Solution 1: MR Quiz

a) False. As the magnetisation vector returns to the z axis, necessarily the transverse component goes to zero.
b) True. If the B_0 field is inhomogeneous, nuclei at different positions will process at different rates and hence the spin system de-phases faster, which results in a shorter free induction decay.
c) False. When the magnetization relaxes, excess energy in the spin system is released to the environment (the lattice).
d) False. A shorter T_1 means that it takes shorter time for the spin system resumes to its thermal equilibrium state.
e) False.
f) A solid-state sample.

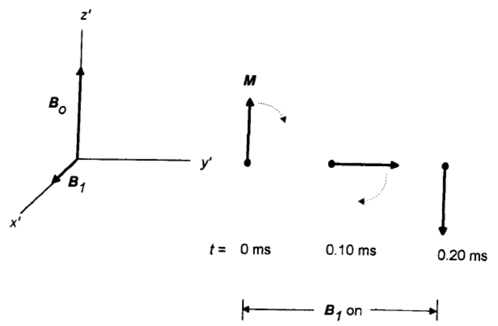
Solution 2: Precession

a) $\nu = \frac{\gamma B_0}{2\pi} = \frac{\left(267.512 \cdot 10^6 \frac{rad}{Ts}\right) \cdot 5.87\ T}{2 \cdot 3.14\ rad} = 250\ Mhz$
Since $\mathbf{B_1}$ is $10^{-5}\mathbf{B_0}$, ^1H nuclei will precess around $\mathbf{B_1}$ at 10^{-5}*(250MHz)=2.5kHz

b) The cycle time t_0=1/ν=0.400ms.

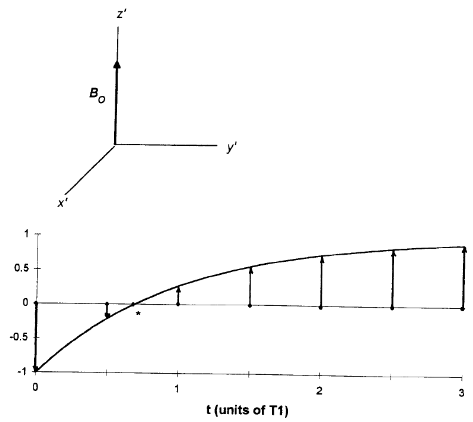
c) Since one complete cycle requires 0.4ms, **M** will rotate 360^0 in 0.4ms. The tip angle will start at 0^0 at t=0. After 0.1ms it will have rotated one-fourth of a cycle (90^0) and 180^0 after 0.2ms.

d) See figure:



e) A tip angle of 180^0 results in a perfectly inverted M, where the population of down spins now outnumbers up spins. No T_2-controlled relaxation is needed since M already lies along the –z axis. But the normal Boltzmannn distribution must be reestablished by longitudinal relaxation (controlled by T1).

See figure below.



Solution 3: Longitudinal Relaxation

a) The z-component of magnetization is given by:

$$M_z(\tau) = M_z(\tau = 0)(1 - 2e^{-\tau/T_1})$$

By setting the left hand side equal to zero, and solving for τ we obtain:

$$\tau = T_1\, ln(2)$$

b) Inversion recovery experiment. A 180° pulse is applied, then a relaxation delay t_{ir} is applied before flipping the relaxed magnetization into the xy plane. The 90° pulse allows to access magnetization along the longitudinal axis, which means the relaxation at time t_{ir}. Redoing this experiment several times varying the relaxation delay t_{ir} allows to rebuild the relaxation curve given by the equation in point a). Especially, you can determine the point such as $M_z(\tau) = 0$ and then calculate relaxation time T_1.

c) The experiment would consist in applying a 180° pulse and waiting a time T_{IR} such that the CSF would be in the xy plane (but not the WM magnetization that has a different T_1) before applying a 90° pulse. The basic idea of the experiment is that after the 90° pulse, the CSF magnetization would return along the z axis and wouldn't be measured while the remaining WM along the z axis would be flip into the xy plane. Using the equation found in a) :

$$T_{IR} = T_1\, ln(2)$$

As $T_{1,CSF}$ = 2800ms, T_{IR} = 1941ms.

At time T_{IR} the WM magnetization (along the z axis) will be :

$$M_{z,WM}(T_{IR}) = M_{z,WM}(t = 0)\left(1 - 2e^{-\frac{T_{IR}}{T_{1,WM}}}\right) = 0.82\, M_{z,WM}(t = 0)$$

Using this experiment CSF signal would be cancelled while WM signal would have fully recovered!!!

Solution 4: Liver Experiment

Signal decrease (in xy plane) is governed by T_2 relaxation. It is described by the following equation : $S(t) = S_0\, e^{-t/T_2}$

So for t = 40ms

$$S(40ms) = S_0\, e^{-40/40} = 0.37\, S_0$$

And for t = 500ms

$$S(500ms) = S_0\, e^{-500/40} = 3.7 \times 10^{-6}\, S_0$$

Solution 5: Excitation Pulses

The excitation profile is given by:

$$H(f) = \int_0^{2\tau} A e^{-i2\pi f t} dt = A \int_0^{2\tau} \cos(2\pi f t)\, dt - iA \int_0^{2\tau} \sin(2\pi f t)\, dt$$

$$= \frac{A}{2\pi f} \sin(2\pi f t)\,\Big|_0^{2\tau} + \frac{iA}{2\pi f} \cos(2\pi f t)\,\Big|_0^{2\tau} = \frac{A}{2\pi f}\,[\sin(4\pi f \tau) + i\,(\cos(4\pi f \tau) - 1)]$$

$$= \frac{A}{2\pi f}\sqrt{2 - 2\cos(4\pi f \tau)}\, \mathrm{e}^{\mathrm{i}\varphi} = \frac{A}{2\pi f}\sqrt{4\sin^2(2\pi f \tau)}\, \mathrm{e}^{\mathrm{i}\varphi} = \frac{A}{\pi f}\,|\sin(2\pi f \tau)|\, \mathrm{e}^{\mathrm{i}\varphi}$$, with

$\varphi = \mathrm{arctg}\left(\frac{\cos(4\pi f \tau) - 1}{\sin(4\pi f \tau)}\right) = -2\pi f \tau$: a phase modulation

$\frac{A}{\pi f}\,|\sin(2\pi f \tau)|$: the amplitude of each frequency contributing to the signal

The excitation bandwidth is given by the frequency interval between the two first zeros of the sinc function: $\frac{A}{\pi f}\,|\sin(2\pi f \tau)|\, \mathrm{e}^{-\mathrm{i}2\pi f \tau}$.

Thus the excitation bandwidth is $\frac{1}{\tau}$.