# 3: Interaction of ionizing radiation with tissue

1. What is the basis of contrast for x-ray imaging ?
2. By which mechanisms does ionizing radiation interact with matter ?
    Rayleigh scattering
    Compton scattering
    Photoelectric effect
    Pair production
3. How does this interaction depend on the tissue ?
    Energy dependence and effective atomic number $Z_{eff}$
4. How can we protect ourselves against the biological effects of ionizing radiation ?
    A radiation protection primer

After this course you
  1. Know the definition of linear and mass attenuation coefficient
  2. Understand the major mechanism of x-ray absorption in tissue
  3. Understand the dependence of these mechanisms on photon energy and tissue composition
  4. Are able to perform contrast-to-noise calculations using effective Z
  5. Understand and are able to apply the basic principles of radiation protection
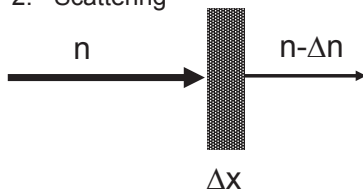
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## 3-1. How can we describe attenuation of x-rays?
### Linear attenuation coefficient $\mu$

Fates of the photon (other than transmission):
  1. Absorption (transfer of $h\nu$ to lattice)
  2. Scattering



$\Delta x$

Photons are removed according to probability law:

The number of absorbed/scattered photons $\Delta n$ in a layer with thickness of $\Delta x$

$$\Delta n \equiv -\mu n \Delta x$$

$\mu$ : **linear attenuation coefficient**
Unit: [cm$^{-1}$]

$\mu = f(E_\nu, Z, \rho)$

Consider situation where $\Delta x \rightarrow 0$, and $n = f(x)$

$$dn(x) = -\mu n(x) dx$$

$$\frac{dn(x)}{dx} = -\mu n(x)$$

Solution ?

$$n(x) = N_0 e^{-\mu x}$$

(provided $\mu$ is constant in x )

**Definition**

Half value layer (HVL) $\equiv$ The thickness of a material allowing to pass one half of photons:

$n(x_{HVL}) = N_0/2$

$$N_0/2 = N_0 e^{-\mu(HVL)}$$

$$HVL = 0.693/\mu$$

Typical HVL values: several cm for tissues, 1-2 cm for aluminum, 0.3 cm for lead

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# What are typical attenuation coefficients ?

| Material | Density (g/cm³) | Electrons per Mass (e/g) × 10²³ | Electron Density (e/cm³) × 10²³ | μ @ 50 keV (cm⁻¹) |
|---|---|---|---|---|
| Hydrogen | 0.000084 | 5.97 | 0.0005 | 0.000028 |
| Water vapor | 0.000598 | 3.34 | 0.002 | 0.000128 |
| Air | 0.00129 | 3.006 | 0.0038 | 0.000290 |
| Fat | 0.91 | 3.34 | 3.04 | 0.193 |
| Ice | 0.917 | 3.34 | 3.06 | 0.196 |
| Water | 1 | 3.34 | 3.34 | 0.214 |
| Compact bone | 1.85 | 3.192 | 5.91 | 0.573 |

$\mu/\rho$ (water) = $\mu/\rho$ (ice)

**Definition**

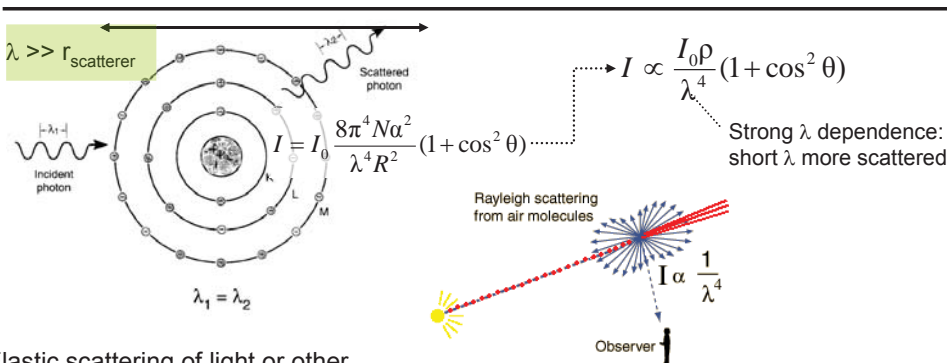**Mass attenuation coefficient** $\mu/\rho$

Unit : [cm²/g]

(constant for all forms of the same chemical substance, e.g. water)

Why do we need more sunscreen in the mountains ?

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## 3-2. What are the 4 basic interactions of x-rays with biological tissue ?
### I. Rayleigh scattering & Compton

$\lambda \gg r_{scatterer}$

$\lambda_1 = \lambda_2$

$I = I_0 \dfrac{8\pi^4 N\alpha^2}{\lambda^4 R^2}(1+\cos^2\theta)$

$I \propto \dfrac{I_0\rho}{\lambda^4}(1+\cos^2\theta)$

Strong $\lambda$ dependence: short $\lambda$ more scattered

Rayleigh scattering from air molecules

$I \alpha \dfrac{1}{\lambda^4}$

Observer

Elastic scattering of light or other electromagnetic radiation by particles **much smaller than the wavelength** of the light, $x = 2\pi r/\lambda$; $x \ll 1$.

Elastic: photon energy = constant

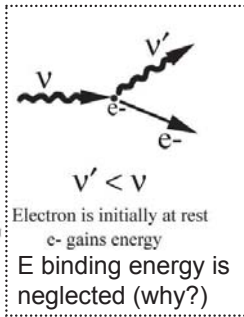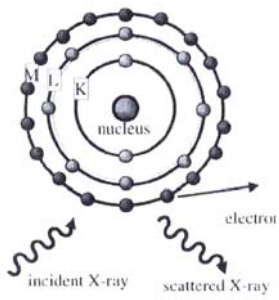No ionization occurs ("classical" scattering)

Scattered X-rays $\Rightarrow$ deleterious effect on image quality

very low probability (< 5%) in x-ray imaging

$E(keV)=1.2/\lambda(nm)$

$\lambda=1.2/E = 0.012$nm (@100keV)

# II. Compton scattering



v' < v

Electron is initially at rest
e- gains energy
E binding energy is neglected (why?)



Arthur Holly Compton

Physics, 1927

Occurs at the outer shell electrons

$\Rightarrow$ ionization

Scattered photon: subject to subsequent interactions
(Rayleigh, Compton scattering or photoelectric effect)

Probability increases with

**photon energy**

electron density

Electron/mass density in tissues ~ constant (independent of Z)
→ proportional to the **density of the material**

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# Relativistic linear momentum
## a brief tour back to 1$^{st}$ year physics

From the definition p≡mv (which is true at any velocity) it follows

$$\vec{p} = m(v)\vec{v} = m_0 \frac{\vec{v}}{\sqrt{1-v^2/c^2}}$$

$$\vec{p} \cdot \vec{p} = m_0^2 \frac{\vec{v}\cdot\vec{v}}{1-v^2/c^2}$$

The value of p is

Relativistic kinetic energy E=m(v)c$^2$

$$E^2 - m_0^2 c^4 = \underbrace{m_0^2 \frac{c^4}{1-v^2/c^2} - m_0^2 \frac{c^4(1-v^2/c^2)}{1-v^2/c^2}}$$

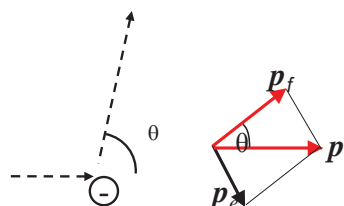$$m_0^2 \frac{v^2}{1-v^2/c^2} c^2 = (pc)^2$$

$$\vec{p} \cdot \vec{p} = \frac{E^2}{c^2} - m_0^2 c^2$$

NB. Light carries energy, but moves at the speed of light (!) c.
**Photon with energy E**:
particle with rest mass (m$_0$=0)
(otherwise its energy would be infinite, since v=c) :

$$\left|\vec{p}\right| = \frac{E}{c}$$



θ

$p_f$

$p_i$

$p_e$

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# Compton scattering
## The basic Equations

A simple elastic collision:

**Conservation of energy**

$E_e = (E_i - E_f) + m_e c^2$

$E_i + m_e c^2 = E_f + E_e$

before — after

$$p_e^2 = \frac{E_e^2}{c^2} - m_e^2 c^2$$

**Conservation of linear momentum:**

$$\vec{p}_i = \vec{p}_f + \vec{p}_e$$

before — after

$$(\vec{p}_e)^2 = (\vec{p}_i - \vec{p}_f)^2$$

$$E_f = \frac{E_i}{(1-\cos\theta)E_i / m_e c^2 + 1}$$

With $\lambda = c/\nu$  $E = hc/\lambda$ it follows:

$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1-\cos\theta)$$
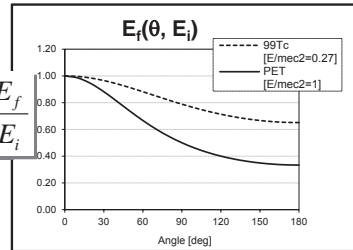
$E_f$ is

Maximal if $\theta=0$:  $E_f^{max} = E_i$

Minimal, if $\theta=\pi$:  $E_f^{min} = \dfrac{E_i}{2E_i / m_e c^2 + 1}$
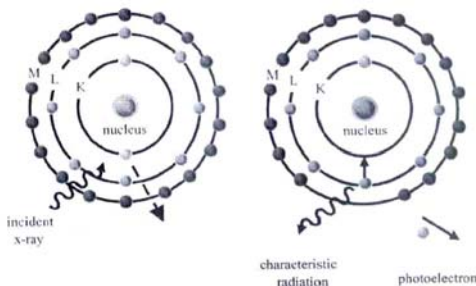
$E_i = m_e c^2$ :

$$E_f = \frac{m_e c^2}{(2-\cos\theta)}$$

$\dfrac{E_f}{E_i}$

**$E_f(\theta, E_i)$**

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# 3-3. Interaction of photons with tissue II
## Photoelectric effect, pair production & summary

Photoelectric absorption effect:

Abruptly increases when $E$ is slightly above $I_K$ – **absorption edges**

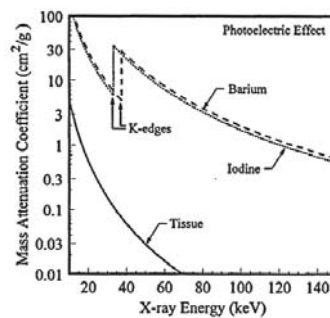Absorption edge energy increases with $Z$ (very low for H, C, N, O)

Inner shell $e^-$ is removed
$\Rightarrow$ energy of the incident X-ray quantum $E_i$
> ionization energy of an electron $I_K$

$$E_i = E_e^K + I_K$$

Vacancy in K-shell: filled with outer shell e
$\Rightarrow$ cascade of emitting *characteristic* X-ray quanta
(or *Auger electrons*, but not so frequent in diagnostic imaging of soft tissues with low Z)

Albert Einstein
Physics, 1921

## IV. Pair production
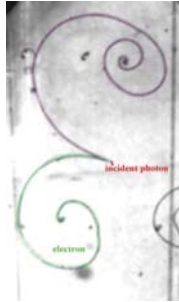
When the photon energy > $2m_ec^2$ = 1.02MeV

It interacts with the electric field of the nucleus of an atom

$\nu \rightarrow e^- + e^+$
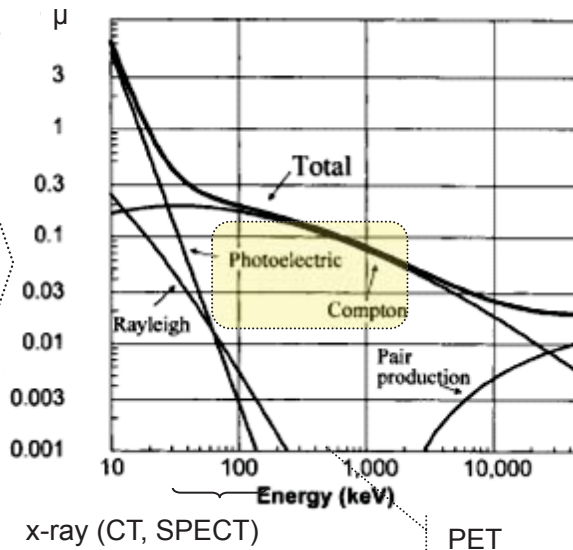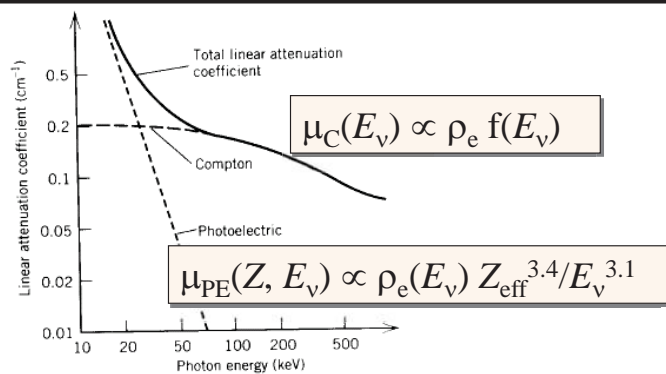
The kinetic energy of the produced particles is

**$E = E_\nu - 2m_ec^2$**

> Not important in Bio-imaging



μ

Total

Photoelectric

Rayleigh

Compton

Pair production

Energy (keV)

x-ray (CT, SPECT)

PET

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# 3-4. What affects the linear attenuation coefficient ?
## effective $Z$, $Z_{eff}$



$$\mu_C(E_\nu) \propto \rho_e\, f(E_\nu)$$

$$\mu_{PE}(Z, E_\nu) \propto \rho_e(E_\nu)\, Z_{eff}^{3.4}/E_\nu^{3.1}$$
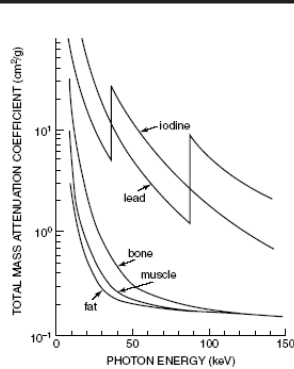
Linear attenuation coefficient of water and the contribution of each interaction to the total attenuation of X-rays as a function of energy.

Compton:

    depends mainly on the e density $\rho_e$

    modestly on energy of x-rays $E_\nu$

Photoelectric effect:

    depends strongly on atomic number Z

Biology:

empirical $Z_{eff}$

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# What is the effective atomic number $Z_{eff}$ of biological tissue ?

Necessary for estimating µ of the photoelectric effect

Empirical relationship for compound materials such as biological tissue:

$$Z_{eff} = \left( \sum_{\substack{all\ tissue \\ components\ i}} \lambda_i Z_i^{3.4} \right)^{1/3.4}$$

$$\lambda_i = \frac{P_i Z_i / A_i}{\sum_{\substack{all\ tissue \\ components\ j}} P_j Z_j / A_j}$$

P : percentage weight

Z : atomic number

A : atomic weight

Example: Estimation of $Z_{eff}$ for water

$^1$H: $Z=A=1$, P=11%,

$^{16}$O: $Z=8$, A=16, P=89%

Denominator of $\lambda$ : $\sum P_j \dfrac{Z_j}{A_j} = 11\dfrac{1}{1} + 89\dfrac{8}{16} = 55.5$

Protons: $\lambda$ = 11/55.5=0.20
Oxygen: $\lambda$ = 44.5/55.5=0.80.

$Z_{eff}$=(0.2·1$^{3.4}$+0.8·8$^{3.4}$)$^{1/3.4}$
=(0.2+1180·0.8) $^{1/3.4}$

=944$^{1/3.4}$=7.5        How good were we ?

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# What is the % mass composition $P_i$ of select biological tissues ?

| % Composition (by Mass) | Adipose Tissue | Muscle (Striated) | Water | Bone (Femur) |
|---|---|---|---|---|
| Hydrogen | 11.2 | 10.2 | 11.2 | 8.4 |
| Carbon | 57.3 | 12.3 | | 27.6 |
| Nitrogen | 1.1 | 3.5 | | 2.7 |
| Oxygen | 30.3 | 72.9 | 88.8 | 41.0 |
| Sodium | | 0.08 | | |
| Magnesium | | 0.02 | | 7.0 |
| Phosphorus | | 0.2 | | 7.0 |
| Sulfur | 0.06 | 0.5 | | 0.2 |
| Potassium | | 0.3 | | |
| Calcium | | 0.007 | | 14.7 |

$Z_{eff}$ ~ 6          7.4          12
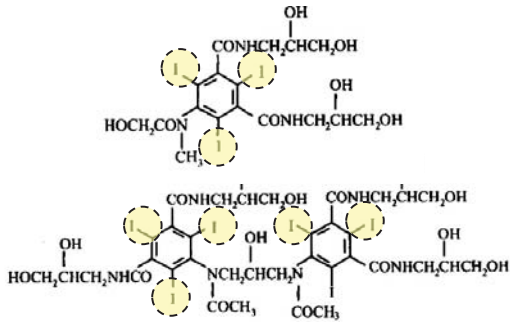
Carbon:

Z/A=6/12
(same for O, 8/16)

# What are X-ray contrast agents ?
## Exogenously administered substance (by infusion/ingestion)

modifying $Z_{eff}$
$\Rightarrow$ use high Z compounds

e.g., compounds with multiple iodine atoms, lanthanides etc.



$Z_{eff}$ of (water+10 mmol/kg iodine) = ?

Iodine:
$P_I$ = 10[mmol/kg]×127[mg/mmol] = 0.127%
$Z_I$ = 53
$A_I$ = 127

Calculation of $Z_{eff}$:

$$Z_{eff} = \left( \sum_{\substack{all\ tissue \\ components\ i}} \lambda_i Z_i^{3.4} \right)^{1/3.4} \qquad \lambda_i = \frac{P_i Z_i / A_i}{\sum_{\substack{all\ tissue \\ components\ j}} P_j Z_j / A_j}$$

~ denominator $\lambda$ of pure $H_2O$

Denominator of $\lambda$ :

$$\sum_i P_i \frac{Z_i}{A_i} = 11\frac{1}{1} + 89\frac{8}{16} + 0.13\frac{53}{127} = 55.6$$

$\rightarrow$ $\lambda$ of H and O are as for water:

$\lambda_H$=0.2
$\lambda_O$=0.8

$$\lambda_I = \frac{0.127 \cdot 53/127}{55.6} = \frac{0.053}{55.6} = 9.5 \times 10^{-4}$$

$$Z_{eff}^{3.4} = \underbrace{0.2 \cdot 1^{3.4} + 0.8 \cdot 8^{3.4}}_{944} + \underbrace{9.5 \cdot 10^{-4} \cdot 53^{3.4}}_{690}$$

$Z_{eff} = 8.8$

$\mu_{PE} \propto Z_{eff}^{3.4}$

$\mu_{PE}(H_2O) \propto 944$
$\mu_{PE}(H_2O+I) \propto 1650$

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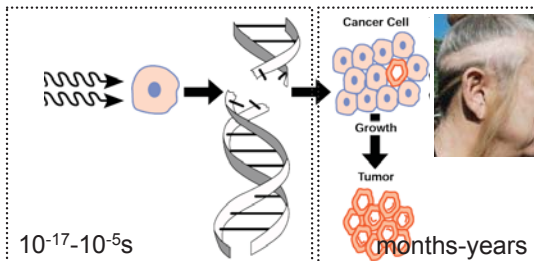
# 3-5. What are the biological effects of ionizing radiation ?

Ionization effects:
instantaneous ($10^{-17}$-$10^{-5}$s)

1. Produce free radicals

2. Break chemical bonds

3. Produce new chemical bonds and cross-linkage between macromolecules

4. Damage molecules that regulate vital cell processes (e.g. DNA, RNA, proteins)



$10^{-17}$-$10^{-5}$s     months-years

Biological effects are delayed:

Cataract (months to years)

Cancer (years-decades)

Tissue sensitivity to radiation
proportional: rate of cell proliferation
inversely prop.: degree of cell differentiation

Pregnancy vs. old age …



sensitivity
Blood-forming organs
Reproductive organs
Skin
Bone and teeth
Muscle
Nervous system

**Radicals** (unpaired valence $e^-$) and reactive oxygen species, e.g. 2 $OH^- \rightarrow H_2O_2$   produced by the body (e.g. oxygen consumption)
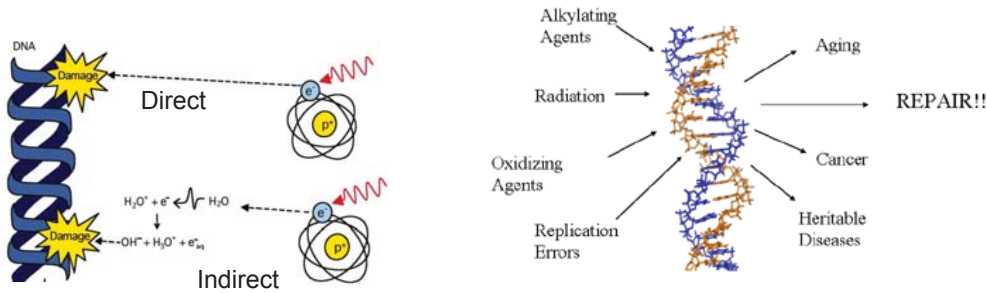
# How does the tissue defend itself against radiation damage ?
## DNA repair



Direct

Indirect

Alkylating Agents → Aging

Radiation → REPAIR!!

Oxidizing Agents → Cancer

Replication Errors → Heritable Diseases

### Healing

| Repair | Regeneration |
|---|---|
| Replace damaged cells with different cell type (fibrosis) | Replace damaged cells by same |
| Organ not returned to original state | Organ returned to original state |
| Radioresistant tissues (muscle, brain): only repair possible | Radiosensitive tissues (skin, digestive system, bone marrow) |

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# How are the effects of ionizing radiation quantified ?
## Three forms of radiation dose

**Absorbed dose D:**
energy deposited by ionizing radiation per unit mass of material:

$D$ = Energy/mass

Units: [Gray[Gy]=1J/kg]

**Equivalent dose *H*:**
= D corrected for effectiveness of radiation to produce biological damage
($w_R$ = radiation weighting factor)

$H = Dw_R$

Units: [Sievert[Sv] = 1J/kg]

**Effective dose *E* :**
H corrected for sensitivity of tissue T
($w_T$ = tissue weighting factor)

$$E(\text{Sv}) = \sum w_T H_T (\text{Sv})$$

Absorbed dose D depends on

1. Intensity of incident x-ray

2. Duration of exposure

$w_R$ = 1 (x-rays, $\beta$ particles)

[20: for $\alpha$ particles]

Bio-imaging:
Equivalent dose H = Absorbed dose D



tissue weighting factor $w_T$

| gonads | bone marrow | colon | lung | stomach | bladder | breast | liver | oesophagus | thyroid | skin | bone surface |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2 | 0.12 | 0.12 | 0.12 | 0.12 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.01 | 0.01 |
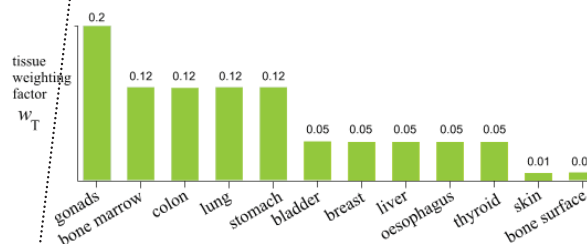
# 3.6 How can we protect ourselves against x-rays ?
## Exposure time, distance and HVL

Absorbed dose depends on intensity $I_0$ of radiation (activity in Bq for radioactive materials)    1 Becquerel [(Bq) = 1 decay/s]

NB. β-emitters ($^{14}$C, $^3$H):
- e$^-$ do not penetrate low $Z$ material (plastic, glass, avoids Bremsstrahlung)
- high energy β-emitters ($^{32}$P): low $Z$ shielding + lead (attenuate Bremsstrahlung)
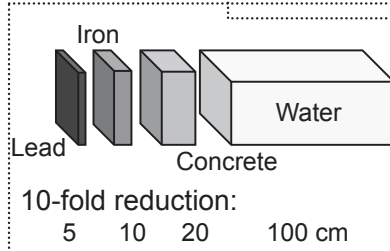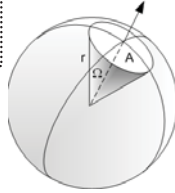
Three basic elements of radioprotection:
1. **Time**
2. **Distance**
3. **Shielding**

Occupancy:
fraction of time of working day (8h), human is in area exposed to radiation

Received radiation = Fraction of **solid angle** occupied by human surface A = A/4πd²

$\sim I_0/d^2$ ($d$=distance to source)

Iron

Water

Lead

Concrete

10-fold reduction:
5    10    20    100 cm

Depends on energy and type of radiation

| HVL | Iron or steel | Brick | Concrete | Dirt | Ice | Wood (soft) | Snow |
|---|---|---|---|---|---|---|---|
| (cm) | 1.8 | 5.1 | 5.8 | 8.4 | 17 | 22 | 52 |

Half-value layers

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# What are typical radiation exposures ?
## Natural, artificial and some examples

| Natural | mrem/yr | mSv/yr | %total |
|---|---|---|---|
| Radon | 200 | 2.0 | 55% |
| Cosmic | 27 | 0.27 | 8% |
| Terrestrial | 28 | 0.28 | 8% |
| Internal | 39 | 0.39 | 11% |
| **Total** | **300** | **3** | **82%** |

| Other | | | |
|---|---|---|---|
| Occupational | 0.9 | <0.01 | <0.3 |
| Nuclear Fuel Cycle | <1 | <0.01 | <0.03 |
| Fallout | <1 | <0.01 | <0.03 |
| Misc. | <1 | <0.01 | <0.03 |

| Artificial | | | |
|---|---|---|---|
| Medical X ray | 39 | 0.39 | 11% |
| Nuclear Med. | 14 | 0.14 | 4% |
| Consumer products | 10 | 0.1 | 3% |
| **Total** | **63** | **0.63** | **18%** |

| Source of Exposure | |
|---|---|
| X-rays from TV set (3 cm) | 0.500 mrem/hour |
| Airplane ride (12km) | 0.500 mrem/hour |
| Radionuclides in the body (i.e., K) | 39 mrem/year |
| Building materials (concrete) | 3 mrem/year |
| Drinking Water | 5 mrem/year |
| Pocket watch (radium dial) | 6 mrem/year |
| Eyeglasses (containing thorium) | 6 - 11 mrem/year |
| Transatlantic Airplane roundtrip | 5 mrem |

| | |
|---|---|
| Chest x-ray, dental x-ray, head & neck | 5-20 mrem |
| Lumbar spinal x-rays | 130 mrem |
| Pelvis x-ray | 44 mrem |
| Hip x-ray | 83 mrem |
| CT (head and body) | 1,100 mrem |

Useful to know:

100 Röntgen equivalent man (REM) = 1 Sievert

http://newnet.lanl.gov/info/dosecalc.asp

http://www.epa.gov/radiation/understand/calculate.html

# Appendix: Derivation of the Compton Relationship

**Situation:** A photon with energy $E_i$ collides with e of mass $m_e$ at rest. One wants to know the energy of the photon after the collision.

**Conservation of energy**

$$E_e = (E_i - E_f) + m_e c^2$$

$$E_e^2 = (E_i - E_f)^2 + (m_e c^2)^2 + 2(E_i - E_f)\, m_e c^2$$

$$E_e^{\,2} = p_e^{\,2} c^2 + m_e^{\,2} c^4 \quad\cdots\cdots\quad p_e^{\,2} = \frac{E_e^{\,2}}{c^2} - m_e^{\,2} c^2$$

$$p_e^2 c^2 + (m_e c^2)^2 = (E_i - E_f)^2 + (m_e c^2)^2 + 2(E_i - E_f)\, m_e c^2$$

$$p_e^2 c^2 = (E_i - E_f)^2 + 2(E_i - E_f)\, m_e c^2$$

$$p_e^2 c^2 = E_i^2 + E_f^2 - 2E_i E_f + 2(E_i - E_f)\, m_e c^2$$

**Conservation of momentum:**

$$\left(\vec{p}_e\right)^2 = \left(\vec{p}_i - \vec{p}_f\right)^2$$

$$\left(\vec{p}_e\right)^2 = p_i^{\,2} + p_f^{\,2} - 2 p_i p_f \cos\theta$$

$$p_i = \frac{E_i}{c} \qquad p_f = \frac{E_f}{c}$$

$$\left(\vec{p}_e\right)^2 = \left(\frac{E_i}{c}\right)^2 + \left(\frac{E_f}{c}\right)^2 - 2\frac{E_i E_f}{c^2}\cos\theta$$

$$p_e^{\,2} c^2 = E_i^{\,2} + E_f^{\,2} - 2E_i E_f \cos\theta$$

$$-2E_i E_f + 2(E_i - E_f)\, m_e c^2 = -2E_i E_f \cos\theta$$

$$E_i E_f - (E_i - E_f)\, m_e c^2 = E_i E_f \cos\theta$$

$$E_i E_f + E_f m_e c^2 - E_i E_f \cos\theta = E_i\, m_e c^2$$

$$E_f (E_i + m_e c^2 - E_i \cos\theta) = E_i\, m_e c^2$$

$\theta$

$(-)$

$$\Longrightarrow \quad E_f = \frac{E_i m_e c^2}{E_i\left(1 - \cos\theta\right) + m_e c^2} \quad \text{3-20}$$