## Problem 1 – SNR vs. Contrast

Notice that the field of view (FOV) and the matrix size is unchanged for all the cases a, b and c represented in this problem. It essentially means that spatial resolution is the same for three cases in-plane. But due to increasing slice thickness, through-plane resolutions in the third dimension are different.

1) SNR is given by $\frac{Signal}{Noise}$

   Case (b): $SNR_{lesion} = \frac{10}{1} = 10$ whereas $SNR_{tissue} = \frac{5}{1} = 5$

   Case (c): $SNR_{lesion} = \frac{15}{1} = 15$ whereas $SNR_{tissue} = \frac{10}{1} = 10$

   Case (d): $SNR_{lesion} = \frac{25}{1} = 25$ whereas $SNR_{tissue} = \frac{20}{1} = 20$

   As we can see for both lesion and tissue regions, case (d) provides maximal SNR.

2) Contrast between two regions A and B in an image is given by $Contrast_{A-B} = \frac{Signal_A - Signal_B}{Signal_A + Signal_B}$

   Case (b): $Contrast_{lesion-tissue} = \frac{10-5}{10+5} = 0.33$

   Case (c): $Contrast_{lesion-tissue} = \frac{15-10}{15+10} = 0.2$

   Case (d): $Contrast_{lesion-tissue} = \frac{25-20}{25+20} = 0.11$

3) CNR between two regions A and B in an image is given by $CNR_{A-B} = \frac{Signal_A - Signal_B}{Noise}$

   Case (b): $CNR_{lesion-tissue} = \frac{10-5}{1} = 5$

   Case (c): $CNR_{lesion-tissue} = \frac{15-10}{1} = 5$

   Case (d): $CNR_{lesion-tissue} = \frac{25-20}{1} = 5$

We see how the contrast and CNR are affected by the choice of slice thickness. If the slice is too thick, we get a good SNR but the contrast is reduced by a partial volume effect (mixing of the signals of the lesion and background). If the slice is too thin, the SNR may be too low to visualize the details clearly.

## Problem 2 – Voxel Size vs. SNR

a) Assuming that molecule density is constant over the sample, the signal strength is proportional to the voxel size (proportional to the number of molecules). It means that a :

   1) 20%, 2) 50%, 3) 80%

   reduction of the edge size is equivalent to a :

   1) $1-0.8^3 = 49\%$, 2) $1-0.5^3 = 88\%$, 3) $1-0.2^3 = 99\%$ (!)

reduction of voxel size respectively → reduction of signal strength.

b) SNR can be kept by averaging the signal over many acquisitions:

Averaging affects noise, not the signal, which keeps the same intensity. In fact, averaging the signals allow to decrease the noise variance and, consequently the noise, so that SNR is increased. However this method is really time consuming and can cause many problems for clinical applications!!

## Problem 3 – Optimising CNR

a) CNR is the difference in signal intensity relative to noise between two tissues. In other words, it describes the ability to distinguish different tissues. Difference in signal intensities arises from the different relaxation times ($T_1$) between tissues. CNR formula is given by:

$$\mathrm{CNR(t)} = \frac{1}{noise}(S_{GM}(t)-S_{WM}(t)) = \frac{1}{noise}\left(e^{-\frac{t}{T_{1,WM}}} - e^{-\frac{t}{T_{1,GM}}}\right)$$

The difference between CNR and SNR is that CNR evaluates the difference in signal intensity between two tissues comparing to the noise while SNR evaluates the absolute signal intensity comparing to the noise.

b) Maximum CNR is found when :

$$\frac{d}{dt}\mathrm{CNR(t)} = 0 = \frac{1}{noise}\left(\frac{-1}{T_{1WM}}e^{-\frac{t}{T_{1WM}}} + \frac{1}{T_{1GM}}e^{-\frac{t}{T_{1GM}}}\right)$$

$$\frac{1}{T_{1WM}}e^{-\frac{t}{T_{1WM}}} = \frac{1}{T_{1GM}}e^{-\frac{t}{T_{1GM}}}$$

$$\frac{T_{1GM}}{T_{1WM}} = e^{-\frac{t}{T_{1GM}}+\frac{t}{T_{1WM}}} = e^{\frac{(T_{1GM}-T_{1WM})t}{T_{1GM}T_{1WM}}}$$

$$\frac{(T_{1GM}-T_{1WM})}{T_{1GM}T_{1WM}}\,t = \ln\left(\frac{T_{1GM}}{T_{1WM}}\right)$$

$$t = t_{max} = \frac{T_{1GM}T_{1WM}}{(T_{1GM}-T_{1WM})}\ln\left(\frac{T_{1GM}}{T_{1WM}}\right)$$

c) If SNR is low for all the tissues, the whole image might have a level of noise high enough to be higher or in the same range than the signal intensity differences between different tissues, which means a bad CNR. That's why a good CNR necessitates at least one of the tissues to have a good SNR.

However, a good SNR doesn't mean a good CNR in every case. In fact, if the properties of different tissues ($T_1$ in the previous example) are quite similar, their signal intensities won't be much different, and no good CNR could be obtained even if their SNR is good.

In order to differenciate different tissues, the CNR is the most important factor, although a good CNR cannot be obtained without a relatively good SNR.