

4: From x-ray to image – Computed Tomography

1. What factors influence contrast in x-ray imaging ?
 - Beam hardening
 - Sensitivity and resolution considerations
2. What influences CNR of x-ray imaging ?
3. What is the fundamental basis for image reconstruction using x-ray absorption ?
 - Radon Transform
4. How can x-ray images be reconstructed?
 - Sinogram
 - Backprojection vs. filtered backprojection
 - Central Slice Theorem
5. Examples & Summary

After this course you

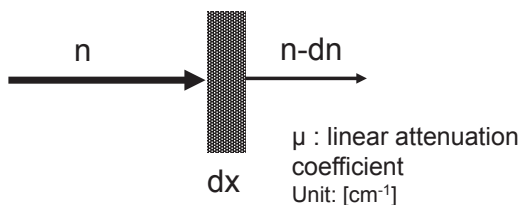
1. Understand the consequences of the Bremsstrahlung continuum on image contrast
2. Understand how Compton scattering reduces image contrast and how its influence can be reduced
3. Are familiar with the Radon transform
4. Understand the principle of matrix reconstruction and backprojection
5. Understand the major mechanisms leading to CT contrast

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4-1

4-1. What does absorption in the real world imply ?

Linear attenuation coefficient μ



Contrast is “well-defined”
for monochromatic x-rays

$$\text{But, } \mu = f(E_v, Z, \rho)$$

If μ is constant in x

$$n(x) = N_0 e^{-\mu x}$$

The measurement that is **wanted**:

$$\mu(x, y)$$

What is **measured**: $n(x)$

$$\ln\left(\frac{n(x)}{N_0}\right) = -\mu x$$

(μ for a homogeneous object of thickness x)

Two consequences:

Beam hardening

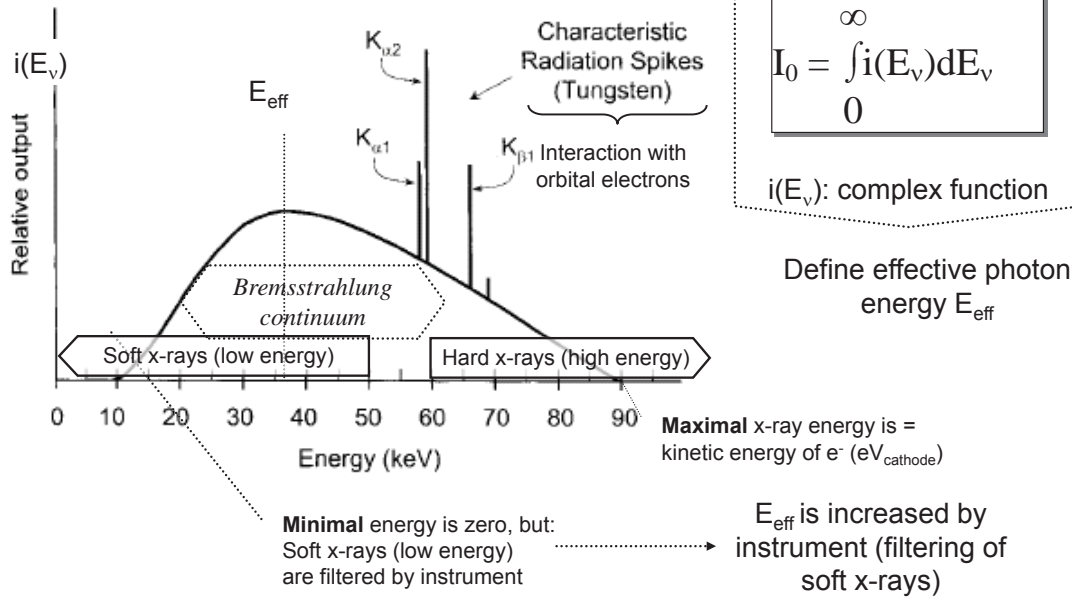
Depth dependent contrast

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4-4

What does the Energy Spectrum of an x-ray tube really look like ?

filtered Bremsstrahlung and characteristic emission



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4-5

What is the consequence of energy-dependent absorption ?

Beam Hardening - Effective energy depends on depth

A similar consequence arises in tissue:

Ideal:

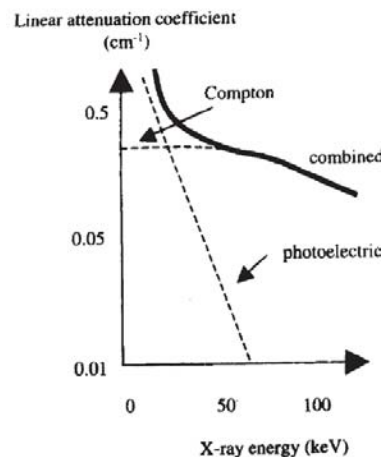
Monochromatic x-rays
($E_v(\lambda) = \delta(\lambda_0)$)

Reality:

Polychromatic, multienergetic $i(E_v)$

Absorption is not uniform with E_v

- Contrast changes with large objects and depth
- Excessive radiation dose to superficial tissue



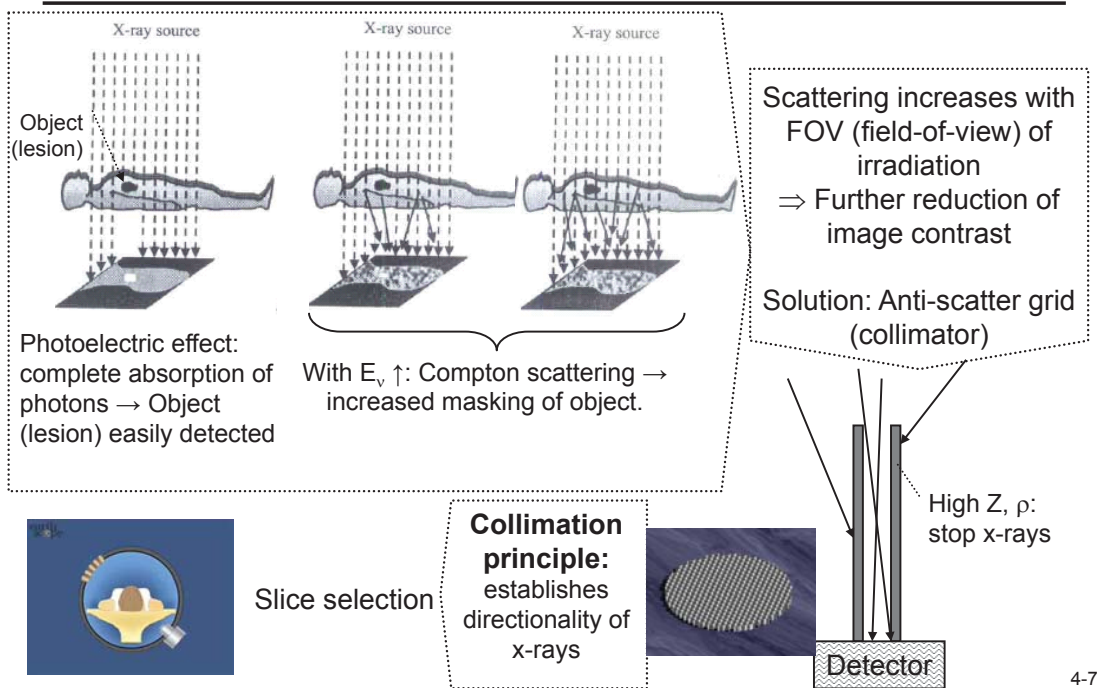
"Solution": Reduce $i(E_v)$ for soft x-rays

(e.g. 3mm Al eliminates 90% of 20keV photons)

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4-2. How does x-ray scattering impact CNR ?



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How is CNR quantified ?

Signal:

- $\propto I(d)$ (no. photons detected)
- $\propto I_0$ (no. photons irradiated)

CT intensity can be measured in absolute terms (CT-number)

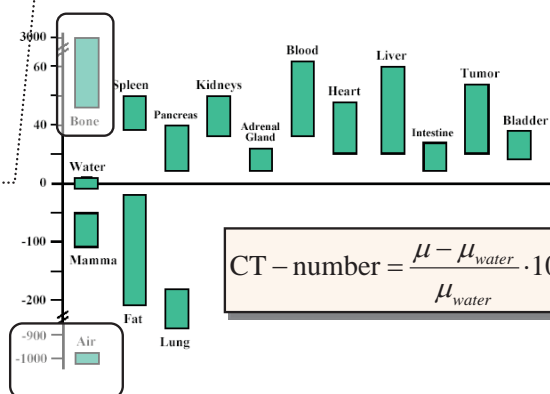
Soft tissue: Typically has weak contrast (small **Hounsfield units**)

Contrast: $\Delta I(d)$ due to $\mu(d)$ differences

- $i(E_v), \mu_C$ produces reduced contrast
- Compton scattering: Antiscatter grid

HU : attenuation normalized to water ($\equiv 0$)
range from -1000 (air) to +3000 (bone and contrast agents)
soft tissues: -300 to +100

CT-numbers of tissue in Hounsfield units (HU)

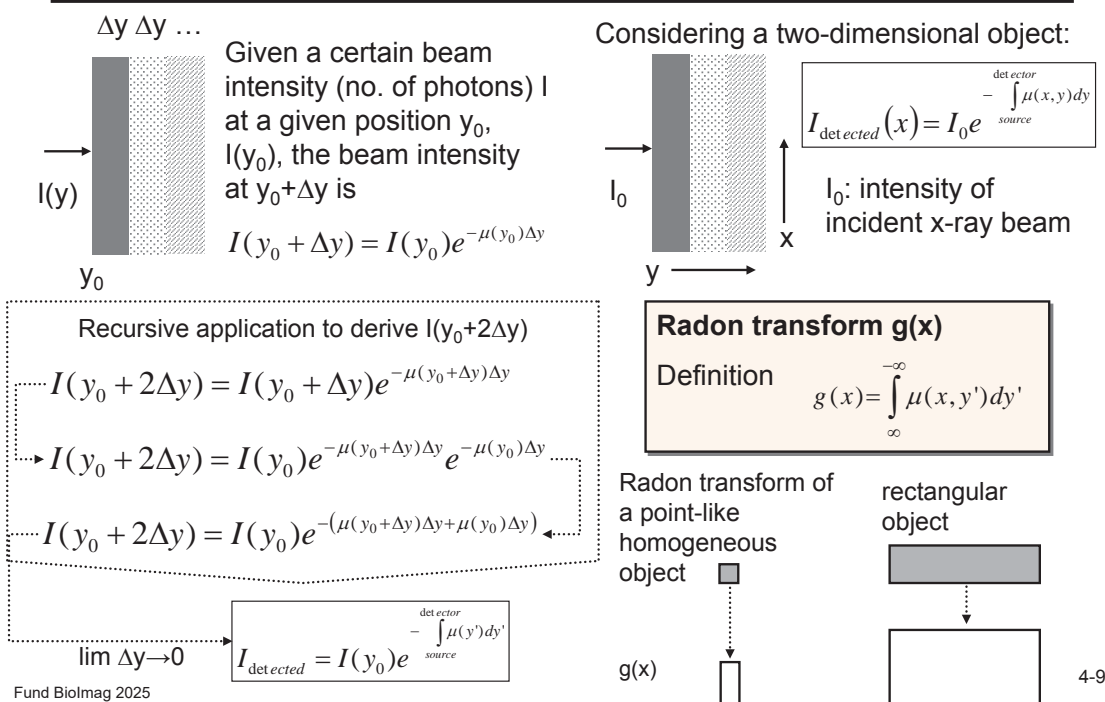


$$CT - number = \frac{\mu - \mu_{water}}{\mu_{water}} \cdot 1000$$

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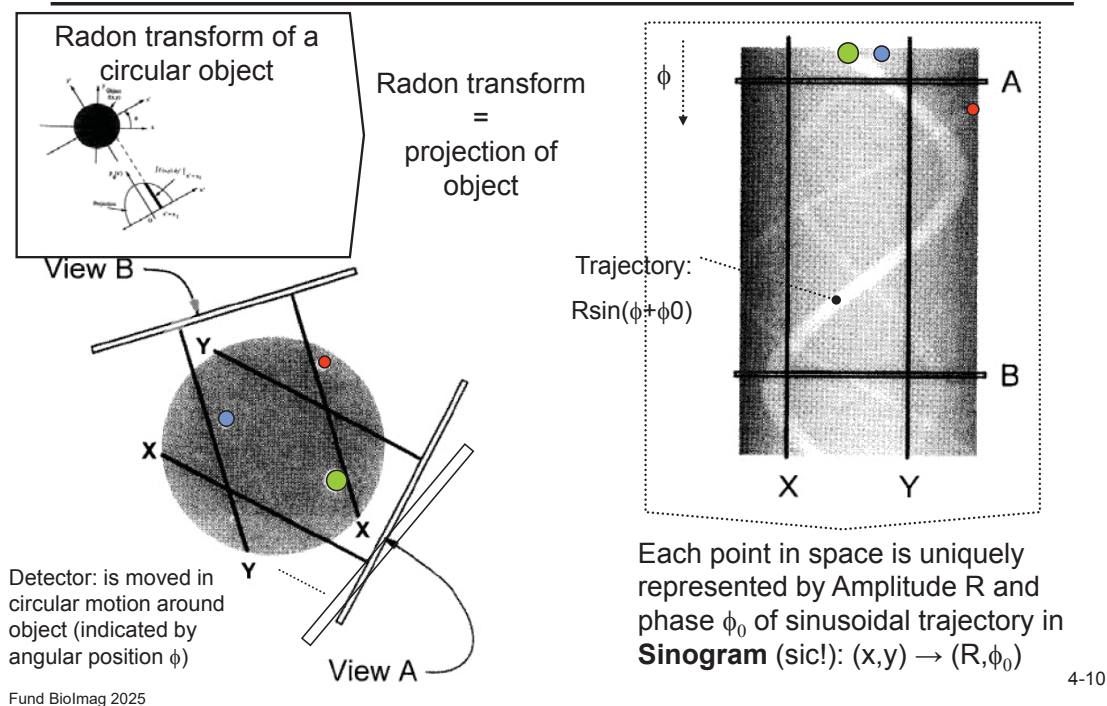
4-3. What is the basis of image reconstruction ?

The Radon transform



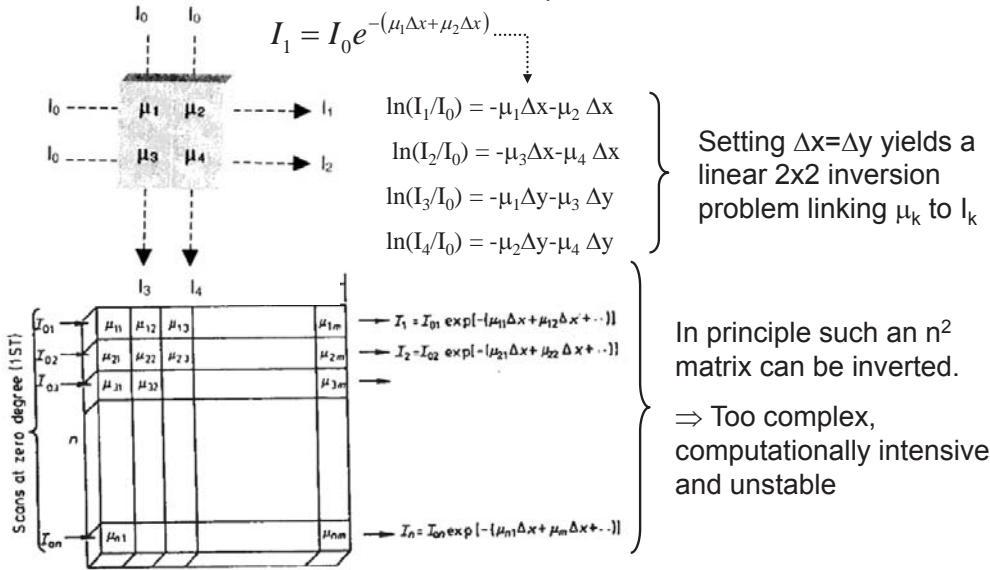
Does each pixel have a unique trajectory ?

Sinogram



Can a CT image be constructed by Matrix inversion ?

Decomposing an object into a 2x2 matrix requires a minimum of 4 measurements:



CT was introduced in 1970 \Rightarrow simple reconstruction algorithm!

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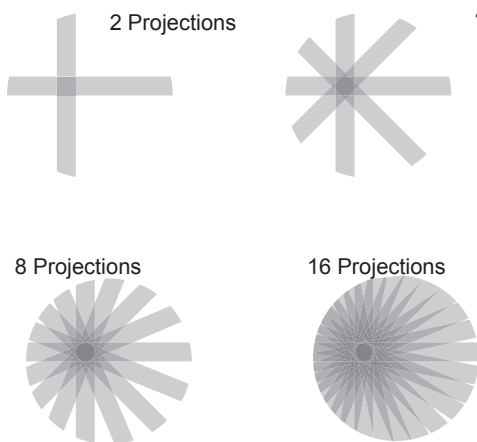
4-4. What algorithm is adapted to 1970's computing power ?

Backprojection reconstruction

Basic reconstruction principle: Along the measured projection direction fill in each pixel constant numbers corresponding to the Radon transform (projection intensity).

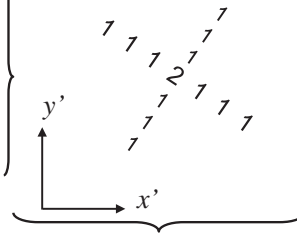
Repeat for next orientation of the projection, sum the values in overlapping pixels.

Illustration with gray shades (point-like object):



$$R_\theta(x') = \int_{-\infty}^{+\infty} f(x' \cos \theta - y' \sin \theta, x' \sin \theta + y' \cos \theta) dy'$$

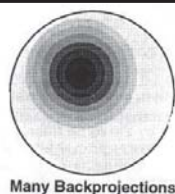
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



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Why does simple Backprojection have poor spatial resolution ?

Backprojection has poor spatial resolution:

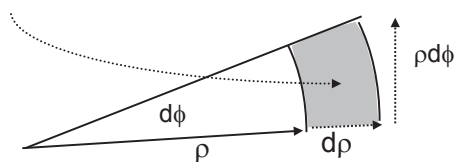


Reconstruction of a point-object falls off with $1/r$

WHY?

The reconstruction falls off with $1/r$
(in analogy to the decrease of light intensity in 2D)

$$dx dy = dA = \rho \sin(d\phi) d\rho \approx \rho d\phi d\rho$$



Number of rays (projections):
constant with $d\phi$

But:

$$\text{pixel size} = dx dy \propto dA = \text{const}$$

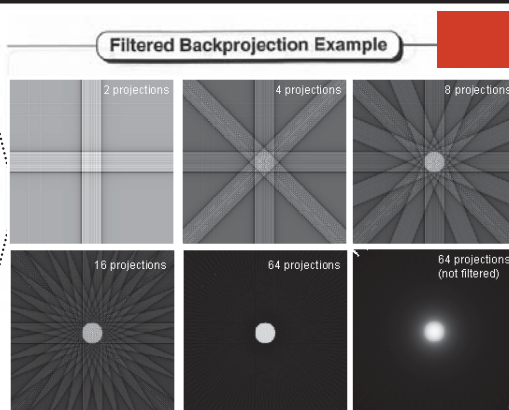
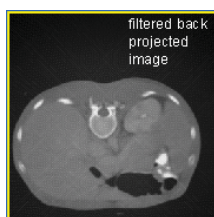
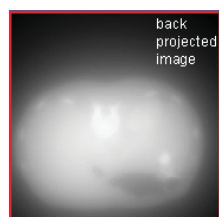
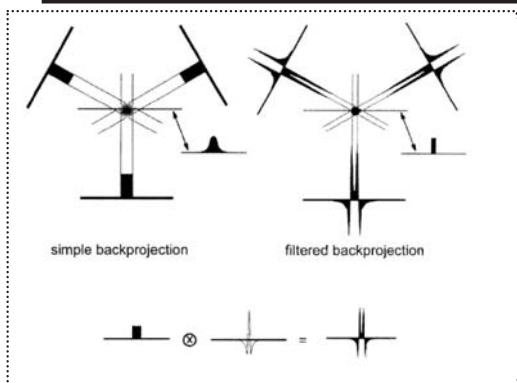
$$\Rightarrow \text{No. of rays} \propto 1/\rho$$

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How can good image resolution be maintained ?

Filtered Backprojection

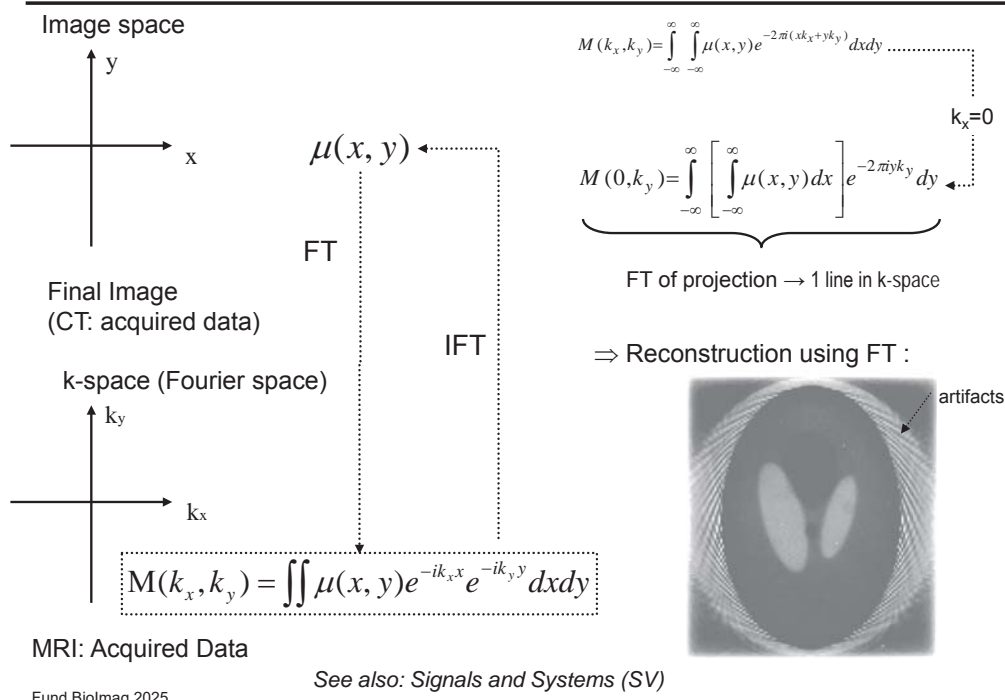


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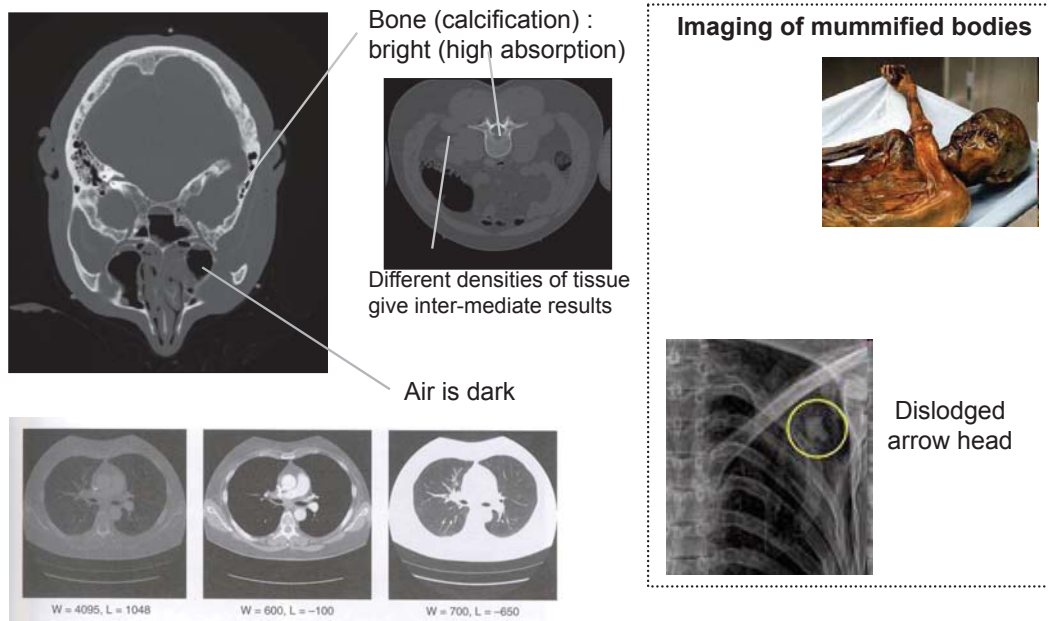
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How is Backprojection linked to Fourier transform ?

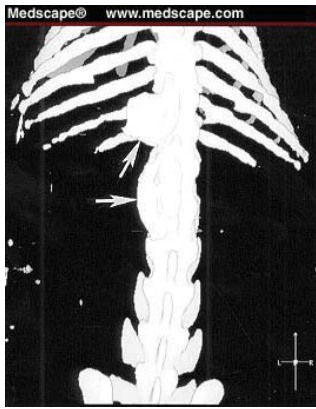
Central Slice Theorem



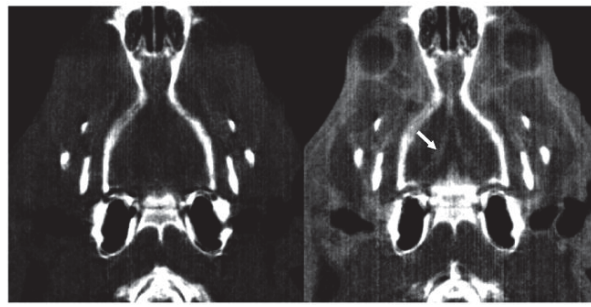
4-5. X-ray CT : Examples (Human)



CT: Examples (mouse)



3D CT scan of rodent spine treated with human mesenchymal stem cells (transduced with the human BMP-9 gene via an adenoviral vector) significant bone formation at the treatment sites (arrows)

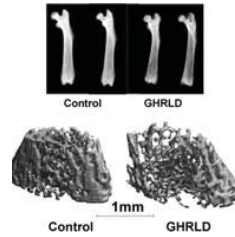


Pre-

Post-contrast



13µm micro CT of mouse placenta vasculature



Micro-CT of mouse femur bone

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CT: Summary

Main **contrast** is bone vs. soft tissue (or air) (calcium content i.e. e^- density ρ)

Contrast agents (increase Z_{eff}) allow depiction of vessel architecture and lesions

SNR and CNR:

1. Intensity can be increased by cathode current
2. High spatial resolution possible (limited only by radiation dose in humans)

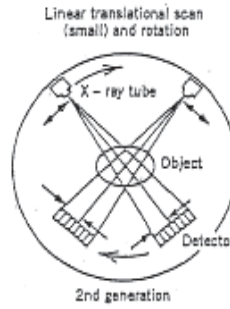
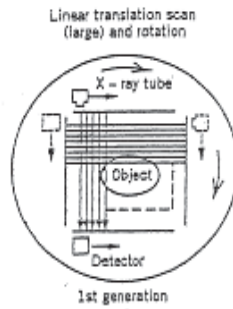
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How have CT scanners evolved ?

Generations of CT scanners

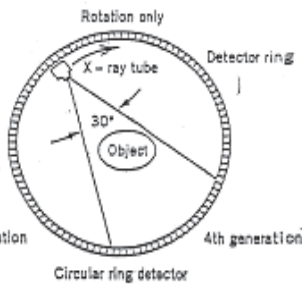
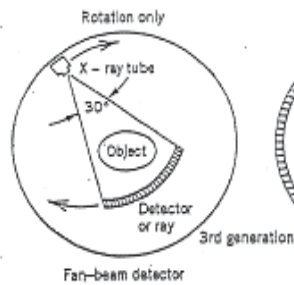
First Generation

- Parallel beam design
- One/two detectors
- Translation/rotation



2nd Generation

- Small fan beam
- Translation/rotation
- Larger no. of detectors



3rd Generation

- Multiple detectors
- Large fan beam

4th Generation

- Detector ring
- Large fan beam