Internal thermal noise in the LIGO test masses: A direct approach

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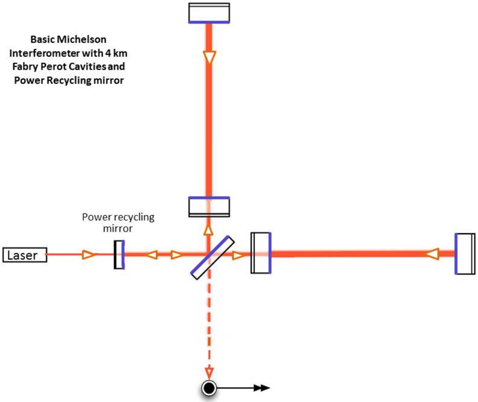
Introduction



- Dual recycled, Fabry-Perot Michelson Interferometer
- Noise in the position of the test masses limits the sensitivity of the measurement

General Method

Reminder: Fluctuation Dissipation Theorem

$$|F_L(\omega)|^2 = 4k_BT\, R(\omega)$$

$$S_{VV}(\omega) = 4k_BT\, \frac{R(\omega)}{|Z(\omega)|^2}$$

$$\text{where } R(\omega) := Re(Z(\omega)), \qquad Z(\omega) := \frac{F(\omega)}{v(\omega)}$$

This implies for the spectral density of the position:

$$S_{XX}(\omega) = \frac{S_{VV}(\omega)}{\omega^2} = \frac{4k_BT}{\omega^2}\frac{R(\omega)}{|Z(\omega)|^2}$$

General Method

- The read out variable of interest is:

$$x(t) = \int f(\vec{r})\, y(\vec{r}, t)\, d^2r$$

$y(\vec{r}, t)$: displacement of boundary along direction of laser beam at time t

$f(\vec{r})$: form factor dependent on monitoring laser beam, which full-fills:

$\int f(\vec{r})\, d^2r = 1$

- We are interested in fluctuations in $x(t)$.
Goal: Find the spectral density associated with these fluctuations, $S_{XX}(f)$.

General Method

- Apply the generalized fluctuation-dissipation theorem:

$$S_{XX}(f) = \frac{k_B T}{\pi^2 f^2} \, |Re(Y(f))|,$$

where $Y(f) := \frac{1}{Z(f)}$ is the complex admittance associated with x(t).

- Consider an oscillating pressure at surface of test mass:

$$P(\vec{r},t) := F(t) f(\vec{r}), \qquad \text{where } F(t) = F_0 \cos(2\pi f t)$$

- Link the response of the system to the power dissipated into the system W_{diss}:

$$W_{diss} = \frac{F_0^2}{2} |Re(Y(f))|$$

General Method

- One obtains:

$$S_{XX}(f) = \frac{2k_B T}{\pi^2 f^2} \frac{W_{diss}}{F_0^2} \tag{1}$$

- Thus, to calculate $S_{XX}(f)$ one can follow these three steps:

(i) Apply an oscillatory pressure $P(\vec{r}, t) = F_0 \cos(2\pi f t) f(\vec{r})$

(ii) Calculate the average power W_{diss} dissipated in the test mass when the oscillating force is applied.

(iii) Use equation (1) to calculate $S_{XX}(f)$.

Homogeneous damping

Application of derived method to fused silica test mass:

- Youngs Modulus of fused silica: $E = E_0(1 + \mathrm{i}\phi(f))$
  $\phi(f)$: material's loss angle

- Assume a Gaussian beam profile: $f(\vec{r}) = \frac{1}{\pi r_0^2} e^{-r^2/r_0^2}$,
  r_0: radius of monitoring laser beam.

- Calculated dissipated power via: $W_{diss} = 2\pi f\, U_{max}\, \phi(f)$
  U_{max} : Energy of elastic deformation when test mass is maximally contracted.

Homogeneous damping

Result

$$S_{XX}(f) = \frac{4k_B T}{f} \frac{1-\sigma^2}{\pi^3 E_0 r_0} \, I \, \phi \cdot (1 + \mathcal{O}(\frac{r_0}{R}))$$

E_0, ϕ: from Young modulus of material
σ: Poisson ratio of material
R: Radius of test mass
$I \approx 1.87322$ for a Gaussian beam

- The analytic expression is more exact for small probe beam sizes ($\frac{r_0}{R} \rightarrow 0$).
- Result is in agreement with the result obtained using the normal mode decomposition. E.g. $S_{XX}(100Hz) \approx 8.7 \cdot 10^{-40}$ m^2/Hz.

Surface Damping

- Surface losses could play a role if the mirror coating dissipates energy or if the mirror surface is polished inadequately.

- The power dissipated at one point of the material is proportional to the square of the stress ($= force\ /\ area$). Since we apply the force at a finite surface, the dissipated power scales as:

$$W_{diss,coating} \propto (\frac{F_0}{r_0^2})^2\ r_0^2 = \frac{F_0^2}{r_0^2}$$

- This scaling is different than the scaling of the homogeneous case ($S_{XX,bulk} \propto \frac{1}{r_0}$). Surface losses could therefore be important for small beam sizes.

Conclusion

- To compute the thermal internal noise of the test mass, we can use the power dissipated into the system if an oscillation pressure is applied.
- Numerical agreement in case of homogeneous damping in a fused silica test mass monitored by a Gaussian laser beam with result obtained via normal mode decomposition.
- The size of the monitoring beam is taken into account via the spatial profile of the auxiliary pressure. This gives an advantage over the normal mode decomposition.

FIG. 1. Identical defects A and B create fluctuating stress in different parts of the test mass. The stress created by defect A will influence the phase shift of the laser beam readout more than the stress created by defect B, although both A and B make identical contributions to the Q's of the test-mass elastic modes.