Statistical Physics IV: Non-equilibrium statistical physics

ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE (EPFL)

Solutions to Exercise No.8

Solution: Crooks fluctuation theorem and Jarzynski equality

(a) Probability distribution of the work computed for both directions and intersection gives free energy.

(b) Jarzynski equality requires to take equilibrium measurements that take very long stabilization and are subject to drift. Moreover, the average to be done is of an exponential that is very sensitive to rare tail events.

Solution: Quantization and coherent states

(a) First show

$$a\,|\alpha\rangle = e^{-|\alpha|^2}\sum_n \alpha^n \frac{a\,|n\rangle}{\sqrt{n!}} = e^{-|\alpha|^2}\sum_n \alpha^n \frac{|n-1\rangle}{\sqrt{(n-1)!}} = \alpha\,|\alpha\rangle$$

Then we see that

$$\langle n\rangle = \langle\alpha|a^\dagger a|\alpha\rangle = |\alpha|^2$$

and

$$\langle n^2\rangle = \langle\alpha|a^\dagger a a^\dagger a|\alpha\rangle = \langle\alpha|(a^\dagger a^\dagger a a + a^\dagger a a)|\alpha\rangle = |\alpha|^4 + |\alpha|^2$$

(b)

$$a^\dagger||\alpha\rangle = \sum_n \alpha^n (n+1)\frac{|n+1\rangle}{\sqrt{(n+1)!}} = \frac{\partial}{\partial\alpha}||\alpha\rangle$$