

# Measurement of the Instantaneous Velocity of a Brownian Particle

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Brownian motion of particles affects many branches of science. We report on the Brownian motion of micrometer-sized beads of glass held in air by an optical tweezer, over a wide range of pressures, and we measured the instantaneous velocity of a Brownian particle. Our results provide direct verification of the energy equipartition theorem for a Brownian particle. For short times, the ballistic regime of Brownian motion was observed, in contrast to the usual diffusive regime. We discuss the applications of these methods toward cooling the center-of-mass motion of a bead in vacuum to the quantum ground motional state.



# Direct observation of kinesin stepping by optical trapping interferometry

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**Do biological motors move with regular steps? To address this question, we constructed instrumentation with the spatial and temporal sensitivity to resolve movement on a molecular scale. We deposited silica beads carrying single molecules of the motor protein kinesin on microtubules using optical tweezers and analysed their motion under controlled loads by interferometry. We find that kinesin moves with 8-nm steps.**

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### Thermal noise in mechanical experiments

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The fluctuation-dissipation theorem is applied to the case of low-dissipation mechanical oscillators, whose losses are dominated by processes occurring inside the material of which the oscillators are made. In the common case of losses described by a complex spring constant with a constant imaginary part, the thermal noise displacement power spectrum is steeper by one power of  $\omega$  than is predicted by a velocity-damping model. I construct models for the thermal noise spectra of systems with more than one mode of vibration, and evaluate a model of a specific design of pendulum suspension for the test masses in a gravitational-wave interferometer.

## Internal thermal noise in the LIGO test masses: A direct approach

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The internal thermal noise in LIGO's test masses is analyzed by a new technique, a direct application of the fluctuation-dissipation theorem to LIGO's readout observable,  $x(t)$  = (longitudinal position of test-mass face, weighted by laser beam's Gaussian profile). Previous analyses, which relied on a normal-mode decomposition of the test-mass motion, were valid only if the dissipation is uniformly distributed over the test-mass interior, and they converged reliably to a final answer only when the beam size was a non-negligible fraction of the test-mass cross section. This paper's direct analysis, by contrast, can handle inhomogeneous dissipation and arbitrary beam sizes. In the domain of validity of the previous analysis, the two methods give the same answer for  $S_x(f)$ , the spectral density of thermal noise, to within expected accuracy. The new analysis predicts that thermal noise due to dissipation concentrated in the test mass's front face (e.g., due to mirror coating) scales as  $1/r_0^2$ , by contrast with homogeneous dissipation, which scales as  $1/r_0$  ( $r_0$  is the beam radius); so surface dissipation could become significant for small beam sizes. [S0556-2821(97)05524-0]

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# Quantum Mechanics of a Macroscopic Variable: The Phase Difference of a Josephson Junction

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Experiments to investigate the quantum behavior of a macroscopic degree of freedom, namely the phase difference across a Josephson tunnel junction, are described. The experiments involve measurements of the escape rate of the junction from its zero voltage state. Low temperature measurements of the escape rate for junctions that are either nearly undamped or moderately damped agree very closely with predictions for macroscopic quantum tunneling, with no adjustable parameters. Microwave spectroscopy reveals quantized energy levels in the potential well of the junction in excellent agreement with quantum-mechanical calculations. The system can be regarded as a “macroscopic nucleus with wires.”

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emphasized, one must distinguish carefully between macroscopic quantum phenomena originating in the superposition of a large number of microscopic variables and those displayed by a single macroscopic degree of freedom. It is the latter that we discuss in this article.

Our usual observations on a billiard ball or Brownian particle reveal classical behavior because Planck's constant  $\hbar$  is so tiny. However, at least in principle there is nothing to prevent us from designing an experiment in which these objects are quantum mechanical. To do so we have to satisfy two criteria: (i) the thermal energy must be small compared with the separation of the quantized energy levels, and (ii) the macroscopic degree of freedom must be sufficiently decoupled from all other degrees of freedom if the lifetime of the quantum states is to be longer than the characteristic time scale of the system ( $I$ ). To illustrate the application of these



## Subrecoil Laser Cooling and Lévy Flights

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Anomalous diffusion processes, dominated by rare events, are shown to exist and to play a central role in certain subrecoil laser cooling schemes. We present a new statistical analysis of these processes, in terms of Lévy flights, which provides a precise analytical description of the cooled atoms in the long time limit, where the standard methods of quantum optics are inappropriate. These analytical predictions are quantitatively checked by comparison with the results of quantum Monte Carlo simulations of the cooling process at intermediate times.

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## Equation of State Calculations by Fast Computing Machines

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A general method, suitable for fast computing machines, for investigating such properties as equations of state for substances consisting of interacting individual molecules is described. The method consists of a modified Monte Carlo integration over configuration space. Results for the two-dimensional rigid-sphere system have been obtained on the Los Alamos MANIAC and are presented here. These results are compared to the free volume equation of state and to a four-term virial coefficient expansion.



## LETTERS

# Verification of the Crooks fluctuation theorem and recovery of RNA folding free energies

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Atomic force microscopes and optical tweezers are widely used to probe the mechanical properties of individual molecules and molecular interactions, by exerting mechanical forces that induce transitions such as unfolding or dissociation. These transitions often occur under nonequilibrium conditions and are associated with hysteresis effects—features usually taken to preclude the extraction of equilibrium information from the experimental data. But fluctuation theorems<sup>1–5</sup> allow us to relate the work along nonequilibrium trajectories to thermodynamic free-energy differences. They have been shown to be applicable to single-molecule force measurements<sup>6</sup> and have already provided information on the folding free energy of a RNA hairpin<sup>7,8</sup>. Here we show that the Crooks fluctuation theorem<sup>9</sup> can be used to determine folding free energies for folding and unfolding processes occurring in weak as well as strong nonequilibrium regimes, thereby providing a test of its validity under such conditions. We use optical tweezers<sup>10</sup> to measure repeatedly the mechanical work associated with the unfolding and refolding of a small RNA hairpin<sup>11</sup> and an RNA three-helix junction<sup>12</sup>. The resultant work distributions are then analysed according to the theorem and allow us to determine the difference in folding free energy between an RNA molecule and a mutant differing only by one base pair, and the thermodynamic stabilizing effect of magnesium ions on the RNA structure.

system usually makes it difficult in practice to extract unfolding free energies using small loading rates (below a few pN s<sup>−1</sup>). Drift effects decrease noticeably for larger pulling speeds, making it possible to obtain more reliable experimental data (and also good statistics as a large number of pulls can be executed in a reasonable time), but at the expense of a more irreversible unfolding process. Here we show that significant improvements can be obtained by using the CFT, which provides a more robust and more rapidly converging method to extract equilibrium free energies from non-equilibrium processes.

The CFT allows us to quantify the amount of hysteresis observed in the values of the irreversible work done to unfold and refold a macromolecule. Let  $P_U(W)$  denote the probability distribution of the values of the work performed on the molecule in an infinite number of pulling experiments along the unfolding (U) process, and define  $P_R(W)$  analogously for the reverse (R) process. For the CFT to be applicable, the unfolding and refolding processes need to be related by time-reversal symmetry, that is, in our experiments, the optical trap used to manipulate the molecule must be moved at the same speeds during unfolding and refolding. Moreover, the molecular transition probed always has to start in an equilibrium state (folded in the unfolding process, and denatured or unfolded in the refolding process) and reach a well-defined final state. The CFT<sup>9</sup> then predicts that:

$$P_U(W) = e^{-(W - \Delta G)/k_B T} P_R(W)$$



## Quantum Theory of Cavity-Assisted Sideband Cooling of Mechanical Motion

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We present a quantum-mechanical theory of the cooling of a cantilever coupled via radiation pressure to an illuminated optical cavity. Applying the quantum noise approach to the fluctuations of the radiation pressure force, we derive the optomechanical cooling rate and the minimum achievable phonon number. We find that reaching the quantum limit of arbitrarily small phonon numbers requires going into the good-cavity (resolved phonon sideband) regime where the cavity linewidth is much smaller than the mechanical frequency and the corresponding cavity detuning. This is in contrast to the common assumption that the mechanical frequency and the cavity detuning should be comparable to the cavity damping.



# A NEW METHOD FOR THE DETECTION OF A PERIODIC SIGNAL OF UNKNOWN SHAPE AND PERIOD

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## ABSTRACT

We present a new method for the detection and measurement of a periodic signal in a data set when we have no prior knowledge of the existence of such a signal or of its characteristics. It is applicable to data consisting of the locations or times of discrete events. We use Bayes's theorem to address both the signal detection problem and the estimation problem of measuring the characteristics of a detected signal. To address the detection problem, we use Bayes's theorem to compare a constant rate model for the signal to models with periodic structure. The periodic models describe the signal plus background rate as a stepwise distribution in  $m$  bins per period, for various values of  $m$ . The Bayesian posterior probability for a periodic model contains a term which quantifies Ockham's razor, penalizing successively more complicated periodic models for their greater complexity even though they are assigned equal prior probabilities. The calculation thus balances model simplicity with goodness of fit, allowing us to determine both whether there is evidence for a periodic signal, and the optimum number of bins for describing the structure in the data. Unlike the results of traditional "frequentist" calculations, the outcome of the Bayesian calculation does not depend on the number of periods examined, but only on the range examined. Once a signal is detected, we again use Bayes's theorem to estimate various parameters of the signal, such as its frequency or the shape of the light curve. The probability density for the frequency is inversely proportional to the multiplicity of the binned events, which is simply related both to the combinatorial entropy of the binned distribution and to the  $\chi^2$  measure of its misfit to a uniform distribution used in the "epoch folding" method for period detection. The probability density for the light-curve shape produces light-curve estimates that are superpositions of stepwise distributions with various phases and number of bins, and which are thus smoother than a simple histogram. Error bars for the light-curve shape are also easily calculated. The method also handles gaps in the data due to intermittent observing or dead time. We apply the method to simulated data generated with both stepwise and sinusoidal light curves and demonstrate that it can sensitively detect such signals and accurately estimate both the signal frequency and its shape, even when the light curve does not have a stepwise shape. We also describe a test for nonperiodic source variability that is a simple modification of our period detection procedure.

*Subject headings:* methods: analytical — methods: numerical



## Decoherence Dynamics of Complex Photon States in a Superconducting Circuit

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Quantum states inevitably decay with time into a probabilistic mixture of classical states due to their interaction with the environment and measurement instrumentation. We present the first measurement of the decoherence dynamics of complex photon states in a condensed-matter system. By controllably preparing a number of distinct quantum-superposed photon states in a superconducting microwave resonator, we show that the subsequent decay dynamics can be quantitatively described by taking into account only two distinct decay channels: energy relaxation and pure dephasing. Our ability to prepare specific initial quantum states allows us to measure the evolution of specific elements in the quantum density matrix in a very detailed manner that can be compared with theory.

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## Quantum Reservoir Engineering with Laser Cooled Trapped Ions

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We show how to design different couplings between a single ion trapped in a harmonic potential and an environment. The coupling is due to the absorption of a laser photon and subsequent spontaneous emission. The variation of the laser frequencies and intensities allows one to “engineer” the coupling and select the master equation describing the motion of the ion. [S0031-9007(96)01762-0]

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