

## Solution 1: Quiz of the Week

Answer the following questions:

- FOV gets bigger. Thinking in 1D, the discrete sampling of the Fourier transform of the signal is similar to multiply this Fourier transform with a Dirac comb of period p. The inverse Fourier transform of this is a convolution of the real signal (object) with a Dirac comb of period 1/p. If the k-space is sampled more densely, p is smaller and 1/p is thus bigger. The obtained image is thus repeated with a larger period, enabling the imaging of bigger objects ( $\leftrightarrow$ larger FOV)
- The area under the pre- and re-phasing gradient lobe has to be equal to obtain an echo. See slide 11-7.
- If you apply three gradient  $G_x, G_y$  and  $G_z$  during the acquisition, the signal that you obtain is :

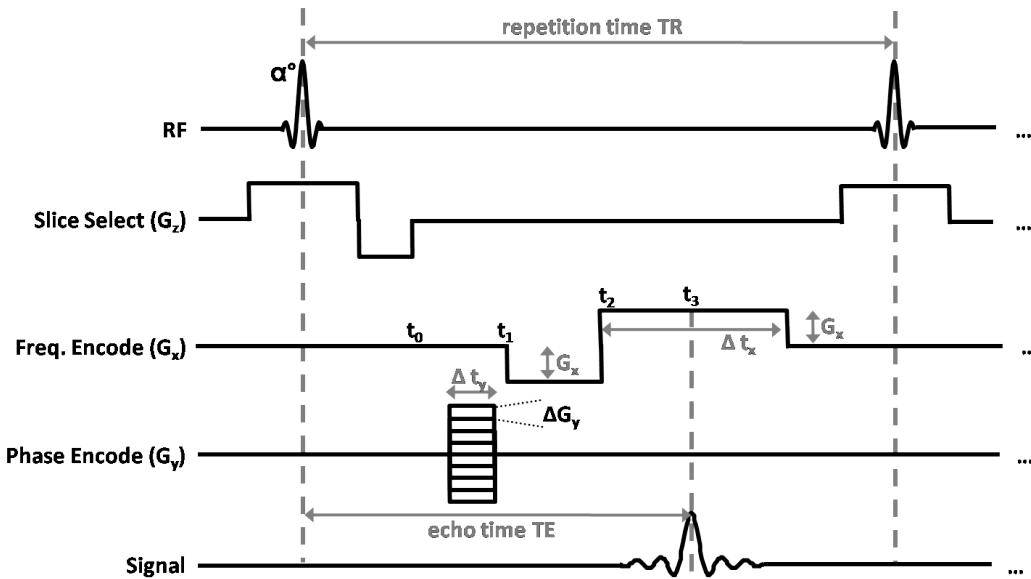
$$S(t) = \iiint_{\text{object}} M_{\perp}(x, y, z) e^{i\gamma G_x x t} e^{i\gamma G_y y t} e^{i\gamma G_z z t} dx dy dz$$

In this case, you cannot scan the k-space independently in x, y and z directions. For a certain value of t, you have a fixed triplet of values for  $k_x, k_y$  and  $k_z$ . You are actually measuring the projection of the signal along a k-space

line parametrically defined by  $\begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix} = \gamma t \begin{pmatrix} G_x \\ G_y \\ G_z \end{pmatrix}$

- In the slice selection dimension, with a same pulse (fixed bandwidth), the exited slice will be thinner for higher gradient. See slide 11-4.  
In the frequency encoded direction, bigger gradients enable to measure a larger frequency range and obtain in that way higher frequency components. The signal is dephased and rephased faster, resulting in less signal loss. Similarly, stronger gradients enable the measurement of higher frequency components in the phase direction in the same amount of time. See slide 11-12  
In summary, stronger gradients improve the resolution in the slice selection direction. In the frequency and phase encoding direction, stronger gradients either enable higher resolution or decreased acquisition time.
- An echo is generally needed because the signal can technically not be sampled directly after a pulse bringing the magnetisation in the transverse plane. The electronic has to be switched from transmitting mode to receiving mode and the encoding gradients must be switched on. This takes time during which a good part of the signal already vanished due to  $T_2$  decay.

## Solution 2: The 2D FLASH Sequence



## I. Slice Selection

a) Suppose the RF bandwidth is 1.0 kHz and a 3-mm-thick slice is desired. What gradient amplitude should be used for the slice selection?

$$G_z = \frac{2\pi\Delta f}{\gamma\Delta z} = \frac{1.0 \cdot 10^3 \text{ Hz}}{42.57 \cdot 10^6 \frac{\text{Hz}}{\text{T}} \cdot 0.003 \text{ m}} = 7.83 \frac{\text{mT}}{\text{m}}$$

b) At 1.5 T, water resonates approximately  $f_{cs}=210$  Hz higher than lipids (i.e. fat). If the amplitude of the slice-selection gradient is +11mT/m, what is the slice-selection offset caused by chemical shift?

$$\delta z = \frac{2\pi f_{cs}}{\gamma G} = \frac{210 \text{ Hz}}{42.57 \cdot 10^6 \frac{\text{Hz}}{\text{T}} \cdot 11 \frac{\text{mT}}{\text{m}}} = 0.45 \text{ mm}$$

c) After slice selection, the phases of the excited spins are de-phased in the direction orthogonal to the slice. This so-called "phase dispersion" can be rewound using a gradient opposite to the one used for selecting the slice. Assume the spins are excited exactly at time point 0.5\*RF\_duration. How big is the maximal phase dispersion of the slice in problem a) above?

$$\text{phase difference in rad: } \delta\varphi = \omega\tau = 2\pi f\tau, \text{ with } \tau = \frac{\text{RF duration}}{2}$$

maximal phase difference if spins are maximally far apart:  $\delta z = 3\text{mm}$

$$\delta\varphi = \omega\tau = \gamma \cdot G_z \cdot \delta z \cdot \tau = 267.5 \cdot 10^6 \frac{\text{rad}}{\text{sT}} \cdot 7.83 \cdot 10^{-3} \frac{\text{T}}{\text{m}} \cdot 3 \cdot 10^{-3} \text{ m} \cdot \tau$$

$$\delta\varphi = 6283.6 \cdot \tau \text{ rad} = 3.14 \text{ rad}$$

## II. Gradient Echo

a) For the sake of simplicity, the gradients in the diagram are drawn as rectangles (instead of trapezoids like in the course). Why is this (i.e. the rectangular case) unrealistic? Give a physical explanation.  
Because slew rates are physically limited. With rectangular gradients, the slew rate would be infinity.

b) Let  $\varphi(t, x, y)$  be the phase of a spin at time  $t$  and position  $(x, y)$  the excited slice. We assume that after slice selection, the phase is zero ( $\varphi(t_0, x, y) = 0 \text{ rad}$ ). Give a mathematical description of the phase at time points  $t_1-t_3$  of the Nth phase encoding step with respect to the imaging gradients amplitudes  $G_x$  and  $\Delta G_y$ .

$$\begin{aligned}\varphi(t_1, x, y) &= \gamma \Delta t_y \cdot N \cdot \Delta G_y \cdot y \\ \varphi(t_2, x, y) &= \varphi(t_1, x, y) - \gamma \frac{\Delta t_x}{2} \cdot G_x \cdot x \\ \varphi(t_3 = TE, x, y) &= \varphi(t_2, x, y) + \gamma \frac{\Delta t_x}{2} \cdot G_x \cdot x = \varphi(t_1, x, y)\end{aligned}$$

At time point  $t_3$ , a gradient echo occurs. As the formula shows, the spins have at that point a phase proportional to their position in y-direction and to the number  $N$  of the current phase encoding step.

c) Suppose our scanner has a maximal gradient amplitude of  $G_{max}=40 \text{ mT/m}$ . A volume of 80 slices, each having a matrix size of  $128 \times 128$  pixels on a field-of-view (FOV) of  $24 \times 24 \text{ cm}^2$  is acquired from this scanner with a sampling bandwidth  $f_s=128 \text{ kHz}$ . Calculate the necessary gradient amplitude  $G_x$  using the diagram below.

Set maximal Larmor frequency of outmost point of the FOV to equal the sampling bandwidth:

$$f_{Larmor,max} = \frac{\gamma}{2\pi} \cdot G_x \cdot x_{max} := f_s$$

This leads to a gradient strength  $G_x$  of

$$G_x = \frac{2\pi \cdot f_s}{\gamma \cdot x_{max}} = 12.5 \frac{\text{mT}}{\text{m}}$$

With the given sampling bandwidth, the frequency encoding takes

$$\Delta t_x = \frac{128}{f_s} = 1 \text{ ms}$$

For the pre-winder, we need only half of the area under the gradient. Since the amplitude stays the same, the time needed for the prephaser is thus 0.5 ms.

The maximal gradient strength in plus/minus y-direction  $|N/2 \cdot \Delta G_y \cdot \Delta t_y|$  is equal to  $\gamma G_x \cdot \Delta t_x$  because the FOV and matrix size are the same in both dimensions. We want to use the minimal time for this gradient, so we take the maximal gradient amplitude, which gives a gradient duration

$$G_{max} \cdot \Delta t_y = G_x/2 \cdot \Delta t_x \rightarrow \Delta t_y = \frac{G_x/2 \cdot \Delta t_x}{G_{max}} = \frac{\frac{12.5 \text{ mT}}{m} \cdot 1 \text{ ms}}{\frac{40 \text{ mT}}{m} \cdot 2} = 156.3 \mu\text{s}$$

Now we add together all the gradient durations:

$$TE_{min} = \text{slice selection} + \text{phase encoding} + \frac{\Delta t_x}{2} + \frac{\Delta t_x}{2} = 1 \text{ ms} + 0.156 \text{ ms} + 1 \text{ ms} = 2.16 \text{ ms}$$

$$TR_{min} = TE_{min} + \frac{\Delta t_x}{2} + \text{slice selection}/2 = 3.16 \text{ ms}$$

$$\text{acquisition time } TA = 80 * 128 * 2.66 \text{ ms} \cong 32 \text{ s}$$

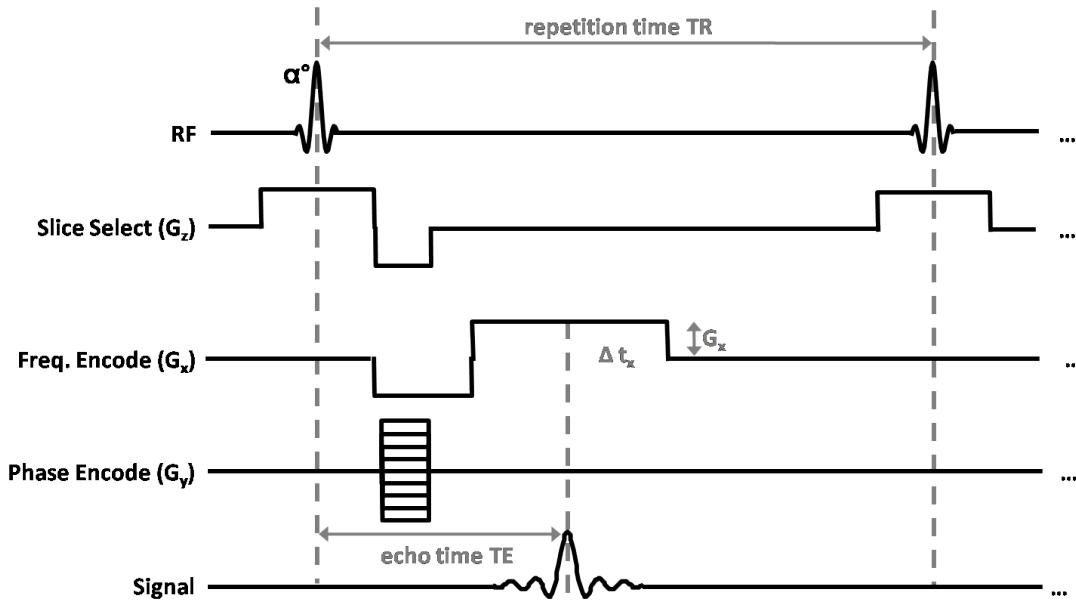
d) In the sequence shown in the diagram, the slice rephase gradient, the phase encode gradient and the frequency encode dephasing gradient are played out one after the other. This prolongs the echo time  $TE$  and the repetition time  $TR$  considerably. To speed up the sequence, the three gradients can be played out at the same time. Prove (taking the considerations from b) that this still creates the same gradient echo at time point  $t_3$ . Sketch also the new sequence diagram and calculate the minimal  $TE$  with the parameters from c).

The formulas in b) are completely independent of the absolute time they are executed. Thus, they can be superimposed to each other to reduce  $TE$ .

If the gradient durations are not changed, the minimal echo time is

$$TE_{min} = 1 \text{ ms} + \frac{\Delta t_x}{2} = 1.5 \text{ ms}$$

The optimised sequence diagram is shown below.



e) *What further possibilities do you see to speed up the sequence?*

Increase the sampling bandwidth. This reduces the time for the frequency encoding gradient. Also the slice rewinder can be made shorter with corresponding higher amplitude.

f) *Sketch the k-space trajectory of this sequence.*

Cartesian trajectory.

g) *Usually, only one k-space line per TR is acquired. Why is that? If we acquired more than one line per excitation, what would we have to do and what problems would we encounter?*

Because the signal decays away (for them with T2). It is done, but you have a T2(\*) filter artefact, i.e. not every k-space line has the same contrast! You could acquire two images though and map T2(\*).

## Solution 3: Creatine bottles

a) The gradient acts as an additional magnetic field which strength varies linearly. Consequence is that creatine spins “see” a different magnetic field depending on their spatial position and their precession frequency vary linearly along gradient direction.

b) The chemical shift differences of the ethanol peaks converted in Hertz are:

$$\delta_{\text{CH}_3} - \delta_{\text{OH}} = 1.18 \text{ ppm} \Rightarrow \nu_{\text{CH}_3} - \nu_{\text{OH}} = 1.18 * 128 \text{ Hz} = 151 \text{ Hz}$$

$$\delta_{\text{OH}} - \delta_{\text{CH}_2} = 1.38 \text{ ppm} \Rightarrow \nu_{\text{CH}_2} - \nu_{\text{OH}} = 1.38 * 128 \text{ Hz} = 177 \text{ Hz}$$

At 3 T a gradient strength of  $0.05 \text{ mT/m}$  creates a frequency shift of  $0.05 * \frac{\gamma}{2\pi} * 10^{-3} \text{ Hz/m} = 2.1 \text{ kHz/m}$ .

Consequently, if a first bottle with concentration  $C_1$  is set at position  $x = 0$  (equivalent to the  $\text{CH}_2$  peak), the second bottle has to be at position  $x = + 177/2100 = 8 \text{ cm}$  with half concentration (the OH group has only half number of the  $\text{CH}_2$  group), and the third bottle at position  $x = + 328/2100 = 15.6 \text{ cm}$  with concentration  $3/2$ .

c) No it's false. Each encoding step creates a phase modulation of the whole sample, whose effect is to encode one spatial dimension, while signal is collected from the whole sample. Consequently, no k space point has a value of zero, which would mean that the signal coming from the whole sample would be zero!

For the same reason frequency encoding (which creates a frequency modulation along the sample) cannot lead to null value of k-space points.