

11: Echo formation and spatial encoding

1. What makes the magnetic resonance signal spatially dependent ?
2. How is the position of an MR signal identified ?
 - Slice selection
3. What is echo formation and how is it achieved ?
 - Echo formation
 - Gradient echo sequence
4. How is a two-dimensional MR image encoded ?

After this course you

1. Understand the principle of slice selection
2. Are familiar with dephasing and rephasing of transverse magnetization and how it leads to echo formation
3. Understand the principle of spatial encoding in MRI
4. Can describe the basic imaging sequence and the three necessary elements
5. Understand the principle of image formation in MRI and how it impacts spatial resolution

11-1

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11-1. What do we know about magnetic resonance so far ?

Adding a 3rd magnetic field

So far

- 1) Excite spins using RF field at ω_L
- 2) Record time signal (Known as FID)
- 3) M_{xy} decays, M_z grows (T_2 and T_1 relaxation)

RF coils measure signal from **entire** body (no spatial information)

B_0 : Static Magnetic Field

Creates equilibrium magnetization

0.1 T to 12 T

» Earth's field is $0.5 \cdot 10^{-4}$ T

B_1 : Radiofrequency Field (RF)

0.05mT, on resonance

Detection of MR signal (RF coils)

Precessional Frequency $\omega_L = \gamma B_0$ Static Magnetic Field

$$B_z(\vec{r}) = B_0 + \vec{G} \cdot \vec{r}$$

e.g. $\mathbf{G} = (G_x, 0, 0)$

How to encode spatial position ?

$$B(x) = B_0 + G_x x \rightarrow \omega_L = f(x)$$

Magnetic field B along z varies spatially with x , y , and/or z :

$$\vec{G} \equiv \frac{dB_z}{d\vec{r}}$$

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How is the gradient field created ?

One coil for each spatial dimension: G_x , G_y , G_z

G: Gradient Field

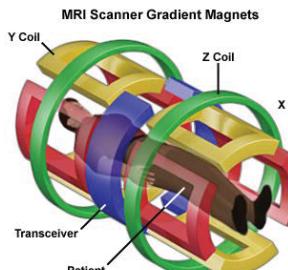
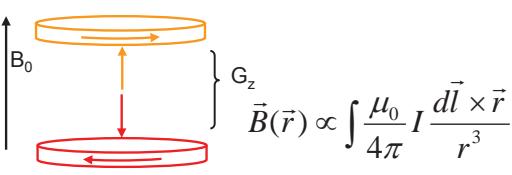
10-50 mT/m in $\sim 100\mu\text{s}$

$$\vec{G} \equiv \frac{dB_z}{dr}$$

Used to determine spatial position of signal (frequency)

Created by a set of 3 additional coils (gradient coil)

Example: z-gradient coil principle (Helmholtz pair)



NB. Why are MRI scans so loud ?

Lorentz-force of B_z (3T) on rapidly switched current in gradient coil (wire)

($\sim 100\text{A}$ in $\sim 100\mu\text{s}$)

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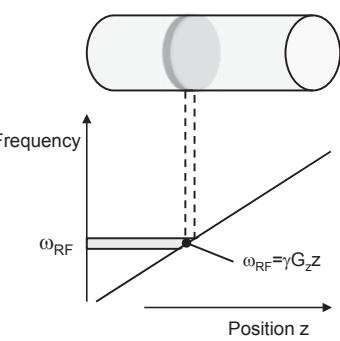
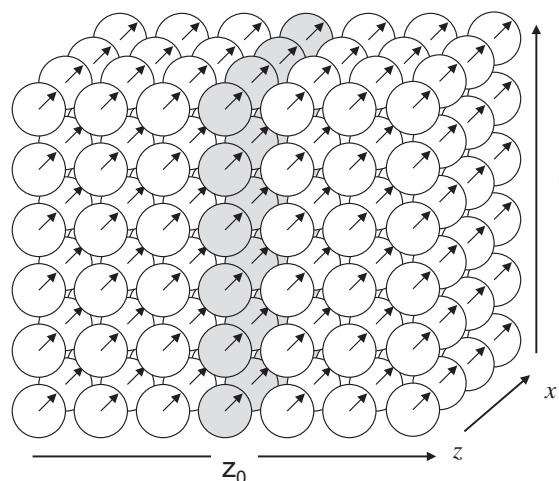
How is slice-selection achieved ?

Only magnetization on-resonance is excited

On-resonance:

Frequency ω_{RF} of RF field B_1 matches the precession frequency of magnetization

Moving Frequency ω_{RF} alters position of slice :



NB. Not to confuse:

(x,y) refers to spatial dimensions

M_{xy} or M_{\perp} refers to transverse magnetization (in magnetization space)

(coordinate systems are different, but share z)

$$\omega_{RF} = \gamma B_0 + \gamma G_z z_0$$

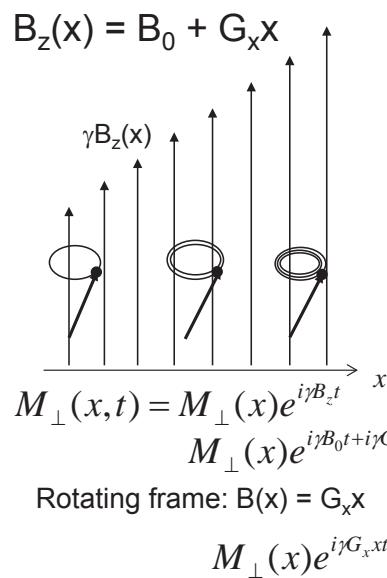
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11-2. What is the basic principle of encoding spatial information ?

frequency encoding - 1D example

Spatial-varying resonance frequency $\gamma B(x)$ during detection



Detected signal = sum of all precessing magnetization:

$$S(t) \propto \underbrace{\int M_{\perp}(x, 0) e^{i\gamma G_x x t} dx}_{\text{object}}$$

What does this resemble ?

$$S(t) \propto \underbrace{\int M_{\perp}(\omega, 0) e^{i\omega t} d\omega}_{\text{object}}$$

= Inverse Fourier Transformation !

$$S(k) \propto \underbrace{\int M_{\perp}(x, 0) e^{ikx} dx}_{\text{object}}$$

For 2D object:

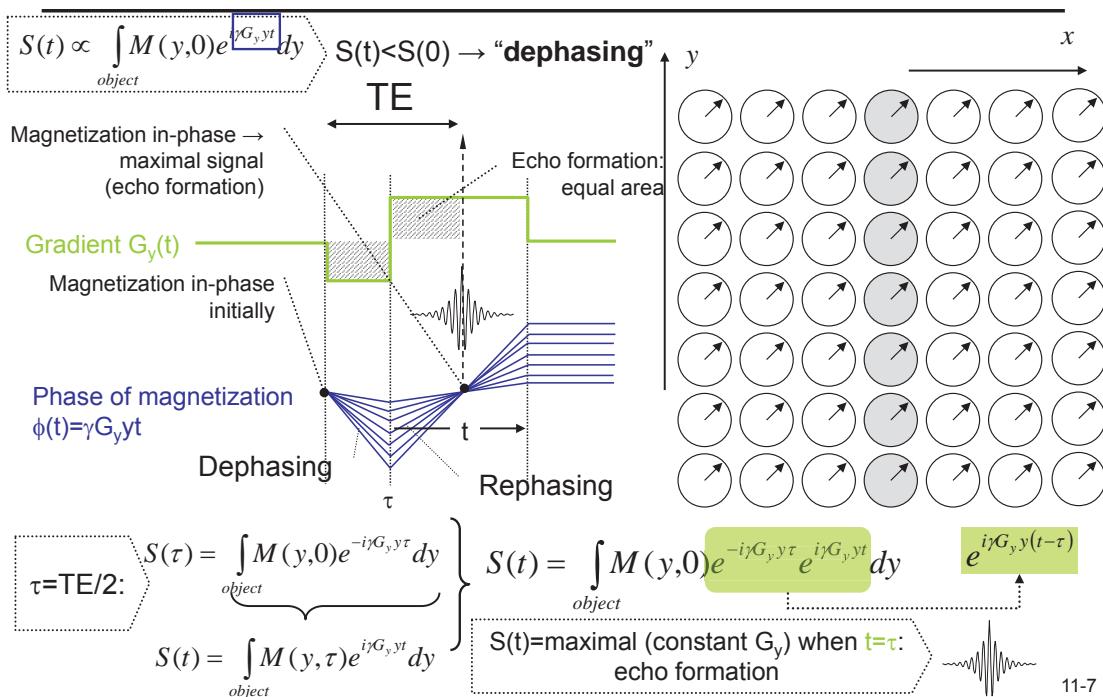
$$M(x) = \int M(x, y) dy$$

= Radon Transform 11-5

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11-3. When is the signal maximal in the presence of G ?

Echo formation: Dephasing and rephasing

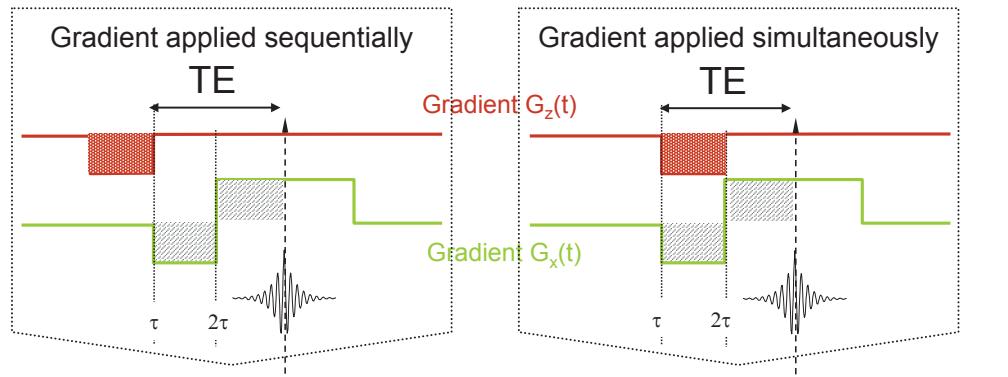


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Is it important when a gradient is applied ?

gradient applied at different time has the same effect on magnetization phase



Question: Is there a difference in effect on echo ?

$$M(\tau) = M(0)e^{-i\gamma G_z z\tau}$$

$$M(2\tau) = M(\tau)e^{-i\gamma G_x x\tau} = M(0)e^{-i\gamma G_x x\tau} e^{-i\gamma G_z z\tau}$$

$$M(\tau) = M(0)$$

$$M(2\tau) = M(\tau)e^{-i\gamma(G_x x + G_z z)\tau} = M(0)e^{-i\gamma G_x x\tau} e^{-i\gamma G_z z\tau}$$

$$\gamma(\vec{G} \cdot \vec{r})\tau$$

$$M(TE) = M(2\tau)e^{i\gamma G_x x TE/2}$$

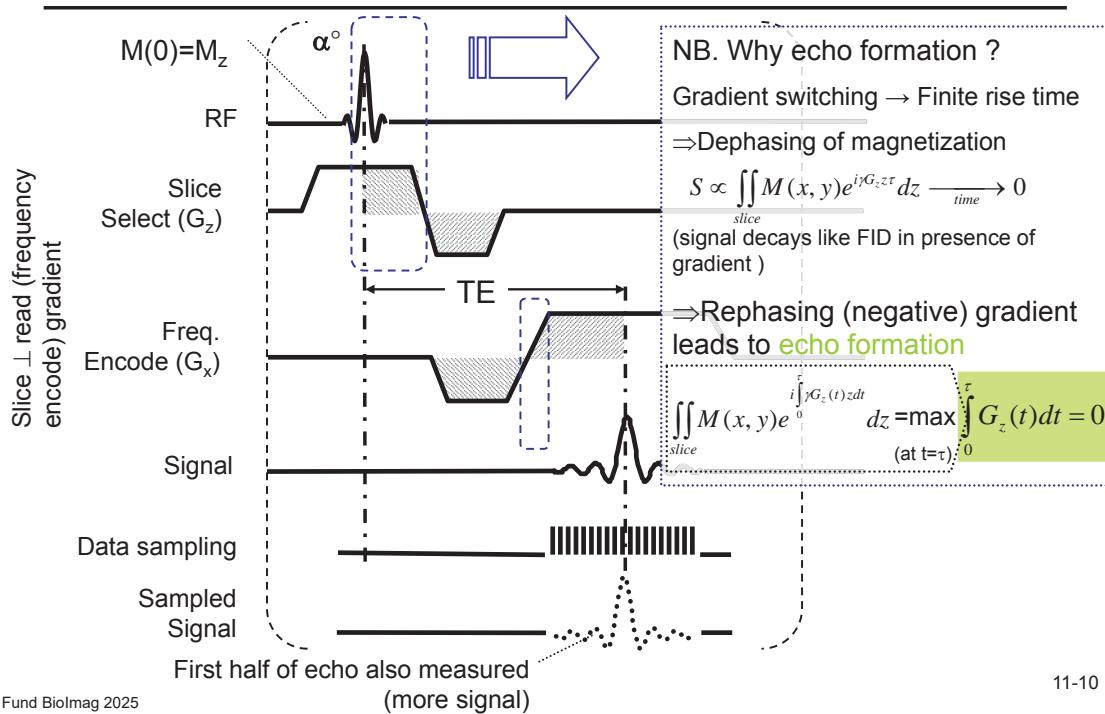
$$M(TE) = M(0)e^{-i\gamma(G_x x + G_z z)\tau} e^{i\gamma G_x x TE/2} = M(0)e^{-i\gamma G_z z\tau}$$

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Application of two orthogonal gradients simultaneously or sequentially generates the same phase for M_{xy}

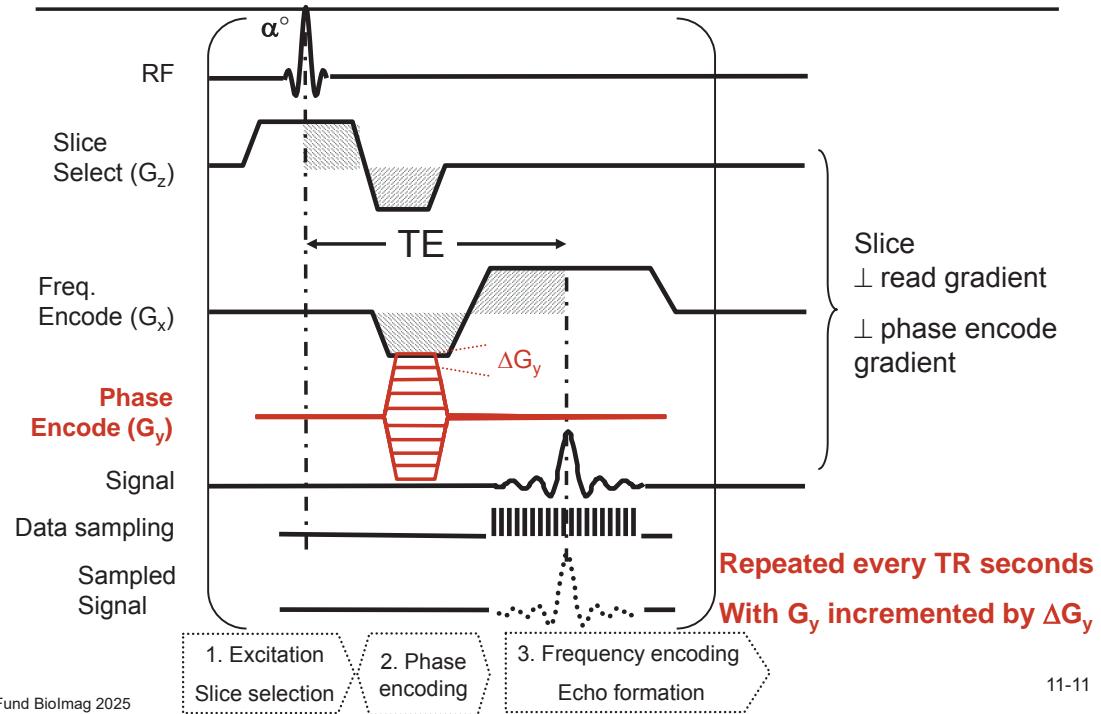
What are the basic elements of the Gradient echo sequence ?



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11-4. How is the 2nd dimension encoded ?

gradient echo imaging sequence

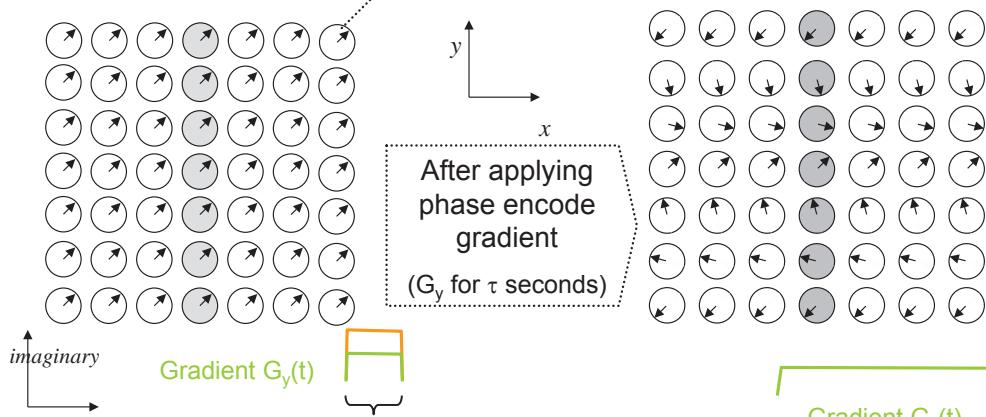


How does the phase encoding gradient encode the 2nd spatial dimension ?

Consider two-dimensional object

Phase encode step 1

Step 2 (twice the gradient strength)



Phase of voxel magnetization $e^{i\phi}$:

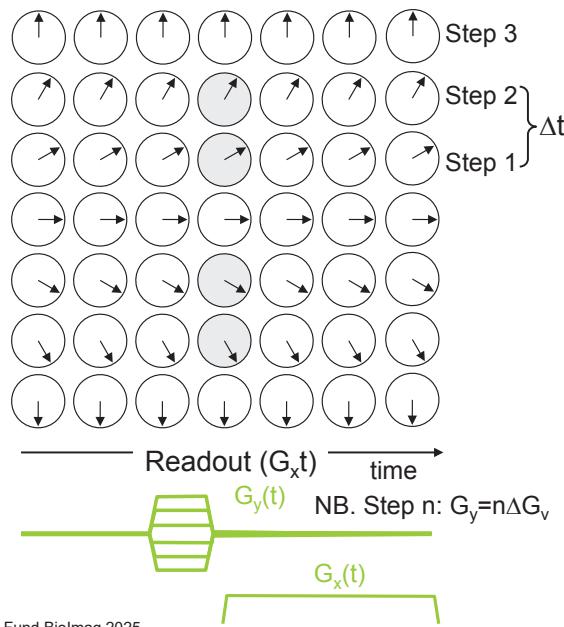
$$e^{i\gamma G_y y \tau}$$

$$e^{i\gamma G_y y \tau} \underbrace{e^{i\gamma G_x x \tau}}_{e^{i(k_x x + k_y y)}} = e^{i(k_x x + k_y y)}$$

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How is incrementing the phase step-by-step (phase encoding) equivalent to frequency encoding ?

Phase ϕ of a single pixel in x,y plane:



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$$\omega \equiv \frac{d\phi}{dt}$$

Signal of the single voxel:

$$S(n, \Delta G_y, t) \propto M_{\perp}(x, y) e^{i\gamma n \Delta G_y y \tau} e^{i\gamma G_x x t}$$

$$\left. \begin{array}{l} k_x \equiv \gamma G_x t \\ k_y \equiv \gamma G_y \tau \end{array} \right\} S(k_x, k_y) \propto M_{\perp}(x, y) e^{i(k_x x + k_y y)}$$

Signal of the entire object :

$$S(k_x, k_y) \propto \iint_{\text{object}} M_{\perp}(x, y) e^{i(k_x x + k_y y)} dx dy$$

What does this resemble ?

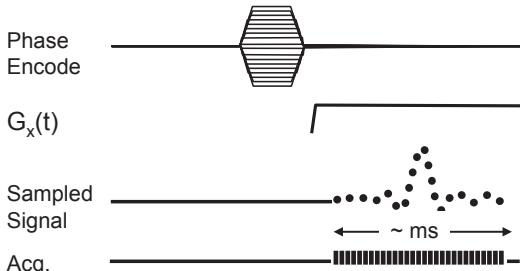
$$M_{\perp}(x, y) \propto \iint_{\text{object}} S(k_x, k_y) e^{-i(k_x x + k_y y)} dk_x dk_y$$

MR image generation:
FT of the signal

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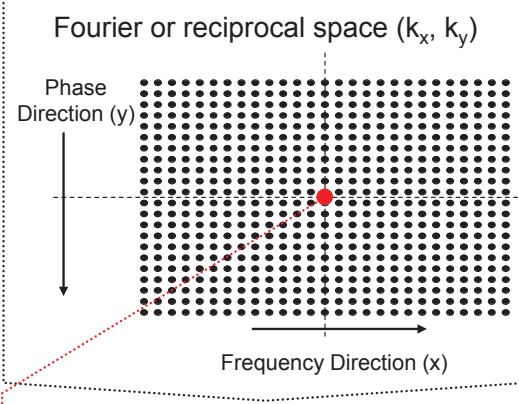
11-5. How is the spatial information encoded in MRI ?

scanning k-space (Fourier or reciprocal space) sequentially

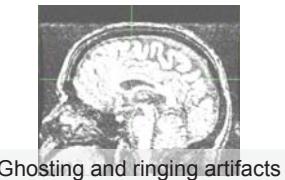


One line of k-space acquired per TR

MR scans are long and motion-sensitive



Subject moved head during acquisition

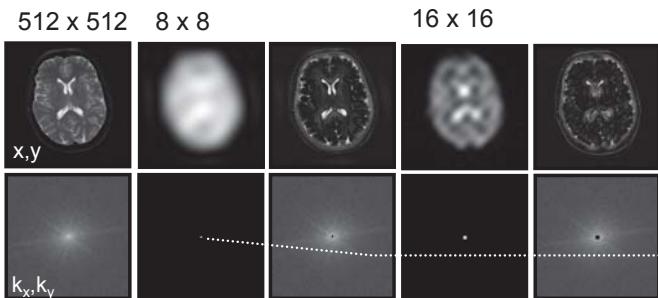


Uniform resolution and sensitivity
(Limited by voxel magnetization)

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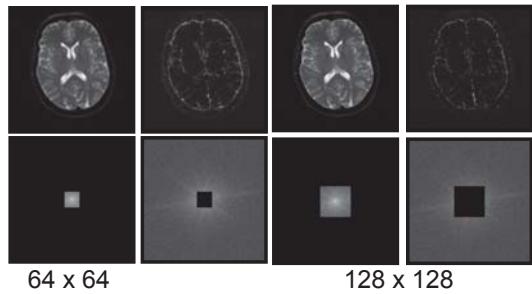
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What are some effects of incomplete sampling ? of Fourier space (k-space)



Time of acquisition of center of k-space point ($k_x, k_y=0$) determines contrast of image:

$$S(0,0) = \left. \iint M_{\perp}(x, y) dx dy \right|_{k_x=0, k_y=0}$$



Discrete FFT

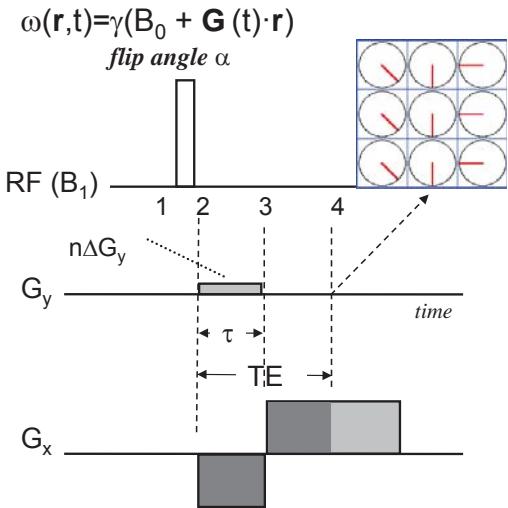
(periodicity + time shift)



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Summary: Spatial encoding with gradients

Phase encoding, echo formation + 2DFT



Magnetization at time points specified:
 1: $(0,0,M_z)$
 rotated by RF pulse by α^0 about x:
 2: $(0,M_z \sin \alpha, M_z \cos \alpha) \equiv (0, M_y, \dots)$
 [now only consider M_{xy}]
 Precesses with $B = -\gamma G_y y - \gamma G_x x$
 3: $M_y(\sin[-\gamma(n\Delta G_y y + G_x x)\tau], \cos[-\gamma(n\Delta G_y y + G_x x)\tau]) \equiv M_y[\sin(-\phi_x), \cos(-\phi_x)]$,
 with $\phi_x = \gamma G_x x \tau$ (rotation by angle $-\phi_x$)
 inverting gradient, i.e. $B = +\gamma G_x x$:
 after another τ , rotates by angle $+\phi_x$
 \Rightarrow maximal signal at $TE=2\tau$ ($\Delta G_y=0$)
 4: $M_y(0,1,\dots) = (0, M_z \sin \alpha, M_z \cos \alpha)$
 \rightarrow **Echo formation**

$$\begin{aligned} \text{Signal } S(\tau, t) &\propto M_{\perp}(t) = \iint m(x, y, t) dx dy \\ &= \iint m(x, y, 0) e^{\gamma(n\Delta G_y y + G_x x)\tau} e^{\gamma G_x x(t+\tau)} dx dy \\ S(k_x, k_y) &\propto \iint m_{\perp}(x, y) e^{-k_x x} e^{-k_y y} dx dy \end{aligned}$$

MRI measures the **2D Fourier transformation** of the object
 (measuring the 2nd dimension requires time!)

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