Statistical Physics IV: Non-equilibrium statistical physics
ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE (EPFL)

Exercise No. 12

12.1 Dissipative coupling induced by a reservoir and emergence of dark and bright modes[1]

Consider the coupling of multiple (two) harmonic oscillators to the *same* heat bath as given by the following Hamiltonians.

$$\hat{H}_{sys} = \hbar\omega_1(\hat{a}_1^\dagger \hat{a}_1 + \frac{1}{2}) + \hbar\omega_2(\hat{a}_2^\dagger \hat{a}_2 + \frac{1}{2})$$

$$\hat{H}_{bath} = \sum_k \hbar\omega_k(\hat{b}_k^\dagger \hat{b}_k + \frac{1}{2})$$

$$\hat{H}_{int,bath} = \sum_k \sum_{i=1,2} \hbar g_k(\hat{b}_k^\dagger \hat{a}_i + \hat{b}_k \hat{a}_i^\dagger)$$

1. Demonstrate that in addition to the standard dissipation we encountered before, the dynamics leads to the emergence of coupling between the harmonic oscillator modes, a process termed *dissipative coupling*. Demonstrate this property by deriving the quantum Langevin equations for the system operators. Explain for which frequencies ω_1,ω_2 this coupling is relevant for the system dynamics.

2. Repeat the treatment for the case in which the two modes have initially the same frequency $\omega_1 = \omega_2$ (i.e. are frequency degenerate) and in addition coupled together, as given by: $\hat{H}_{int,m} = \sum_{i \neq j} \hbar g_i(a_j^\dagger \hat{a}_i + a_j \hat{a}_i^\dagger)$. Show, that in the limit of large mutual coupling g_i (compared to the decay rates of the individual modes, induced by the bath) and for the case where the coupling of both modes to the bath is approximately equal ($g_1 \approx g_2$), a *bright mode* and a *dark mode* emerge, that differ in their energy decay rates. Do so by diagonalizing the system Hamiltonian in the presence of the mutual coupling ($\hat{H}_{int,m}$) and find the decay rates of the new modes (which are symmetric and anti-symmetric superpositions of the bare cavity modes). How can you physically explain the different decay rates of the bright and the dark mode?

12.2 Quantum reservoir engineering*[2]

Explain how Kienzler *et al.* were able to engineer non-classical states in their experiment, by engineering the coupling Hamiltonian to the dissipative environment.

[1] see Toth et al: "A dissipative quantum reservoir for microwave light using a mechanical oscillator", Nature Physics (2017)

[2] See Poyatos, Cirac and Zoller, "Quantum Reservoir Engineering with Laser Cooled Trapped Ions", PRL 1996 for the theory and Kienzler et al., "Quantum harmonic oscillator state synthesis by reservoir engineering", Science 2014 for the experimental realization.