

# 11: Echo formation and spatial encoding

1. What makes the magnetic resonance signal spatially dependent ?
2. How is the position of an MR signal identified ?  
Slice selection
3. What is echo formation and how is it achieved ?  
Echo formation  
Gradient echo sequence
4. How is a two-dimensional MR image encoded ?

After this course you

1. Understand the principle of slice selection
2. Are familiar with dephasing and rephasing of transverse magnetization and how it leads to echo formation
3. Understand the principle of spatial encoding in MRI
4. Can describe the basic imaging sequence and the three necessary elements
5. Understand the principle of image formation in MRI and how it impacts spatial resolution

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## 11-1. What do we know about magnetic resonance so far ?

Adding a 3<sup>rd</sup> magnetic field

So far

- 1) Excite spins using RF field at  $\omega_L$
- 2) Record time signal (Known as FID)
- 3)  $M_{xy}$  decays,  $M_z$  grows ( $T_2$  and  $T_1$  relaxation)

RF coils measure signal from **entire** body (no spatial information)

### **$B_0$ : Static Magnetic Field**

Creates equilibrium magnetization

0.1 T to 12 T

» Earth's field is  $0.5 \cdot 10^{-4}$  T

### **$B_1$ : Radiofrequency Field (RF)**

0.05mT, on resonance

Detection of MR signal (RF coils)

Precessional Frequency  $\omega_L = \gamma B_0$  Static Magnetic Field

How to encode spatial position ?

$$B(x) = B_0 + G_x x \rightarrow \omega_L = f(x)$$

Magnetic field B along z varies spatially with x, y, and/or z:

$$\vec{G} \equiv \frac{dB_z}{d\vec{r}}$$

$$B_z(\vec{r}) = B_0 + \vec{G} \cdot \vec{r}$$

e.g.  $\vec{G} = (G_x, 0, 0)$

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# How is the gradient field created ?

One coil for each spatial dimension:  $G_x$ ,  $G_y$ ,  $G_z$

## G: Gradient Field

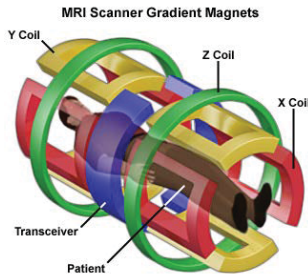
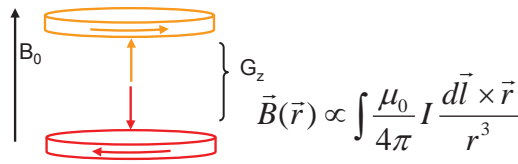
10-50 mT/m in  $\sim 100\mu\text{s}$

Used to determine spatial position of signal (frequency)

Created by a set of 3 additional coils (gradient coil)

$$\vec{G} \equiv \frac{dB_z}{d\vec{r}}$$

Example: z-gradient coil principle (Helmholtz pair)



NB. Why are MRI scans so loud ?

Lorentz-force of  $B_z$  (3T) on rapidly switched current in gradient coil (wire)

( $\sim 100\text{A}$  in  $\sim 100\mu\text{s}$ )

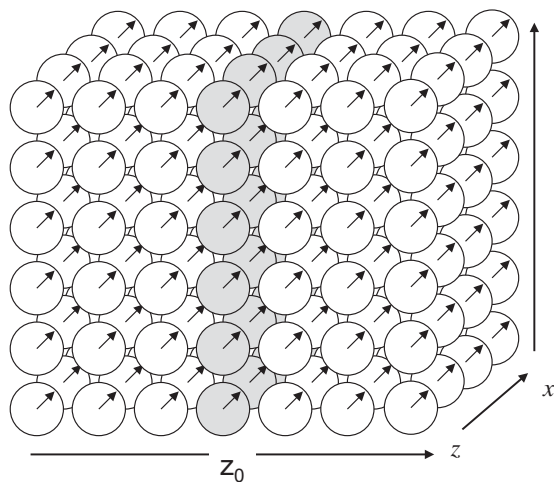
# How is slice-selection achieved ?

Only magnetization on-resonance is excited

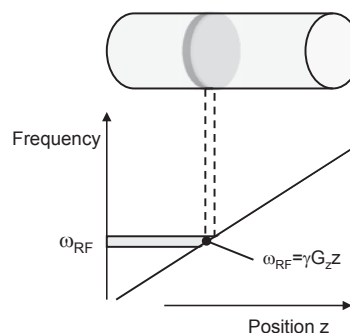
## On-resonance:

Frequency  $\omega_{\text{RF}}$  of RF field  $B_1$  matches the precession frequency of magnetization

Moving Frequency  $\omega_{\text{RF}}$  alters position of slice :



$$\omega_{\text{RF}} = \gamma B_0 + \gamma G_z z_0$$



NB. Not to confuse:

(x,y) refers to spatial dimensions

$M_{xy}$  M or  $M_{\perp}$  refers to transverse magnetization (in magnetization space)

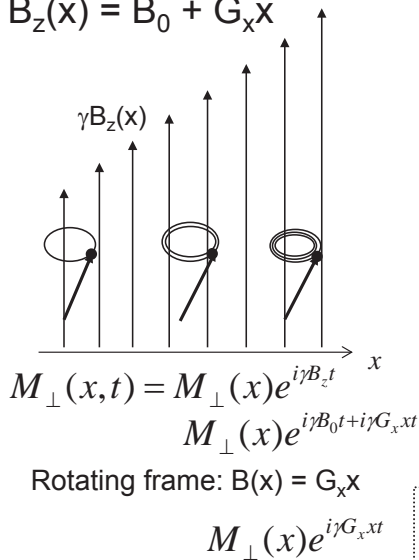
(coordinate systems are different, but share z)

## 11-2. What is the basic principle of encoding spatial information ?

frequency encoding - 1D example

Spatial-varying resonance frequency  $\gamma B_z(x)$  during detection

$$B_z(x) = B_0 + G_x x$$



Detected signal = sum of all precessing magnetization:

$$S(t) \propto \int_{\text{object}} M_{\perp}(x, 0) e^{i\gamma G_x x t} dx$$

What does this resemble ?

$$S(t) \propto \int_{\text{object}} M_{\perp}(\omega, 0) e^{i\omega t} d\omega$$

= Inverse Fourier Transformation !

$$S(k) \propto \int_{\text{object}} M_{\perp}(x, 0) e^{ikx} dx$$

FT of  $S(t)$  (or  $S(k)$ )  
→  $M(x)$

Reconstruction as in  
CT (in principle)

For 2D object:

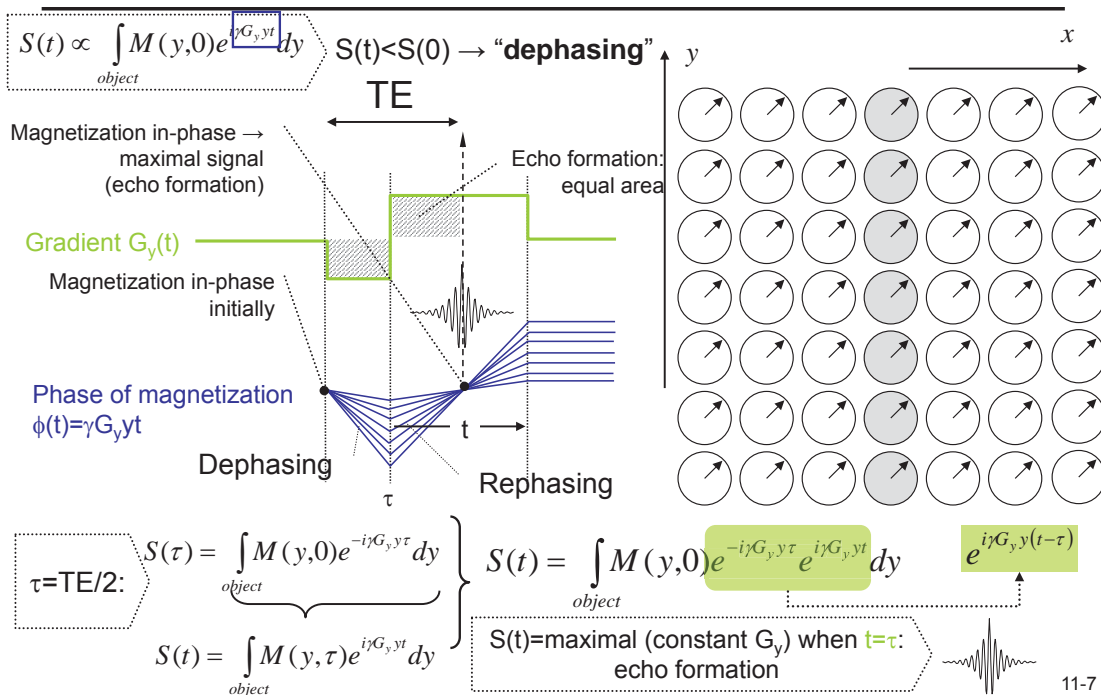
$$M(x) = \int M(x, y) dy$$

= Radon Transform 11-5

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## 11-3. When is the signal maximal in the presence of G ?

Echo formation: Dephasing and rephasing

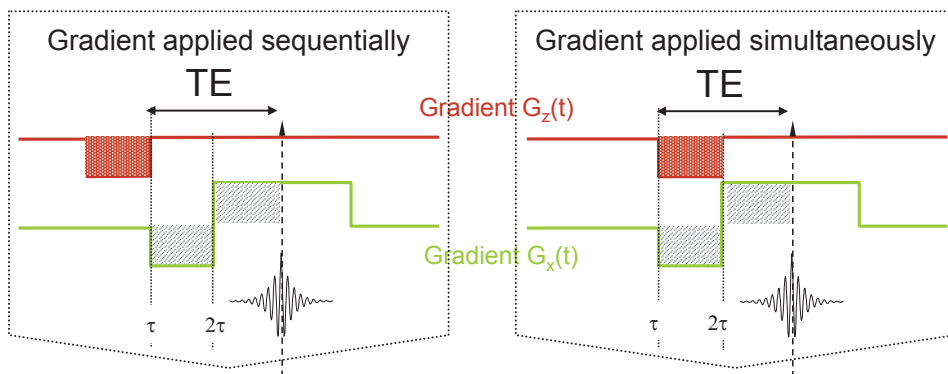


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# Is it important when a gradient is applied ?

gradient applied at different time has the same effect on magnetization phase



Question: Is there a difference in effect on echo ?

$$M(\tau) = M(0)e^{-i\gamma G_z z \tau}$$

$$M(2\tau) = M(\tau)e^{-i\gamma G_x x \tau} = M(0)e^{-i\gamma G_x x \tau} e^{-i\gamma G_z z \tau}$$

$$M(\tau) = M(0)$$

$$M(2\tau) = M(\tau)e^{-i\gamma(G_x x + G_z z)\tau} = M(0)e^{-i\gamma G_x x \tau} e^{-i\gamma G_z z \tau}$$

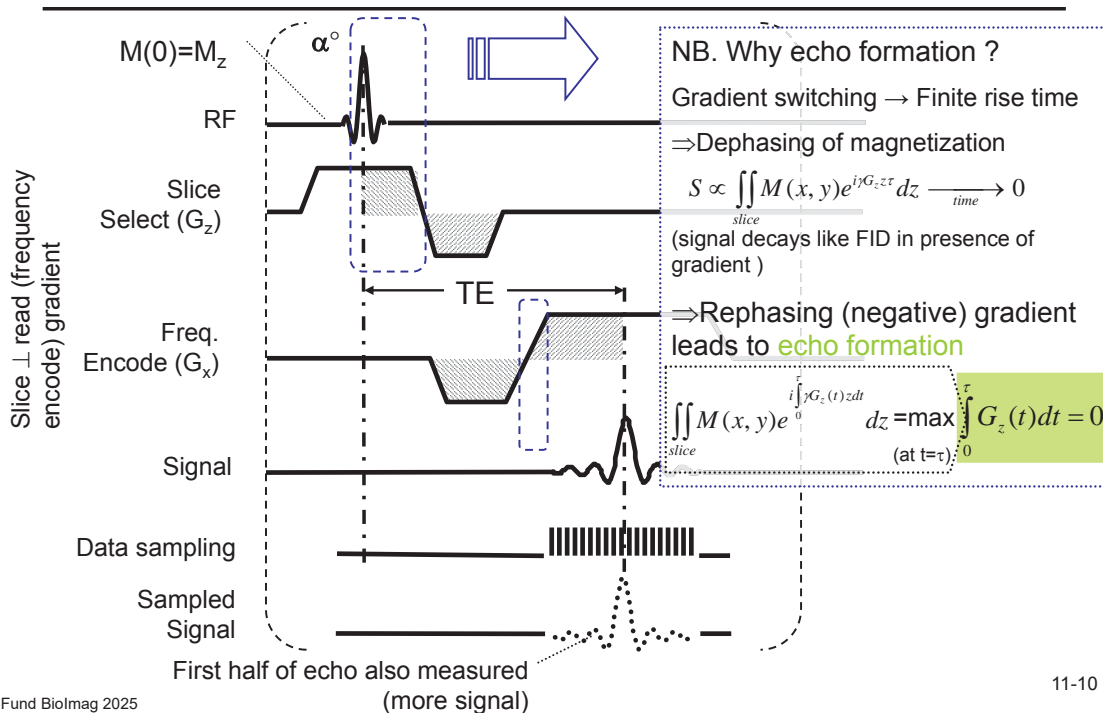
$$M(TE) = M(2\tau)e^{i\gamma G_x x TE/2}$$

$$M(TE) = M(0)e^{-i\gamma(G_x x + G_z z)\tau} e^{i\gamma G_x x TE/2} = M(0)e^{-i\gamma G_z z \tau}$$

Application of two orthogonal gradients simultaneously or sequentially generates the same phase for  $M_{xy}$

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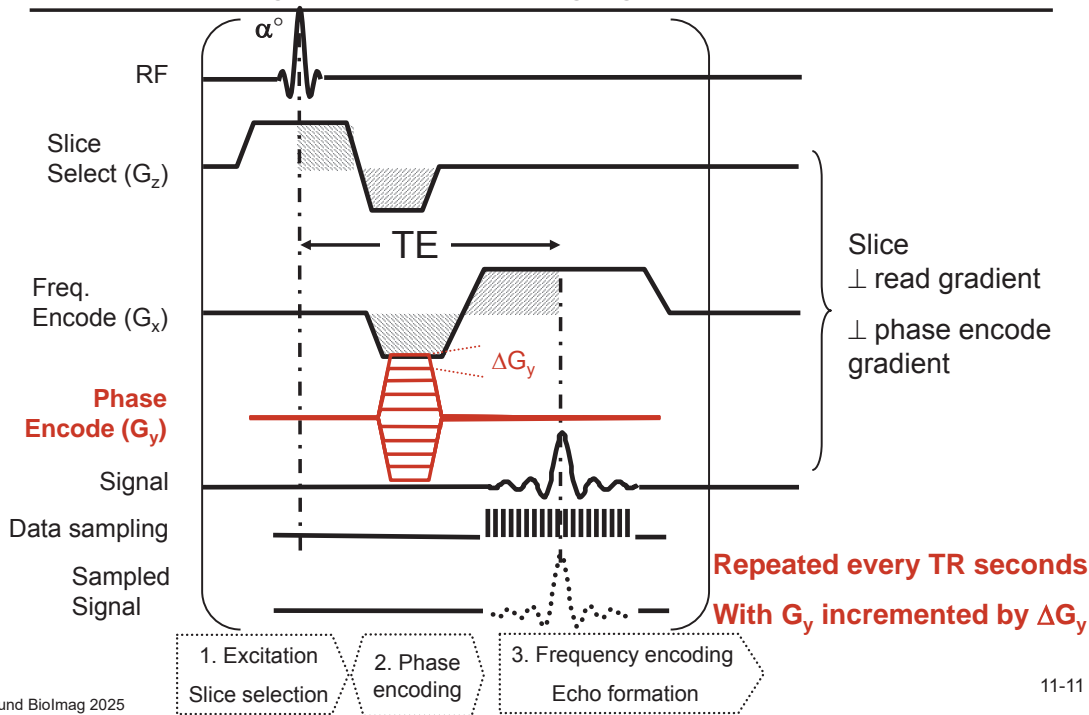
## What are the basic elements of the Gradient echo sequence ?



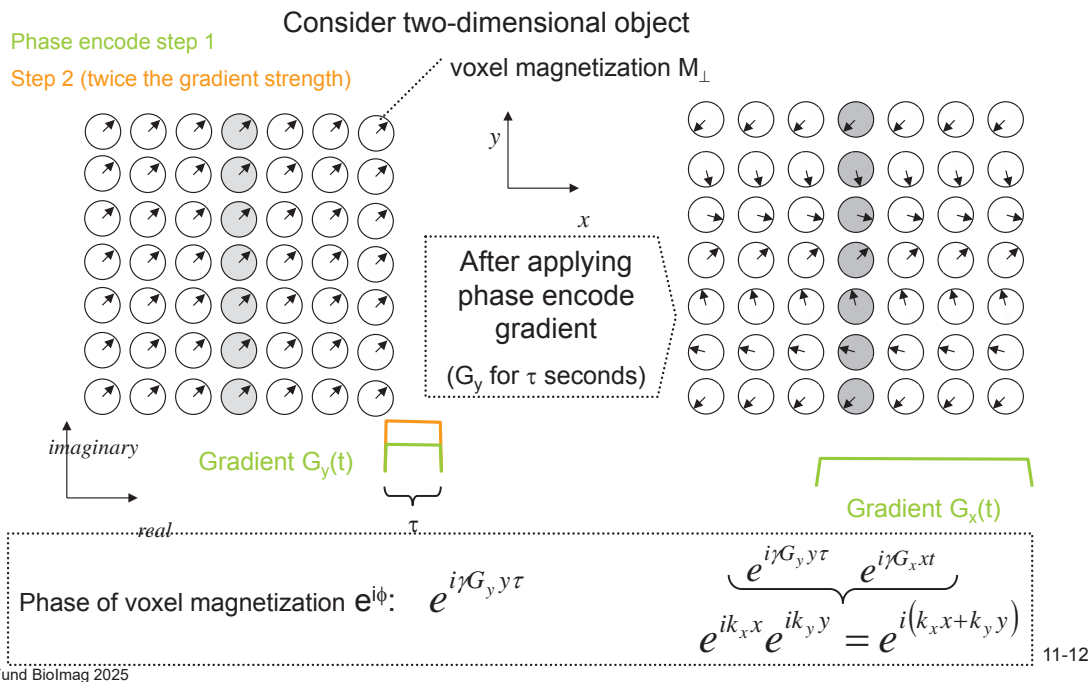
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## 11-4. How is the 2<sup>nd</sup> dimension encoded ?

gradient echo imaging sequence

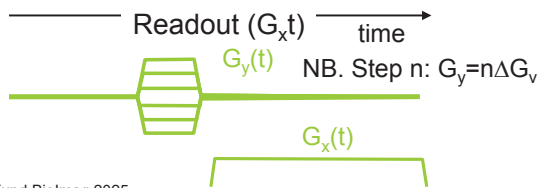
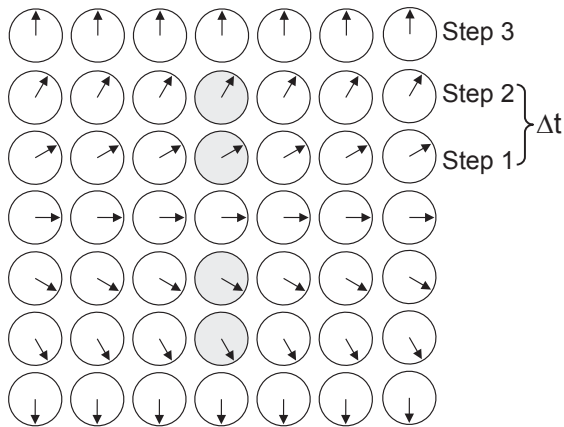


## How does the phase encoding gradient encode the 2<sup>nd</sup> spatial dimension ?



## How is incrementing the phase step-by-step (phase encoding) equivalent to frequency encoding ?

Phase  $\phi$  of a single pixel in x,y plane:



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$$\omega \equiv \frac{d\phi}{dt}$$

Signal of the single voxel:

$$S(n, \Delta G_y, t) \propto M_{\perp}(x, y) e^{i n \Delta G_y y \tau} e^{i \gamma G_x x t}$$

$$\left. \begin{array}{l} k_x \equiv \gamma G_x t \\ k_y \equiv \gamma G_y \tau \end{array} \right\} S(k_x, k_y) \propto M_{\perp}(x, y) e^{i(k_x x + k_y y)}$$

Signal of the entire object :

$$S(k_x, k_y) \propto \iint_{\text{object}} M_{\perp}(x, y) e^{i(k_x x + k_y y)} dx dy$$

What does this resemble ?

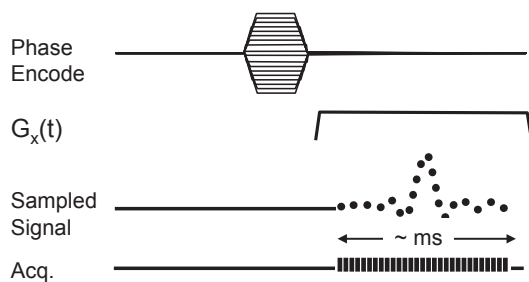
$$M_{\perp}(x, y) \propto \iint_{\text{object}} S(k_x, k_y) e^{-i(k_x x + k_y y)} dk_x dk_y$$

**MR image generation:**  
**FT of the signal**

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## 11-5. How is the spatial information encoded in MRI ?

scanning k-space (Fourier or reciprocal space) sequentially

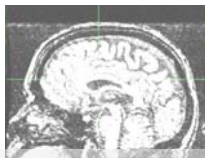


For k-space line every TR=1s:  
256<sup>2</sup> image matrix > 4 min

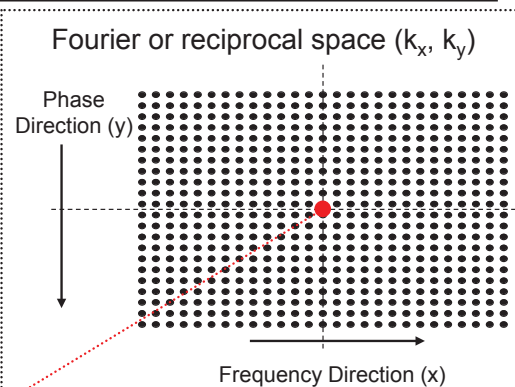
One line of k-space  
acquired per TR

MR scans are long  
and  
motion-sensitive

Subject moved  
head during  
acquisition



Ghosting and ringing artifacts



Maximum  $k_x$  (or  $k_y$ )  $\Leftrightarrow$  Resolution (Nyquist)  
Increment  $\Delta k \Leftrightarrow$  Field-of-view

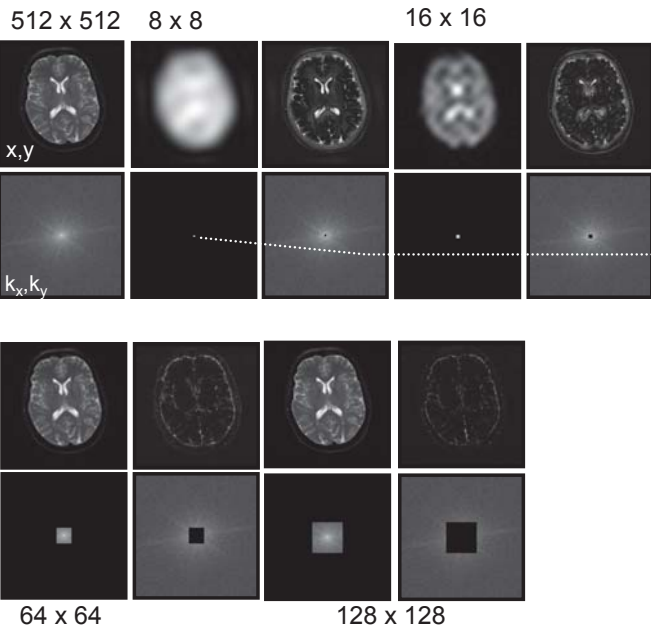
**Uniform** resolution and sensitivity  
(Limited by voxel magnetization)

center of k-space ( $k_x, k_y=0$ )

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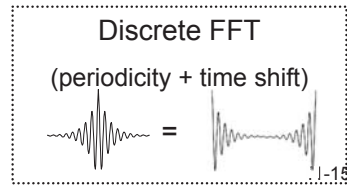
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# What are some effects of incomplete sampling ? of Fourier space (k-space)



Time of acquisition of center of k-space point ( $k_x, k_y=0$ ) determines contrast of image:

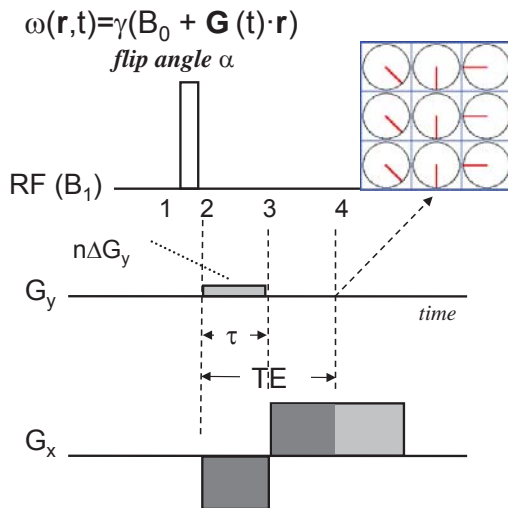
$$S(0,0) = \iint_{\text{object}} M_{\perp}(x,y) dx dy \Big|_{k_x=0, k_y=0}$$



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## Summary: Spatial encoding with gradients

Phase encoding, echo formation + 2DFT



Magnetization at time points specified:

- 1:  $(0,0,M_z)$   
rotated by RF pulse by  $\alpha^0$  about x:
- 2:  $(0, M_z \sin \alpha, M_z \cos \alpha) \equiv (0, M_y, \dots)$   
[now only consider  $M_{xy}$ ]  
Precesses with  $B = -\gamma G_y y - \gamma G_x x$
- 3:  $M_y [\sin[-\gamma(n\Delta G_y y + G_x x)\tau], \cos[-\gamma(n\Delta G_y y + G_x x)\tau]]$   
 $\equiv M_y [\sin(-\phi_x), \cos(-\phi_x)]$ ,  
with  $\phi_x = \gamma G_x x \tau$  (rotation by angle  $-\phi_x$ )  
inverting gradient, i.e.  $B = +\gamma G_x x$ :  
after another  $\tau$ , **rotates by angle  $+\phi_x$**   
 $\Rightarrow$  maximal signal at  $TE=2\tau$  ( $\Delta G_y=0$ )
- 4:  $M_y(0,1,\dots) = (0, M_z \sin \alpha, M_z \cos \alpha)$   
 $\rightarrow$  **Echo formation**

$$\begin{aligned} \text{Signal } S(\tau, t) &\propto M_{\perp}(t) = \iint m(x,y,t) dx dy \\ &= \iint m(x,y,0) e^{-i\gamma(n\Delta G_y y)\tau} e^{-i\gamma G_x x(\tau+t)} dx dy \\ S(k_x, k_y) &\propto \iint m_{\perp}(x,y) e^{-ik_x x} e^{-ik_y y} dx dy \end{aligned}$$

MRI measures the **2D Fourier transformation** of the object  
(measuring the 2<sup>nd</sup> dimension requires time!)

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