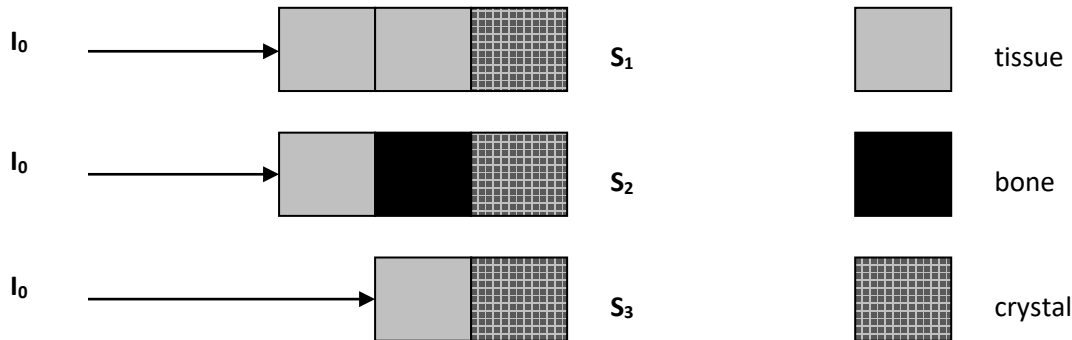


Solution 1 – Crystal Sensors

The value of μ_{tissue} is 0.5 cm^{-1} , μ_{bone} is 1 cm^{-1} , and μ_{crystal} is 2 cm^{-1}



The intensity of signal (light) produced by the crystal is proportional to the attenuation of X-rays within the crystal.

The intensity of signal coming into the crystal is equal to:

$$I_{in_1} = I_0 e^{-0.5 \cdot 1} e^{-0.5 \cdot 1} = I_0 e^{-0.5 \cdot 2}$$

$$I_{in_2} = I_0 e^{-0.5 \cdot 1} e^{-1 \cdot 1}$$

$$I_{in_3} = I_0 e^{-0.5 \cdot 1}$$

The intensity of signal coming out of the crystal is equal to:

$$I_{out_1} = I_{in_1} e^{-2 \cdot 1}$$

$$I_{out_2} = I_{in_2} e^{-2 \cdot 1}$$

$$I_{out_3} = I_{in_3} e^{-2 \cdot 1}$$

Therefore the intensity of signal (light) produced by the crystal is given by:

$$S \propto \text{crystal absorption} = I_{in} - I_{out}$$

$$S_1 \propto I_{in_1} - I_{out_1} = I_{in_1} - I_{in_1} e^{-2 \cdot 1} = I_0 e^{-0.5 \cdot 2} (1 - e^{-2 \cdot 1}) = 0.318 I_0$$

$$S_2 \propto I_{in_2} - I_{out_2} = I_{in_2} - I_{in_2} e^{-2 \cdot 1} = I_0 e^{-0.5 \cdot 1} e^{-1 \cdot 1} (1 - e^{-2 \cdot 1}) = 0.193 I_0$$

$$S_3 \propto I_{in_3} - I_{out_3} = I_{in_3} - I_{in_3} e^{-2 \cdot 1} = I_0 e^{-0.5 \cdot 1} (1 - e^{-2 \cdot 1}) = 0.524 I_0$$

Solution 2 – Radiation Detection

With all the conversion efficiency coefficients given for the different physical processes involved in the radiation detection, the total efficiency of the detection can be obtained by simply multiplying the efficiencies of the subprocesses :

$$\text{Nb of scintillation photons produced in the NaI(Tl) crystal : } 60[\text{keV}] * 30 \left[\frac{\gamma}{\text{keV}} \right] = 1800 \gamma$$

$$\text{Nb of photons absorbed by the photocathode: } 1800 \gamma * 80\% = 1440 \gamma$$

$$\text{Nb of electrons produced in the photocathode: } 1440 \gamma * 0.05 = 72 \text{ electrons}$$

$$\text{Nb of electrons produced after multiplication in the dynodes: } 72 * 3^{10} = 4.25 \cdot 10^6 \text{ electrons}$$

Solution 3 – SNR Considerations

- a) Since the SNR is proportional to the square root of the number of counts, the doubled injected dose increases the SNR by the square root of 2 to give a value of 71:1.
- b) The activity decay is described by $A(t) = A_0 e^{-\lambda t}$.

λ can be derived from the half-life time with the following relation : $\lambda = \frac{\ln(2)}{T_{1/2}} = 3.21 \cdot 10^{-5} s^{-1}$

The number of counts (C) in the experiment is assumed to be proportional to the number of disintegrations (D): $C = \alpha D$ where α is the efficiency of the detection.

The SNR is proportional to the square root of the number of counts. The activity is the infinitesimal number of disintegrations per unit of time. Thus, the number of disintegrations during a given period is the integral of the activity in this period:

$$D_{1,2} = \int_{t_1}^{t_2} A(t) dt = \int_{t_1}^{t_2} A_0 e^{-\lambda t} dt = \frac{-1}{\lambda} A_0 [e^{-\lambda t}]_{t_1}^{t_2} = \frac{A_0}{\lambda} (e^{-\lambda t_1} - e^{-\lambda t_2}) = \frac{1}{\lambda} (A(t_1) - A(t_2))$$

The detected counts during this period are thereby:

$$C_{1,2} = \frac{\alpha}{\lambda} (A(t_1) - A(t_2)) = \frac{\alpha A_0}{\lambda} (e^{-\lambda t_1} - e^{-\lambda t_2})$$

We know now that the SNR is proportional to the number of counts in the measurement. In the case of the 30 min scan, we have:

$$SNR_1 \propto \sqrt{C_{0,30min}} = \sqrt{\frac{\alpha}{\lambda} (A(0) - A(30min))}$$

In the case of the 60 min scan, we have:

$$SNR_2 \propto \sqrt{C_{0,60min}} = \sqrt{\frac{\alpha}{\lambda} (A(0) - A(60min))}$$

In both cases, λ is the same since the radiotracer is the same and α is also the same in both measurements, assuming same detection geometry. Thus, since we are working with proportionalities, we can write:

$$\begin{aligned} SNR_1 &\propto \sqrt{(A(0) - A(30min))} \\ SNR_2 &\propto \sqrt{(A(0) - A(60min))} \end{aligned}$$

$A(0)$ is in both cases 1mCi.

$$A(30min) = A(1800 \text{ seconds}) = A(0) e^{-\lambda(t=1800s)} = A(0) e^{-3.21 \cdot 10^{-5}(t=1800s)}$$

$$\text{Similarly, } A(60min) = A(3600 \text{ seconds}) = A(0) e^{-\lambda(t=3600s)} = A(0) e^{-3.21 \cdot 10^{-5}(t=3600s)}$$

We know that the $SNR_1 = 50:1$. Let's calculate the ratio between SNR_1 and SNR_2 to get free from the proportionalities:

$$\frac{SNR_2}{SNR_1} = \frac{\sqrt{(A(0) - A(60min))}}{\sqrt{(A(0) - A(30min))}} = \frac{\sqrt{A(0)(1 - e^{-3.21 \cdot 10^{-5} \cdot 3600})}}{\sqrt{A(0)(1 - e^{-3.21 \cdot 10^{-5} \cdot 1800})}} = 1.394$$

So, since SNR_1 is 50:1, then SNR_2 is $1.394 \cdot SNR_1 = \boxed{69.7:1}$

- c) We calculate first the energy of the emitted photon :

$$E = h\nu = h \frac{c}{\lambda} = 2.26 \text{ J} = 141 \text{ keV}$$

In the table at the end of the series 5, we see that the mass attenuation coefficient of iron at this energy is :

$$\mu/\rho = 1.96 \cdot 10^{-1} \text{ cm}^2/\text{g}$$

The linear attenuation coefficient can then be calculated using the density of iron:

$$\mu = \mu/\rho \cdot \rho = 1.54 \text{ cm}^{-1}$$

The measured intensity behind 2 cm of iron will be :

$$I_1 = I_0 e^{-\mu x} = I_0 e^{-1.54 \cdot 2} = 0.046 I_0$$

We know that the SNR goes with the square root of the number of counts (or intensity).

For I_0 , we had a SNR of 50:1. We have then :

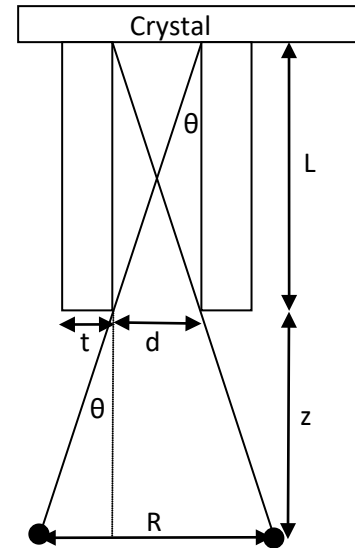
$$\frac{SNR_1}{SNR_0} = \frac{\sqrt{I_1}}{\sqrt{I_0}} = \frac{\sqrt{0.046 I_0}}{\sqrt{I_0}} = 0.21$$

$$SNR_1 = 0.21 * SNR_0 = 11:1$$

Solution 4 – Collimation I

Sizes are displayed in the figure on the right. The resolution R can be defined as the minimum distance at which two point sources can still be separated.

Two triangles with an angle θ can be defined so that $\tan\theta = d/L = \frac{1}{2}(R-d)/z$, so

$$R=(2dz+dL)/L.$$


Solution 5 – Collimation II

- a) The measured sensitivity for ^{99m}Tc is $5.88 \cdot 10^5 / (51.80 \cdot 10^3 \cdot 2) = 5.676 \text{ counts}/(\text{kBq} \cdot \text{min})$. The factory specification is $202 \text{ cnts}/(\mu\text{Ci} \cdot \text{min}) = 202/37 \text{ cnts}/(\text{kBq} \cdot \text{min}) = 5.459 \text{ cnts}/(\text{kBq} \cdot \text{min})$. Here $\mu\text{Ci} = 37 \text{ kBq}$ is used. The measurement thus gives a 4% higher sensitivity.
- b) The surface of the Petri dish that can be seen from P is circular and has a radius $R = 0.5 \cdot D \cdot (H+L)/L$ if H is the distance from the dish to the top of the collimator. The surface is then πR^2 . Only the activity within this surface can be attributed to the number of counts in P. The chance that radiation from this surface goes through a surface dA around P is equal to the relative spatial angle (ω_r) under which dA is seen: $\omega_r = dA / (4\pi \cdot (H+L)^2)$. The number of counts in dA around P is then proportional to $\pi R^2 \cdot \omega_r = \pi [0.5 \cdot D \cdot (H+L)/L]^2 \cdot dA / (4\pi \cdot (H+L)^2) = 0.0625 (D/L)^2 dA = 0.0625 (1.45/24.1)^2 dA = 2.262 \cdot 10^{-4} dA$. This is independent of H. This example can be generalized by looking at a random spatial angle from a random point on the crystal.

- c) The number of crystal parts under the Petri dish is given by: $n = \frac{S}{\pi \frac{D^2}{4}}$
- The part a of the total activity A in front of each hole is: $a = \frac{A}{S} \pi \frac{D^2}{4}$

The radiation density r seen by each crystal part is given by the spatial angle with which a point on the Petri dish "sees" the crystal part:

$$r = a * \frac{\pi \frac{D^2}{4}}{4\pi L^2}$$

Finally, the total radiation density R seen by the complete crystal is:

$$R = n * r = \frac{S}{\pi \frac{D^2}{4}} * a * \frac{\pi \frac{D^2}{4}}{4\pi L^2} = \frac{S}{\pi \frac{D^2}{4}} * \frac{A}{S} \pi \frac{D^2}{4} * \frac{\pi \frac{D^2}{4}}{4\pi L^2} = A * \underbrace{\frac{\pi \frac{D^2}{4}}{4\pi L^2}}_{\text{geometric efficiency } \varepsilon}$$

$$\varepsilon = 2.262 * 10^{-4}$$

- d) The factory specification says 202 cnts/(μ Ci·min) are counted. An activity of 1 μ Ci corresponds to $60 \cdot 3.7 \cdot 10^4$ disintegrations per minute. The fraction of disintegrations that leads to a count (using 1 Ci = 37 GBq, 1 Bq = 1/s) is then:

$$(202 \text{ cnts/min}) / (3.7 \cdot 10^4 \text{ Bq}) = (202 \text{ cnts/ } 60 \text{ s}) / (3.7 \cdot 10^4 \text{ s}) = 9.099 \cdot 10^{-5}.$$

- e) The geometric sensitivity (c.) is higher because (I) in the real measurement not all gammas are detected (only photons that undergo photo-absorption are counted, but there is also Compton scattering), (II) we neglected the surface of lead which decreases the effective crystal surface, (III) there is attenuation in the fluid and the dish, the coating of the collimator and the protection of the NaI crystal, and (IV) ^{99m}Tc only emits a 140 keV photon in 88% of its disintegrations.

However, detection of scattered photons also takes place, which will slightly compensate for the effects mentioned above. Apparently the effects under (I) to (IV) dominate though.