Ultrasound

Solution 1

a. If the two structures are 0.5 mm apart in depth, then the difference in path for backscattered beams of sound is 1 mm.
   A resolution criterion is not an absolute value and might depend on the pulse shape and experimental noise. In the course (see lesson 2, slide 5), the criterion was to have two times the pulse length between two successive echoes to be able to distinguish two structures. In theory, the resolution criterion can be lowered to a less conservative value of one pulse length between two successive echoes. It is the limit case before the pulses are overlapping.
   With this criterion, structures can be seen as apart when the path difference is bigger than the pulse length, which is 2 wave lengths (2λ) here. So $2\lambda < 1$ mm, or $\lambda < 0.5$ mm, and since the frequency is $f = c/\lambda$, it goes that $f > 1580/(0.5\cdot10^{-3}) = 3.16\cdot10^{6}$ Hz = 3.16 MHz.
   (one would get 6.32 MHz with the criterion used in the lesson 2)
b. The attenuation is 1 dB/(MHz·cm). If the depth is 5 cm, the total path of received sound is 10 cm. The attenuation is then 10 dB/MHz. An attenuation of 10^{10} corresponds to $10\cdot\log_{10}(10^{10}) = 100$ dB. The transducer then should be allowed to have a frequency of 10 MHz based on allowable attenuation.
c. Somewhere in between the two frequencies: sufficient resolution with no more attenuation than necessary. In practice the choice will determined by what transducers are available.

Solution 2

Consider the schematic view (figure below) of a profile in depth of a head scanned with ultrasound imaging. The structures are successively skin, bone, brain and bone again, characterized by impedances Z_1, Z_2, and Z_3 (=$1.12\cdot10^6$, $7.8\cdot10^6$ and $1.09\cdot10^6$ respectively). We consider that the transducer is in close contact with the skin and we neglect absorption as well as signal coming from multiple reflections.

a. Since the transducer is assumed to be in close contact with the skin, I_0 is entirely transmitted into the skin layer. At the first interface, the portion of sound wave transmitted through the bone is:

$$R_{skin/bone} = \left(\frac{Z_{skin} - Z_{bone}}{Z_{skin} + Z_{bone}}\right)^2 = 0.56$$

$$T_{skin/bone} = 1 - R_{skin/bone} = 0.44$$

   In the same way, we calculate the reflexion and transmission coefficients between bone and brain.

$$R_{brain/bone} = \left(\frac{Z_{brain} - Z_{bone}}{Z_{brain} + Z_{bone}}\right)^2 = 0.57$$

$$T_{brain/bone} = 1 - R_{brain/bone} = 0.43$$

   Neglecting the absorptions and the multiple reflections, the signal intensity coming back from the brain is then by:

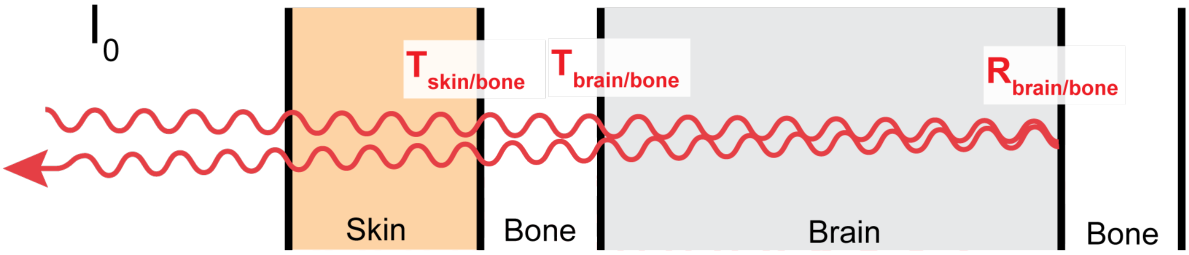
$$I = I_0 \cdot T_{skin/bone} \cdot T_{brain/bone} \cdot R_{brain/bone} \cdot T_{brain/bone} \cdot T_{skin/bone}$$

$$I = 0.02 \cdot I_0$$

   Only 2% of the input signal is coming back from the brain and can be used for imaging.

b. The bone/soft tissue interfaces are especially disadvantageous for ultrasound imaging. The consequence is that a big shadow (with low SNR) is observed behind each bone structure. The amount of signal coming back from the brain is very low, even without considering absorption. Since the brain is surrounded by the skull, ultrasound imaging is inappropriate for brain measurements.

c. We assumed that the transducer is in close contact with the skin. However, in practice, this is not perfectly true, and a layer of air might be present between the transducer and the skin. Considering the very low acoustic impedance of air, the reflexion on the skin would be more than 99%. Gel, with similar impedance than skin, is put between the transducer and the skin to avoid this type of loss through reflections outside of the body.



X-rays interactions and production

Solution 3

a. An electron volt is the energy that an electron would have if it were accelerated by a one volt potential difference :
W = qV = $(1.6{\cdot}10^{-19}$ C) (1.0V) = $1.6{\cdot}10^{-19}$ J = 1 eV.
b. 141 eV ·1000 ·$1.6{\cdot}10^{-19}$ J /eV = $2.26{\cdot}10^{-14}$ J
c. ν = c/λ = [($3{\cdot}10^{8}$ m/s)/400 nm] = $7.5{\cdot}10^{14}$ s-1 = $7.5{\cdot}10^{14}$ Hz
Energy per photon : $E = h\nu = 4.95{\cdot}10^{-19}\, J = 3.1\, eV$
d. E = $m_e c^2$ = $8.19{\cdot}10^{-14}$ J = 0.51 MeV

Solution 4

a. When a radiation is ionizing, it means that it has sufficient energy to interact with atoms by removing one electron. The least energy for that is encountered in the case of the H atom, where the binding energy of the electron is 13.6 eV. If the radiation is ionizing, it can modify the electronic structures of molecules and stimulate chemical reactions by creation of free radicals, which is especially a problem in living tissue, where this can alter the functions of cells and modify their DNA. The ionizing limit is thus an important threshold to quantify the dangerousness of an electromagnetic radiation.
b. The critical value (see course slides 14-15) is 13.6eV. From the relationship of energy and wavelength of EM waves (slide 13):

$$E = hc/\lambda$$

we get :

$$\lambda_{ionizing} = \frac{hc}{E_{ionizing}} = 91\, nm$$

(be careful to use SI units)
Since $E_{ionizing}$ is a minimal energy limit, $\lambda_{ionizing}$ is a maximal wavelength limit.
c. By looking on the EM spectrum (slide 15), we see that the ionizing limit is at shorter wavelength than visible light, somewhere in the middle of the UV radiation. Sunscreen is therefore used in case of sun exposure, in order to absorb or reflect some of the sun's ultraviolet radiation on the skin exposed to sunlight and thus helps protect against sunburn and prevents skin cancer.
d. By conservation of energy in the photoelectric effect, the energies of the photoelectrons are the following:
K-shell: 40 - 28 = 12 keV
L-shell: 40 - 5 = 35 keV

Constants

Energy of a Tc_{99m} photon	141 keV
Electron charge	$e = 1.6 \cdot 10^{-19}$ C
Speed of light	$c = 3.0 \cdot 10^{8}$ m/s
Electron mass	$m_e = 9.1 \cdot 10^{-31}$ kg
Planck constant	$h = 6.6 \cdot 10^{-34}$ J·s