Statistical Physics IV: Non-equilibrium statistical physics

ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE (EPFL)

Solutions to Exercise No.5

5.1 Solution: Proof of Kramers escape rate

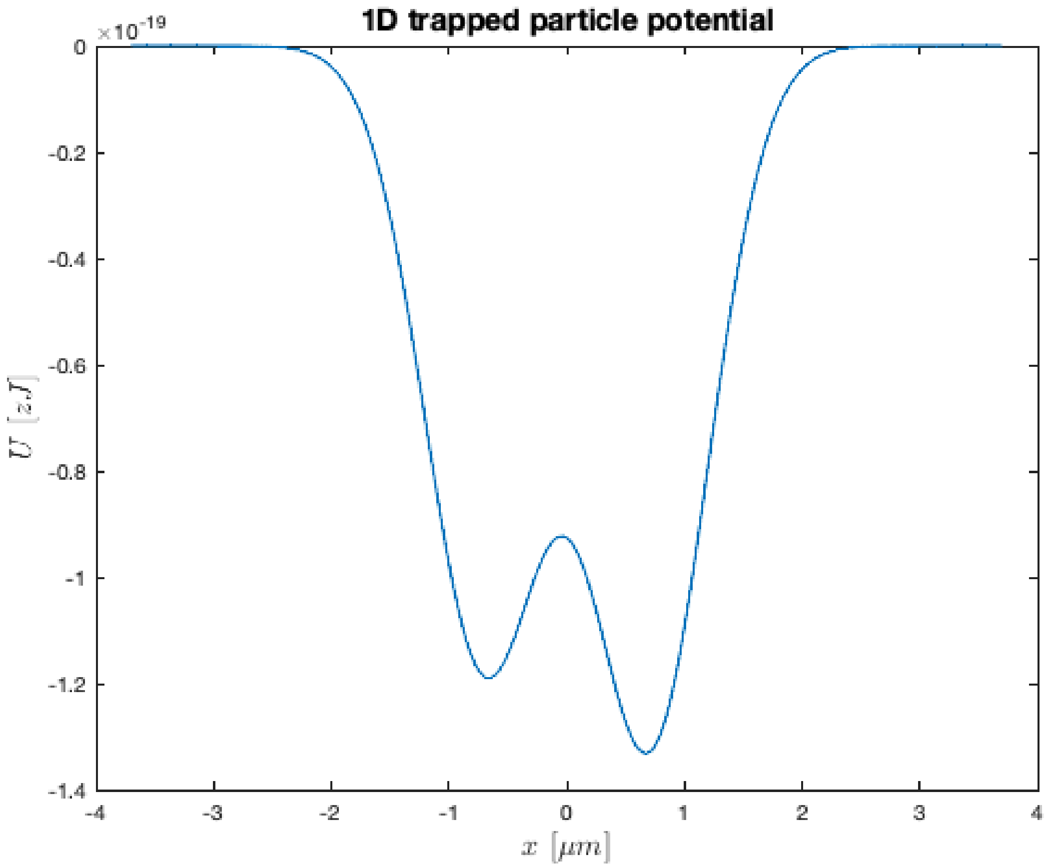
You can find the solution in Risken, "The Fokker-Planck Equation: Methods of Solution and Applications", Second edition, Pg. 123-124. The book is available on the course Moodle page.

5.2 Solution: Application of the Kramers escape rate formula: optical tweezers

See the supplementary Matlab code below:

```
%Stat Phys IV exercise: estimation of Kramers transition rate between two
%optical tweezer traps


clear all; close all;

w0=1e-6; %Common beam waist
x10=-7e-7; x20=-x10; %Beam centers positions
P1=8e-2; P2=9e-2; %Beam powers
I10=(2*P1/pi/(w0^2)); %Calculation of peak intensities
I20=(2*P2/pi/(w0^2));
% Factor for conversion of optical intensities to potential
% energy (in SI units)
k=-2.28e-30;
D=2.47e-12; %Diffusion coefficient (in SI units)
kBT=4.14e-21; %Boltzmann factor (in SI units)

x=linspace(-3*w0+x10,3*w0+x20,10002);
xinc=(x(end)-x(1))/(length(x)-1);
I1=I10*exp(-2*(x-x10).^2/(w0^2));
I2=I20*exp(-2*(x-x20).^2/(w0^2));
U=k*(I1+I2); %Potential landscape
dU=diff(U)/xinc;
d2U=diff(dU)/xinc;
x=x(2:end-1); U=U(2:end-1); dU=dU(1:end-1); %Rescaling to the size of d2U
d2U_peaks=findpeaks(abs(d2U));
% Peaks corresponding to the desired potential bareer:
d2_bottom_well=d2U_peaks(2); d2_top_well=d2U_peaks(3);
U_top=findpeaks(U,'NPeaks',1); U_bottom=-findpeaks(abs(U),'NPeaks',1);
% Kramers escape rate:
r=D*sqrt(d2_bottom_well*d2_top_well)*exp((U_bottom-U_top)/kBT)/(2*pi*kBT);


plot(x*1e6,U);
title('1D trapped particle potential','FontSize',14);
xlabel('$x\ [\mu m]$','Interpreter','latex','FontSize',14);
ylabel('$U\ [zJ]$','Interpreter','latex','FontSize',14);
disp(['Transition rate: ', num2str(r*1000), '[mHz]']);
```

Transition rate: 56.6581[mHz]



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5.4 Solution: First passage time and the backward Fokker-Planck equation

1. For the derivation of the backward Fokker-Planck equation see The Fokker-Planck Equation Methods of Solution and Applications, H. Risken, Second Edition, Chapter 4.2
2. For the calculation of the first passage time see Handbook of Stochastic Methods for Physics, Chemistry and Natural Sciences, C. W. Gardiner, 2nd ed, Chapter 5.2.7
3. For the simulation see the supplementary Jupyter notebook.