Quantum Field Theory

Set 13

Exercise 1: Unpolarized $e^+e^- \to \mu^+\mu^-$ scattering

The aim of this exercise is to compute the differential cross section $\frac{d\sigma}{d\cos\theta}$ for the process $e^+e^- \to \mu^+\mu^-$. Proceed through the following steps.

- Recall the QED Lagrangian and extract the relevant Feynman rules.
- Write all the Feynman diagrams contributing to the scattering.
- Guess by dimensional analysis the high-energy behavior of the cross section.
- Write down the amplitude according to the Feynman rules; neglect the electron mass.
- Square the amplitude and sum/average over polarizations.
- Consider the cross section in the center of mass frame: include flux factor and phase space factor.
- Parametrize the momenta and write the cross section as a function of the total center-of-mass-energy squared; check with the dimensional analysis guess.

Recommended complementary reading: Peskin-Schroeder pages 141-148, to understand in detail the helicity structure of this scattering.

Exercise 2: Parity violation in polarized Z decay

Consider the term describing the interaction between a massive neutral vector boson Z_μ and a lepton-antilepton pair:

$$\mathcal{L}_{\text{int}} = Z_\mu \bar{l}\gamma^\mu(g_V + g_A\gamma^5)l.$$

Show that if both g_V and g_A are non zero it doesn't exist a parity assignment for the field Z_μ which makes the Lagrangian invariant under parity transformations.
Consider the decay $Z \to e^+e^-$ where the initial Z is polarized in the $\hat{z}$ direction. Call θ the angle between the momentum of the electron and the $\hat{z}$ direction. Show that if the interaction is parity preserving the decay amplitude is invariant under $\theta \to \pi - \theta$.
Define the forward-backward asymmetry:

$$A = \frac{N_+ - N_-}{N_+ - N_-},$$

where N_+ (N_-) is the number of electrons emitted in the upper (lower) half space w.r.t. the $\hat{z}$ direction. This quantity must be zero in a parity invariant theory. Compute A given that:

$$A = \frac{\Gamma_{[0,\pi/2]} - \Gamma_{[\pi/2,\pi]}}{\Gamma_{[0,\pi/2]} + \Gamma_{[\pi/2,\pi]}},$$

with

$$\Gamma_{[a,b]} \equiv \int_a^b \frac{d\Gamma}{d\theta} d\theta.$$

Exercise 3: Higgs mechanism

Consider the lagrangian of a $SU(2) \times U(1)$ gauge theory, containing a scalar field H in the $(\mathbf{2}, 1)$ representation of the gauge group

$$\mathcal{L} = -\frac{1}{4g^2} W^I_{\mu\nu} W^{I\,\mu\nu} - \frac{1}{4{g_Y}^2} B_{\mu\nu} B^{\mu\nu} + (D_\mu H)^\dagger D^\mu H + V(H)\,,$$

where the covariant derivative is

$$D_\mu H = \left(\partial_\mu + \frac{i}{2} \sigma^I W^I_\mu + \frac{i}{2} B_\mu \right) H$$

and the potential for the scalar is given by

$$V(H) = -m^2 H^\dagger H + \frac{\lambda}{2} (H^\dagger H)^2\,.$$

$v = \frac{m}{\sqrt{\lambda}}$ is the vacuum expectation value of $|H|$. Using the parametrization $H(x) = U(x) \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$, we choose the unitary gauge and set $U(x) = \mathbb{1}$.

Substitute this into the lagrangian, and rescale the gauge fields to get canonically normalized kinetic terms. Diagonalize the mass matrix of vectors and identify the massive $W^\pm_\mu, Z_\mu$ and the massless A_μ fields, and their masses.