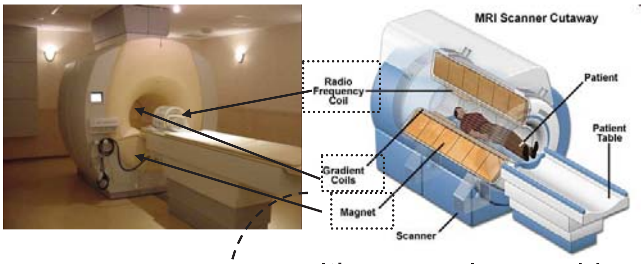
8: Introduction to Magnetic Resonance

1. What are the components of an MR scanner ?
2. What is the basis of the MR signal ?
3. How is nuclear magnetization affected by an external magnetic field ?
4. What affects the equilibrium magnetization ?
5. How do we best describe the motion of magnetization (in the rotating frame of reference) ?

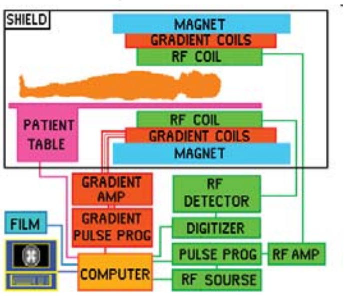
After this week you
1. Are familiar with the prerequisites for nuclear spin
2. know the factors determining nuclear magnetization
3. Can compare magnetizations for different nuclei and magnetic field
4. Know the equation of motion for magnetization
5. Are able to describe the motion of magnetization in lab and rotating frame
6. Understand that MRI has complex mechanisms

8-1. What are the essential components of an MRI scanner ?



It's a complex machine …

Cut-open in real life



Schematic depiction of all MRI components

This course focuses on the major elements of MRI :
- Nucleus
- Magnet
- RF coil
- Gradient coil

What are the risks of the scanner being never off ?

Superconducting wires cooled to lHe temperature (4K)

Current stays for 1000 years …

It's a powerful magnet …

Magnetic field B_0 [unit: Tesla, T]

Earth's magnetic field ~ 5 10^{-5} T

Electromagnets < 1.5 T

MRI 1-7 T

8-2. What is the basis of Nuclear Magnetism ?

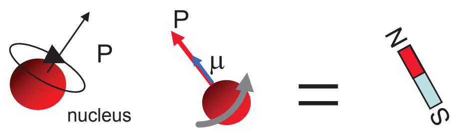
Classical and quantum-mechanical view

Nucleus → angular momentum L (here called P)

⇒ Rotation of electrical charge (nucleus)

⇒ Rotating current

⇒ Dipole moment



Magnetic moment μ of individual spin in induction field B_o $\vec{\mu} = \gamma \vec{P}$

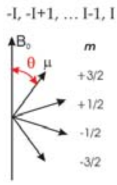
γ: **gyromagnetic ratio** (empirical constant)

The angular momentum P of a nucleus is quantized:

P_z has 2I + 1 values (m):

$$P_z = \frac{h}{2\pi} \cdot m_I$$

$$|\vec{P}| = \frac{h}{2\pi} \cdot \sqrt{I \cdot (I+1)}$$



Spin ½: P=h√3/4π

NMR-active isotopes and their **gyromagnetic ratio** γ

I = 0 (^{12}C, ^{16}O, etc.)
Even mass # & Even atomic #
No Nuclear spin

I = ½ (^{1}H, ^{13}C, ^{15}N etc.)
Spherical charge distribution in nucleus

I > ½ (^{2}H, ^{11}B, ^{23}Na etc.)
Odd mass # & Odd atomic # (I = ½ integer)
Even mass # & Odd atomic # (I=whole integer)
Ellipsoidal charge distribution in nucleus
gives ***quadrapolar electric field***

Isotope	Net Spin (I)	gyromagnetic ratio γ/2π [MHz T^{-1}]	Abundance / %
^{1}H	1/2	42.58	99.98
^{2}H	1	6.54	0.015
^{31}P	1/2	17.25	100.0
^{23}Na	3/2	11.27	100.0
^{15}N	1/2	4.31	0.37
^{13}C	1/2	10.71	1.108
^{19}F	1/2	40.08	100.0

What is the basis for nuclear magnetization ?

Unequal population of Energy levels

Energy of a magnetic dipole in magnetic field B_0 (classical)

$$E = -\vec{\mu} \cdot \vec{B}_0 = -\mu \cdot \cos\theta \cdot B_0 = -\mu_z \cdot B_0$$

Energy is minimal, when $\mu || B_0$

(Where is that used ?) $\vec{\tau} = \vec{\mu} \times \vec{B}_0$

Quantum mechanical description:

$$E_I = -\gamma \cdot \frac{h}{2\pi} \cdot m_I \cdot B_0 \qquad m_I = -I, \ldots, I$$

$$E_1 = +\gamma \cdot \frac{h}{4\pi} \cdot B_0$$ m=-1/2 (N_1 spins)

$$\Delta E = \gamma \cdot \frac{h}{2\pi} \cdot B_0$$

$$E_2 = -\gamma \cdot \frac{h}{4\pi} \cdot B_0$$ m=1/2 (N_2 spins)

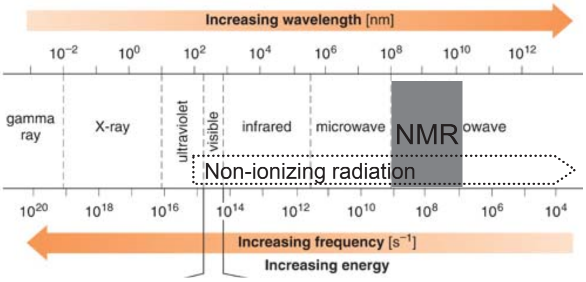
Boltzmann statistics/distribution:
Unequal population of energy levels

$$\frac{N_1}{N_2} = e^{-\frac{\Delta E}{kT}}$$

k : Boltzmann's constant (1.4×10^{-23} J/Kelvin)
NB. At 310K : ~1 in 10^6 excess protons in low energy state (1Tesla)
→ $N_1 \sim N_2 \sim N/2$ (N = no of spins)

Transitions between E_1 and E_2 induced by photons

$h\nu = \Delta E$



8-3. How to classically describe the motion of magnetization ?

View each spin as a magnetic dipole μ
(a tiny bar magnet).
Classically: torque τ of a dipole μ in B

$$\vec{\tau} = \vec{\mu} \times \mathrm{B}$$

2nd law of rotations (P: angular momentum)

$$\vec{\tau} = \frac{d\vec{P}}{dt} \qquad \vec{\mu} = \gamma\vec{P}$$

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma\vec{B}$$

Sum over all μ_k → Magnetization $\vec{M} \equiv \sum \vec{\mu}_k$

Larmor equation

$$\frac{d\vec{M}}{dt} = -\gamma\vec{B} \times \vec{M}$$

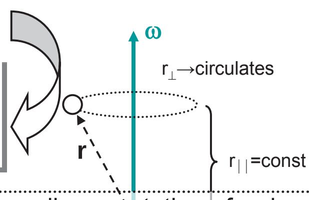
What motion does the Larmor equation describe ?

A brief tour back to rotational kinematics

$v = \omega r = |\Delta r_\perp / \Delta t|$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$

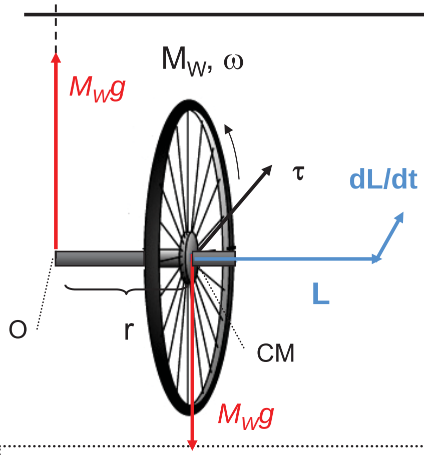


Describes a rotation of **r** about **ω** with frequency $f = \omega/2\pi$

⇒ valid for any vector entity **M**,**L** instead of **r**

Precession of **M** about **B** with frequency $\gamma B/2\pi$

What is precession ?



From Newton's 2nd law (rotations):

$$\frac{d\vec{L}}{dt} = \vec{\tau} = -M_W \vec{g} \times \vec{r} = -M_W r \vec{g} \times \frac{\vec{r}}{r} = -M_W r \vec{g} \times \frac{\vec{L}}{L}$$

$$\frac{d\vec{L}}{dt} = -\frac{M_W r \vec{g}}{L} \times \vec{L} \qquad L = I\omega \qquad \frac{\vec{L}}{L} \cong \frac{\vec{r}}{r}$$

$\Rightarrow$ Precession frequency $\Omega = \frac{r}{I\omega} M_W g$

Observation: The motion of the axis of the wheel with mass M_W is circular about O with constant angular velocity $\mathbf{\Omega} \perp$ to L dictated by τ

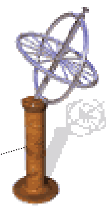
$$\frac{d\vec{L}}{dt} = \vec{\Omega} \times \vec{L}$$

What is the value of Ω?

Precession frequency increases with

1. mass M_W of the wheel
   → **gyro**magnetic ratio γ
2. gravitational pull g
   → magnetic field B_0

Just like a spinning **Gyro**scope in gravity



$$\frac{d\vec{M}}{dt} = -\gamma \vec{B}_0 \times \vec{M}$$

8-4. What are the essentials of Magnetic Resonance ?

nucleus & magnetic field

Nuclear equilibrium magnetization M_0

$$M_0 = (N_2 - N_1)\mu \qquad N_2\left(1 - \frac{N_1}{N_2}\right)\mu = N_2\left(1 - e^{-\frac{\Delta E}{kT}}\right)\mu$$

N_2~N/2 $\qquad \frac{N_1}{N_2} = e^{-\frac{\Delta E}{kT}}$

$$M_0 = \frac{h\mu}{4\pi kT}\gamma B_0 N \qquad e^{-x} \cong (1-x)\ (x << 1)$$

Magnetization increases with

1. No. of spins N (molecules)
2. magnetic field B_0
3. gyromagnetic ratio γ

MRI: ^1H_2O !

Nucleus with non-zero spin and high *gyromagnetic ratio* γ: ^{1}H

Magnet to create magnetic field B_0 || z
- $(N_2-N_1)\mu_z$ results in *equilibrium magnetization M_0*
- ΔE is small (~μeV)
- ⇒ **Non-ionizing e.m. fields**

$$\frac{d\vec{M}}{dt} = -\gamma \vec{B}_0 \times \vec{M} \Rightarrow$$

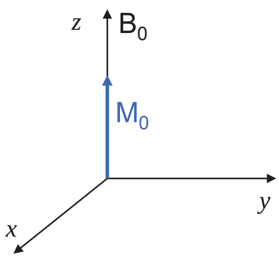
Larmor frequency

$f_L = \gamma B_0 / 2\pi$

$\omega_L = \gamma B_0$

Convention in magnetic resonance:

Static magnetic field B_0 || z

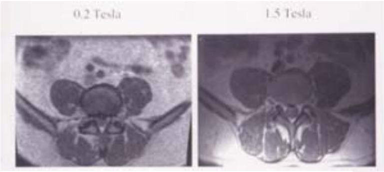


⇒ thermodynamic equilibrium: M_0 || z

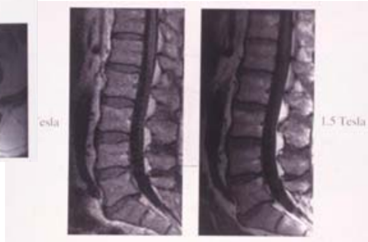
MR is safe, but insensitive

How can the sensitivity be increased ?

magnetic field strength B_0

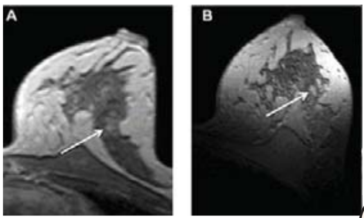


MRI of the lower abdomen



MRI of the spine

MRI of the breast (1.5 vs 3 Tesla)



http://medicalphysicsweb.org/cws/article/research/38414

maximum possible MR signal:

determined by

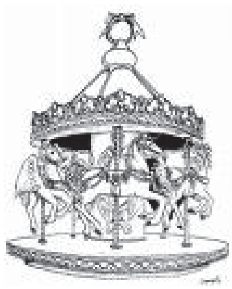
equilibrium nuclear magnetization M_0

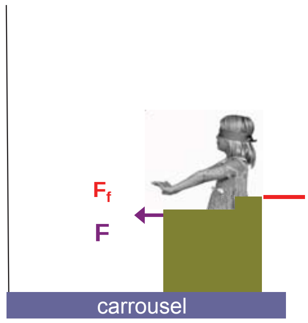
8-5. Why use a Rotating frame of reference to describe the motion of magnetization ?

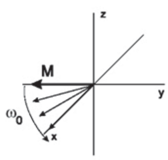
Rotating frame: A reference frame which rotates about z of the laboratory frame at frequency ω_{RF}

Why use a rotating reference frame ?

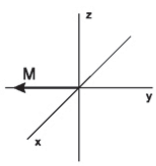
$$\frac{d}{dt}\vec{M} = \vec{M}\times\gamma\vec{B}$$







Lab Frame



Rotating Frame

What is the equation of motion for magnetization in the rotating reference frame ?

Larmor frequency in reference frame rotating with ω_{RF} : $\Omega = \omega_L - \omega_{RF}$

$\Omega = \gamma \Delta B$

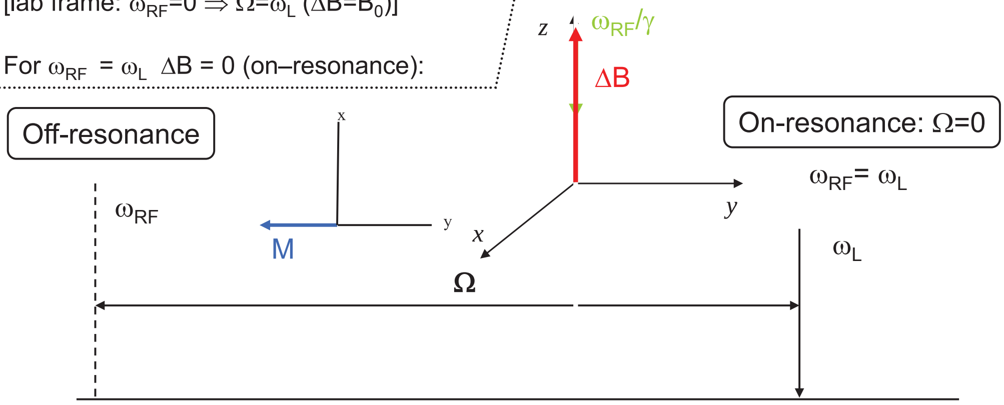
$\Rightarrow \Delta B = \Omega / \gamma = B_o - \omega_{RF} / \gamma$

[lab frame: $\omega_{RF}=0 \Rightarrow \Omega=\omega_L$ ($\Delta B=B_0$)]

For $\omega_{RF} = \omega_L$ $\Delta B = 0$ (on–resonance):

$$\frac{d\vec{M}}{dt} = \vec{\Omega} \times \vec{M}$$

(fictitious) magnetic field ω_{RF}/γ is progressively subtracted from B_0



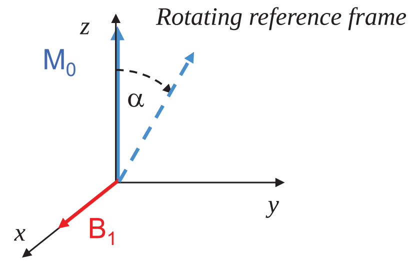
Ex. Flipping magnetization over

in the rotating reference frame

Start with thermodynamic equilibrium magnetization M_0

Reference frame rotating with ω_L (on-resonance)

Apply *additional*, constant magnetic field with magnitude B_1 (in xy plane) for time τ



Definition **Flip angle** = angle of rotation α induced by B_1 applied for τ seconds

What motion can be observed for M ?

$$\frac{d\vec{M}}{dt} = -\gamma \vec{B}_1 \times \vec{M}$$

M_0 precesses about B_1

Magnetization rotates about B_1 with angular velocity γB_1

Frequency $\gamma B_1/2\pi$

$\rightarrow$ period $T = 2\pi/\gamma B_1$

Special cases of α:

90^0: Full **excitation** (all M_0 is rotated into transverse plane, xy, i.e. $M_0 \rightarrow M_{xy}$)

180^0: **Inversion** ($M_z \rightarrow -M_z$)

B_1 = **radiofrequency (RF) field** (why?)

Lab frame: $\mathbf{B_1}(t)=B_1(\cos\omega_L t, \sin\omega_L t)$

γ ~ 42MHz/Tesla $\rightarrow \omega_L/2\pi$ ~ 100MHz

Supplement: Why there is only equilibrium magnetization along B_0 ?

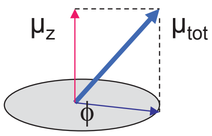
Random Phase approximation

Quantized magnetic moment μ

$$|\vec{\mu}| = \gamma \frac{h}{2\pi} \cdot \sqrt{I \cdot (I+1)}$$

$$\mu_z = \gamma \frac{h}{2\pi} \cdot m_I$$

$\mu_z < \mu_{tot}$

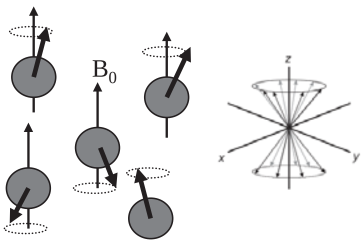


Individual spin is never aligned with B_0 …

But … Equilibrium $M_0 \parallel B_0$

Phase φ of μ_{xy} is random (random phase approximation):

No net μ_{xy}



Bulk Nuclear Magnetization:

$$\vec{\mathbf{M}} = \sum \vec{\boldsymbol{\mu}}$$