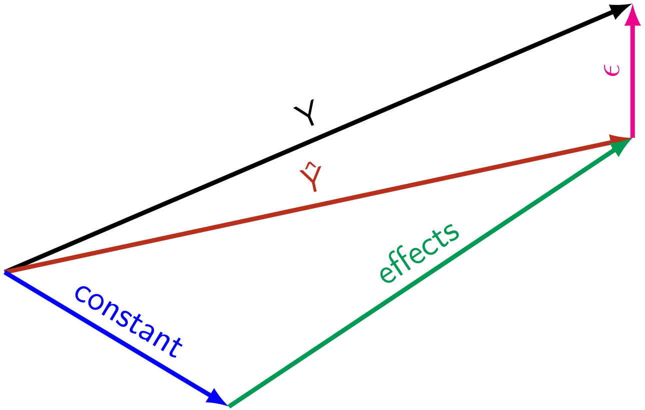


6.6 Constant vs random coefficients

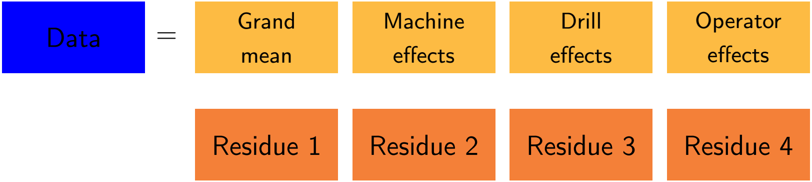
- Constant coefficients
  - → which states of the factors optimise the response
  - → Example : Modelling the performance of a workshop
  - → The testing of the whole population is necessary
  - → $H_o : \tau_i = 0, \forall i$
- Random coefficients
  - → Which factors dominate the phenomenon
  - → Example : A model about learning best practices
    What is the best practice for preparing exams ?
    1. To solve supplementary exercises,
    2. To draw mind map of the different chapters,
    3. To color the textbook.
  - → The testing of a sample of the population is sufficient
  - → $H_o : \sigma_\tau = 0$

6.7 Inference of the effects

$$Y_{mhoi} = \mu + \alpha_m + \beta_h + \gamma_o + \epsilon_{mhoi} \quad m, h, o = 1, \ldots, 3$$

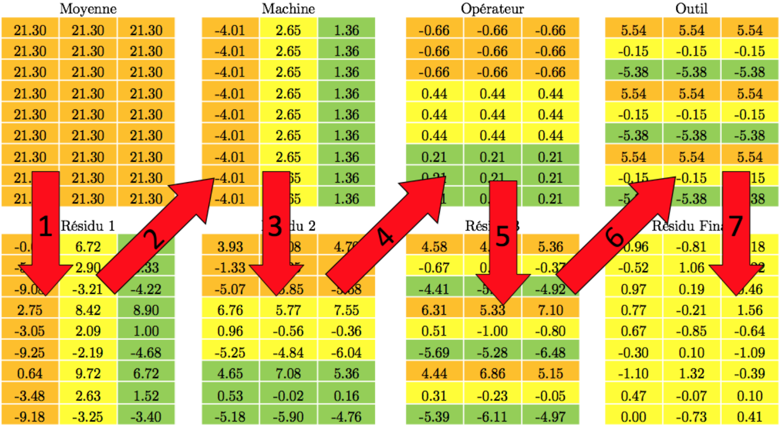


6.8 Sweeping - basic scheme



1. Compute the grand mean μ of all the results
2. Compute residues ϵ_1
3. From residues ϵ_1, compute means α_m for each category of factor m
4. Compute residues ϵ_2
5. From residues ϵ_2, compute means β_h for each category of factor h
6. Etc.

6.9 Sweeping on a spreadsheet



Moyenne

21.30	21.30	21.30
21.30	21.30	21.30
21.30	21.30	21.30
21.30	21.30	21.30
21.30	21.30	21.30
21.30	21.30	21.30
21.30	21.30	21.30
[illegible]	21.30	21.30
[illegible]	21.30	21.30

Machine

-4.01	2.65	1.36
-4.01	2.65	1.36
-4.01	2.65	1.36
-4.01	2.65	1.36
-4.01	2.65	1.36
-4.01	2.65	1.36
-4.01	2.65	1.36
-4.01	[illegible]	1.36
-4.01	[illegible]	1.36

Opérateur

-0.66	-0.66	-0.66
-0.66	-0.66	-0.66
-0.66	-0.66	-0.66
0.44	0.44	0.44
0.44	0.44	0.44
0.44	0.44	0.44
0.21	0.21	0.21
[illegible]	[illegible]	0.21
[illegible]	[illegible]	0.21

Outil

5.54	5.54	5.54
-0.15	-0.15	-0.15
-5.38	-5.38	-5.38
5.54	5.54	5.54
-0.15	-0.15	-0.15
-5.38	-5.38	-5.38
5.54	5.54	5.54
-0.15	-0.15	[illegible]
[illegible]	-5.38	[illegible]

Résidu 1

[illegible]	6.72	[illegible]
[illegible]	2.90	[illegible]
[illegible]	-3.21	-4.22
2.75	8.42	8.90
-3.05	2.09	1.00
-9.25	-2.19	-4.68
0.64	9.72	6.72
-3.48	2.63	1.52
-9.18	-3.25	-3.40

[illegible]du 2

3.93	[illegible]	[illegible]
-1.33	[illegible]	[illegible]
-5.07	[illegible]	[illegible]
6.76	5.77	7.55
0.96	-0.56	-0.36
-5.25	-4.84	-6.04
4.65	7.08	5.36
0.53	-0.02	0.16
-5.18	-5.90	-4.76

Rési[illegible]

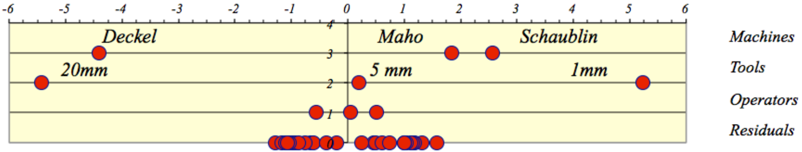
4.58	[illegible]	5.36
-0.67	[illegible]	[illegible]
-4.41	[illegible]	-4.92
6.31	5.33	7.10
0.51	-1.00	-0.80
-5.69	-5.28	-6.48
4.44	6.86	5.15
0.31	-0.23	-0.05
-5.39	-6.11	-4.97

Résidu Fina[illegible]

[illegible]	-0.81	[illegible]
-0.52	1.06	[illegible]
0.97	0.19	[illegible]
0.77	-0.21	1.56
0.67	-0.85	-0.64
-0.30	0.10	-1.09
-1.10	1.32	-0.39
0.47	-0.07	0.10
0.00	-0.73	0.41

6.10 Model and dot plot

$$\hat{Y}_{mho} = 21.3 + \begin{Bmatrix} -4.01 \\ 2.65 \\ 1.368 \end{Bmatrix} + \begin{Bmatrix} 5.54 \\ 0.15 \\ -5.38 \end{Bmatrix} + \begin{Bmatrix} -0.66 \\ 0.44 \\ 0.24 \end{Bmatrix}$$



6.11 Sweeping - Estimation of the interactions

$$Y_{mhoi} = \mu + \alpha_m + \beta_h + \gamma_o + \color{red}{\alpha\beta_{mh} + \alpha\gamma_{mo} + \beta\gamma_{ho}} + \epsilon_{mhoi}$$

Tool effects

	machine 1	machine 2	machine 3
tool 1	5.23	5.23	5.23
tool 2	0.19	0.19	0.19
tool 3	-5.42	-5.42	-5.42
tool 1	5.23	5.23	5.23
tool 2	0.19	0.19	0.19
tool 3	-5.42	-5.42	-5.42
tool 1	5.23	5.23	5.23
tool 2	0.19	0.19	0.19
tool 3	-5.42	-5.42	-5.42

Residual Final

	0.46	1.58	-1.03
operator 1	-0.98	-0.66	1.03
	1.32	-0.61	-1.1
	-1.27	-0.75	-1.07
operator 2	-0.95	0.49	0.75
	1.19	0.61	1
	1.16	1.12	-0.2
operator 3	0.25	-0.92	1
	-1.16	-0.87	-0.38

Machine x operator

	machine 1	machine 2	machine 3
	0.27	0.10	-0.37
operator 1	0.27	0.10	-0.37
	0.27	0.10	-0.37
	-0.34	0.12	0.23
operator 2	-0.34	0.12	0.23
	-0.34	0.12	0.23
	0.08	-0.22	0.14
operator 3	0.08	-0.22	0.14
	0.08	-0.22	0.14

tool 1	0.19	1.48	-0.66
tool 2	-1.25	-0.76	1.40
tool 3	1.05	-0.71	-0.73
tool 1	-0.93	-0.87	-1.30
tool 2	-0.61	0.37	0.52
tool 3	1.53	0.49	0.77
tool 1	1.08	1.34	-0.34
tool 2	0.17	-0.70	0.86
tool 3	-1.24	-0.65	-0.52

Machine x tool

machine 1	machine 2	machine 3
0.11	0.65	-0.77
-0.56	-0.36	0.93
0.45	-0.29	-0.16
0.11	0.65	-0.77
-0.56	-0.36	0.93
0.45	-0.29	-0.16
0.11	0.65	-0.77
-0.56	-0.36	0.93
0.45	-0.29	-0.16

0.08	0.83	0.10
-0.68	-0.40	0.47
0.61	-0.42	-0.57
-1.04	-1.52	-0.53
-0.04	0.74	-0.40
1.09	0.78	0.93
0.96	0.69	0.43
0.73	-0.33	-0.07
-1.69	-0.36	-0.36

Tool x operator

0.34	0.34	0.34
-0.21	-0.21	-0.21
-0.13	-0.13	-0.13
-1.03	-1.03	-1.03
0.10	0.10	0.10
0.93	0.93	0.93
0.69	0.69	0.69
0.11	0.11	0.11
-0.80	-0.80	-0.80

-0.26	0.49	-0.23
-0.48	-0.20	0.68
0.74	-0.29	-0.44
-0.01	-0.49	0.50
-0.14	0.64	-0.50
0.15	-0.15	0.00
0.27	0.00	-0.27
0.62	-0.44	-0.18
-0.89	0.45	0.44

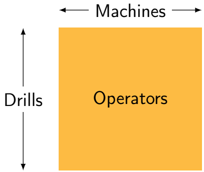
6.12 Algorithmic perspective

- The data set can be represented by a pseudo-tensor y_{ijk} with for example i and j representing P and Q levels of two variables, and k the R replicates
- The reduced means are then
  - $\mu_{ij} = \frac{1}{R} \sum_k x_{ijk}$
  - $\mu_i = \frac{1}{QR} \sum_{j,k} x_{ijk} = \frac{1}{Q} \sum_j \mu_{ij}$
  - $\mu_j = \frac{1}{PR} \sum_{i,k} x_{ijk} = \frac{1}{P} \sum_i \mu_{ij}$
  - $\mu = \frac{1}{PQR} \sum_{i,j,k} x_{ijk}$
- If the model is $y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \epsilon_{ijk}$ then
  - $\alpha_i = \mu_i - \mu$
  - $\beta_j = \mu_i - \mu$
  - $\alpha\beta_{ij} = \mu_{ij} - \mu_i - \mu_j + \mu$

6.13 Cost-benefit ratio

- The model counts 10 coefficients
- The regression has 7 degrees of freedom
- The residue has 20 degrees of freedom
- Cost-benefit ratio ~ 0.37
- **Let's try to find something better**

6.14 Latin square 3×3



a	b	c
b	c	a
c	a	b

	Deckel	Schaublin	Maho
1 mm	Charlie	Pierre	Louis
5 mm	Pierre	Louis	Charlie
10 mm	Louis	Charlie	Pierre

6.15 Sweeping

Mean

21.25	21.25	21.25
21.25	21.25	21.25
21.25	21.25	21.25

Machine

-4.73	2.04	2.68
-4.73	2.04	2.68
-4.73	2.04	2.68

Tool

5.30	5.30	5.30
0.14	0.14	0.14
-5.44	-5.44	-5.44

Operator

-0.04	0.51	-0.48
0.51	-0.48	-0.04
-0.48	-0.04	0.51

Residual 1

0.95	7.80	7.15
-4.42	2.12	2.73
-10.71	-3.79	-1.83

Residual 2

5.67	5.75	4.47
0.31	0.08	0.05
-5.98	-5.83	-4.52

Residual 3

0.37	0.45	-0.83
0.16	-0.07	-0.10
-0.54	-0.39	0.93

Residual 4

0.41	-0.06	-0.35
-0.35	0.41	-0.06
-0.06	-0.35	0.41

6.16 Comparison of the two models

Model infered with 27 data points (factorial design)

$$\hat{Y}_{mho} = 21.3 + \begin{Bmatrix} -4.01 \\ 2.65 \\ 1.368 \end{Bmatrix} + \begin{Bmatrix} 5.54 \\ 0.15 \\ -5.38 \end{Bmatrix} + \begin{Bmatrix} -0.66 \\ 0.44 \\ 0.24 \end{Bmatrix}$$

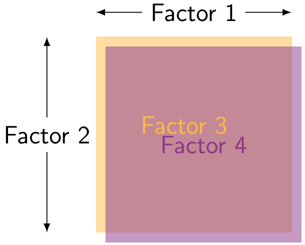
Model infered with 9 data points (Latin square)

$$\hat{Y}_{mho} = 21.90 + \begin{Bmatrix} -3.71 \\ 2.26 \\ 1.45 \end{Bmatrix} + \begin{Bmatrix} 4.94 \\ 0.63 \\ -5.57 \end{Bmatrix} + \begin{Bmatrix} 0.18 \\ -0.17 \\ -0.01 \end{Bmatrix}$$

6.17 Graeco-Latin squares 3×3

$$\begin{array}{|ccc|} \hline \mathrm{A} & \mathrm{B} & \mathrm{C} \\ \mathrm{B} & \mathrm{C} & \mathrm{A} \\ \mathrm{C} & \mathrm{A} & \mathrm{B} \\ \hline \end{array} \quad \text{et} \quad \begin{array}{|ccc|} \hline \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{|ccc|} \hline A\alpha & B\beta & C\gamma \\ B\gamma & C\alpha & A\beta \\ C\beta & A\gamma & B\alpha \\ \hline \end{array}$$

- Factor 1 : by columns
- Factor 2 : by lines
- Factor 3 : by Latin letters
- Factor 4 : by Greek letters



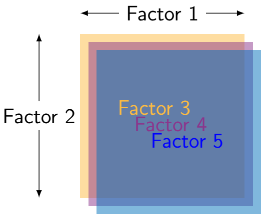
6.18 Hyper Graeco-Latin squares 4×4

A	B	C	D
B	A	D	C
C	D	A	B
D	C	B	A

A	B	C	D
D	C	B	A
B	A	D	C
C	D	A	B

A	B	C	D
C	D	A	B
D	C	B	A
B	A	D	C

- Factor 1 : by columns
- Factor 2 : by lines
- Factor 3 : by first square
- Factor 4 : by second square
- Factor 5 : by third square



6.19 Hyper Graeco-Latin squares 5×5

A	B	C	D	E
C	D	E	A	B
E	A	B	C	D
B	C	D	E	A
D	E	A	B	C

A	B	C	D	E
D	E	A	B	C
B	C	D	E	A
E	A	B	C	D
C	D	E	A	B

A	B	C	D	E
E	A	B	C	D
D	E	A	B	C
C	D	E	A	B
B	C	D	E	A

- Factor 1 : by columns
- Factor 2 : by lines
- Factor 3 : by first square
- Factor 4 : by second square
- Factor 5 : by third square

6.20 ANOVA for 27 and 9 runs

27-experiment set :

Source	SS	DF	MS	F	p
Constant	12'465.7	1	12'465.70		
Machine	264.5	2	132.23	109.3	0.000%
Drill	511.0	2	255.50	211.19	0.000%
Operator	5.2	2	2.62	2.2	14.1%
Residue	24.2	20	1.21	1	
Total	13'270.6	27			

9-experiment set :

Source	SS	DF	MS	F	p
Constant	4'064.9	1	4'064.91		
Machine	101.1	2	50.55	113.7	0.000%
Drill	173.2	2	86.61	194.7	0.000%
Operator	1.5	2	0.74	1.7	21.5%
Residue	0.9	2	0.44	1	
Total	4'341.6	9			

6.21 Dotplot for 27 and 9 runs

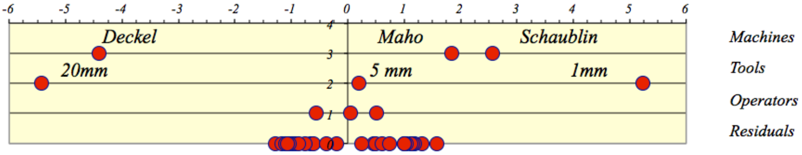


Figure – 27-experiment set

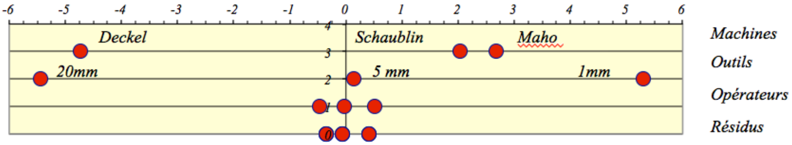


Figure – 9-experiment set

6.22 Routines on Matlab

```
[p,table,stats] = anovan(data,{m,d,o},...
    'display','on',...
    'varnames',{'Machine';'Tool';'Operator'},...
    'model','linear') ;
```

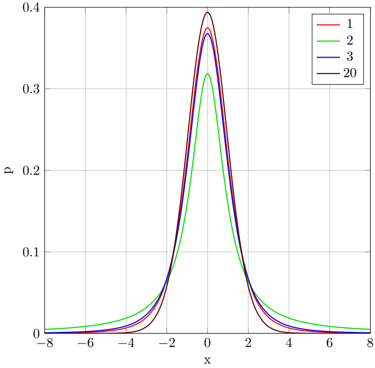
Available information

- stats.resid : residues
- stats.coeffs : coefficients
- stats.terms : terms of the model
- stats.coeffnames : names of the coefficients
- stats.varnames : names of the variables

6.23 The Student T distribution

If the observations X_i are independent identically distributed (IID),

then $\left(\dfrac{\overline{X} - \mu}{s/\sqrt{n}}\right) \sim T(n-1)$



	Parent distribution	sampling dist. for $\bar{y}$
Mean	η	η
Variance	σ^2	$\frac{\sigma^2}{n}$
Std dev.	σ	$\frac{\sigma}{\sqrt{n}}$
Form	$\sim$ any	more nearly Normal

6.24 Matlab : ANOVAN routine

```
[p,table,stats] = anovan(data,{m,d,o},...
    'display','on',...
    'varnames',{'Machine';'Tool';'Operator'},...
    'model','linear') ;
```

Analysis of Variance

Source	Sum Sq.	d.f.	Mean Sq.	F	Prob>F
Machine	206.577	2	103.289	126.39	0
Tool	544.079	2	272.039	332.88	0
Operator	0.381	2	0.191	0.23	0.794
Error	16.345	20	0.817		
Total	767.382	26			

Constrained (Type III) sums of squares.

```
stats_factorial = struct with fields:
         source: 'anovan'
          resid: [27×1 double]
         coeffs: [10×1 double]
            Rtr: [7×7 double]
       rowbasis: [7×10 double]
            dfe: 20
            mse: 0.8172
    nullproject: [10×7 double]
          terms: [3×3 double]
        nlevels: [3×1 double]
     continuous: [0 0 0]
         vmeans: [3×1 double]
       termcols: [4×1 double]
      coeffnames: {10×1 cell}
```

6.25 The rational of a statistical test

- One is testing an hypothesis H_o against an alternate hypothesis H_1.It must be binary :
  - H_o is that the effect of the variable τ is negligible : $\tau_1 = \tau_2 = \ldots = 0$
  - H_1 is that the above is not true for atleast one τ_i.
- For taking the decision above H_o or H_1, a criteria is chosen based on the result of a calculation (a statistic), x in this case.
- In the ANOVA, the statistic is the ratio x between the mean square related to the variable τ,MS_τ, and the mean square of the residue, MS_E.The law of x is the Fisher distribution

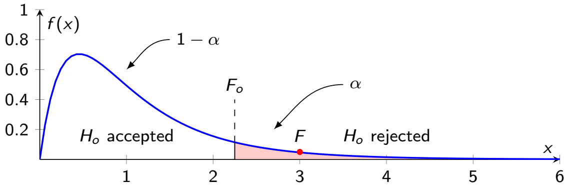
$$x = \frac{MS_\tau}{MS_E} \sim F_{\nu_1,\nu_2} \tag{13}$$

- The standard criteria is that H_o is rejected if $x \geqslant F_o$ with F_o being the ordinate of F_{ν_1,ν_2} defined so that $P(x \geqslant F_o) = \alpha = 5\%$.

6.26 Type I error : rejection of a true H_0

- When fixing a threshold α and then rejecting hypothesis H_o, there is a risk α that H_o is, in fact, correct.
- Example : a test is done to determine if the choice of the tool has an effect on the quality of the production.
  - H_o : "The choice of the tool is negligible"
  - H_1 "At least one tool has a detectable effect"
  - If $\alpha = 5\%$, it determines a limit value F_o to reject H_o.
  - When performing the ANOVA, if $x > F_o$, then H_o is rejected.
  - If the tools have, in fact, no effect, it would be a **false positive**.

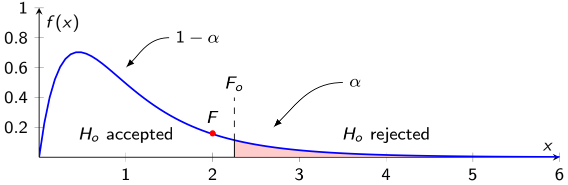
Fisher distribution: $\nu_1 = 4$, $\nu_2 = 20$



6.27 Type II error : acceptance of a wrong H_0

- When detecting an effect, depending on its magnitude there is a probability β that the effect does not exist.
- Same example
  - If $x < F_o$, H_o is accepted
  - If, in fact, one tool at least has an effect, it would be a **false negative**

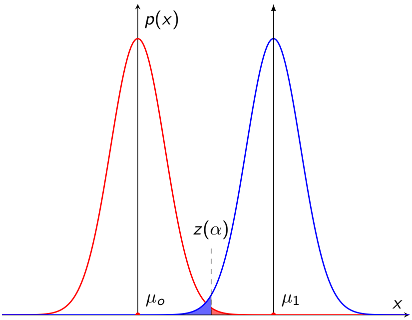
Fisher distribution: $\nu_1 = 4$, $\nu_2 = 20$



6.28 Probability of type I and II errors

Hypothesis		decision	
		don't reject	reject
H_o	true	$1-\alpha$	α
	false	β	$p = 1-\beta$

- The threshold α is chosen usually at 5%, implying a confidence level of 95% and a 5% risk for error type I.
- This risk of type I is the **risk of the producer** in the sense that if a lot is rejected, there is 5% risk of rejecting a product that is good.



- The probability β depends of α but also of other elements :
  - the sample variance
  - the number of samples
  - the magnitude of the effect

6.29 The concept of contrast

- Often the standard hypothesis $H_o : \mu_1 = \mu_i = 0$ is not answering the question of the investigator
- What is important is the comparison between treatments such as $H_o : \mu_3 = \mu_4$
- It is equivalent to $H_o : \mu_3 - \mu_4 = 0$
- A contrast is defined as (a is the nb of treatments)

$$\Gamma = \sum_{i=1}^{a} c_i \ \mu_i \tag{14}$$

- The t-statistics is then

$$t_o = \frac{\sum_{i=1}^{a} c_i \ \bar{y}_{i.}}{\sqrt{\frac{MS_E}{n} \sum_{i=1}^{a} c_i^2}} \tag{15}$$

6.30 Contrast confidence interval and LSD

- The confidence interval (CI) of a contrasts can be evaluated by

$$\Delta = t_{\alpha/2,\nu} \quad \sqrt{\frac{MS_E}{n} \sum_{i=1}^{a} c_i^2} \tag{16}$$

  ν being the DF of the model and then

$$\sum_{i=1}^{a} c_i \ \bar{y}_{i.} - \Delta \leqslant \Gamma \leqslant \sum_{i=1}^{a} c_i \ \bar{y}_{i.} + \Delta \tag{17}$$

  n being the number of samples for each treatment, N being the total number of observations
- The least significant difference (LSD) is defined as

$$LSD = t_{\alpha/2,\nu} \quad \sqrt{\frac{2MS_E}{n}} \tag{18}$$

6.31 LSD for factorial and latin square design

Factorial design

- Nb of obs $N = 27$
- Nb of obs by level $n = 9$
- Significance $\alpha = 5\%$
- $t_{0.975,20} = 2.1$
- $MS_E = 0.6$

$$LSD \approx 2.1 \times \sqrt{\frac{2 \times 0.6}{9}} \approx 0.42$$

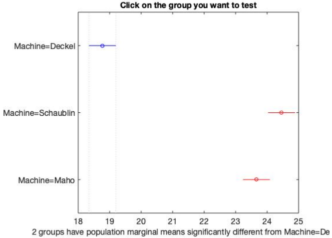
Latin square

- Nb of obs $N = 9$
- Nb of obs by level $n = 3$
- Significance $\alpha = 5\%$
- $t_{0.975,2} = 4.3$
- $MS_E = 0.22$

$$LSD \approx 4.3 \times \sqrt{\frac{2 \times 0.22}{3}} \approx 1.65$$

6.32 Matlab multcompare routine

```
[c1,m1]=multcompare(stats_factorial,...
    "Alpha",0.05,...
    "CType","scheffe",...
    "Dimension",1,...
    "Estimate","column");
```




```
Pair comparison
    1.0000    2.0000   -6.5465   -5.6908   -4.8352    0.0000
    1.0000    3.0000   -5.7470   -4.8914   -4.0357    0.0000
    2.0000    3.0000   -0.0562    0.7995    1.6551    0.0698
Effects
   18.7679    0.2289
   24.4588    0.2289
```

6.33 Summary of ANOVA for CCM

- Decompose Y in orthogonal components
- Compute the sum of the squares
- Determine the degrees of freedom
- Compute the mean squares
- Compare with the residuals
- Disqualify the insignificants effects
- Compute again the error probability
- Analyse pairs of effects to determine significant contrasts

6.34 Youden squares

A Youden square is a type of Latin square where each row and each column contains a unique set of treatments or conditions.

	B_1	B_2	B_3	B_4
C_1	A_1	A_2	A_3	A_4
C_2	A_2	A_3	A_4	A_1
C_3	A_3	A_4	A_1	A_2

6.34 Balanced Designs in DOE

- **Completely Randomized Design (CRD) :**
  - Each treatment is assigned to experimental units completely at random.
  - Ensures that every treatment has the same chance of being applied to any unit.
- **Randomized Block Design (RBD) :**
  - Experimental units are divided into blocks based on a known source of variability.
  - Treatments are randomly assigned within each block, balancing the design across blocks.
- **Latin Square Design :**
  - Controls for two sources of variability.
  - Treatments appear exactly once in each row and each column of a $k \times k$ square matrix.
- **Graeco-Latin Square Design :**
  - An extension of the Latin square design that controls for three sources of variability.
  - Uses two Latin squares overlaid so that each treatment combination appears once.

6.34 Balanced Designs in DOE

- **Youden Square Design :**
  - Derived from a Latin square, but with one fewer column ($k \times (k-1)$).
  - Useful when there is an unbalanced number of treatments or when one treatment is repeated.
- **Factorial Design :**
  - All possible combinations of levels of factors are investigated.
  - Can be balanced by ensuring equal replication of each treatment combination.
- **Balanced Incomplete Block Design (BIBD) :**
  - Not all treatments are applied in every block, but each pair of treatments appears together in the same block an equal number of times.
  - Balances the design even with an incomplete representation of treatments.

6.34 Balanced Designs in DOE

- **Split-Plot Design :**
  - Two levels of randomization : main plots receive one set of treatments, and subplots within main plots receive another set.
  - Ensures balance within the main and subplot treatments.
- **Crossover Design :**
  - Subjects receive multiple treatments in a sequential manner.
  - Balanced by ensuring each treatment is applied in every possible position across different subjects.

6.35 Conclusions