Quantum Field Theory

Exercises in preparation for the exam 2

Exercise 1: two real scalar fields

Consider a theory with two real scalar fields φ_1 and φ_2 and the following Lagrangian :

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi_1\partial^\mu\varphi_1 + \frac{1}{2}\partial_\mu\varphi_2\partial^\mu\varphi_2 + g\partial_\mu\varphi_1\partial^\mu\varphi_2 - \frac{m^2}{2}(\varphi_1^2+\varphi_2^2) - \frac{\lambda}{4!}(\varphi_1^2+\varphi_2^2)^2$$

For what values of g this represents a well defined QFT ? Find the masses of the physical particles.

Exercise 2: Fermi Lagrangian

Consider the Lagrangian of a massive vector field A_μ coupled to a Dirac fermion ψ:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2 A_\mu A^\mu + i\bar{\psi}\not{D}\psi.$$

Derive the equations of motion for the massive vector. Solve them formally making a Fourier transform. Consider the low energy (large distance) solution for A_μ and show that

$$A_\mu(x) \simeq \frac{q}{M^2}\bar{\psi}(x)\gamma_\mu\psi(x).$$

Plug this solution into the equations of motion for the fermion ψ and thus show that the same result can be obtained from the so called *Fermi Lagrangian* that contains an interaction involving 4 fermion fields:

$$\mathcal{L}_F = \bar{\psi}i\not{\partial}\psi - \frac{q^2}{2M^2}\bar{\psi}\gamma^\mu\psi\ \bar{\psi}\gamma_\mu\psi.$$

Exercise 3: Parity and Spinor Representation

Taking into account that under parity rotations and boosts transform as, $J^i \to J^i$ and $K^i \to -K^i$. Then under parity,

$$P : J^i_\pm \to J^i_\mp, \tag{1}$$

which imply that Lorentz representations under parity undergo a swap,

$$P : (j_-, j_+) \to (j_+, j_-). \tag{2}$$

Now defining a spinor field in the usual way,

$$\Psi_D(x) \equiv \begin{pmatrix}\psi_L(x)\\ \psi_R(x)\end{pmatrix}, \tag{3}$$

where $\psi_L = (j_-, j_+)$ and $\psi_R = (j_+, j_-)$, and knowing how a representation of parity acts on the boost generators show,

$$P\Lambda_{L/R}(\vec{\theta}, \vec{\eta})P = \Lambda_{R/L}(\vec{\theta}, \vec{\eta}). \tag{4}$$

With this, how does parity act on our spinor field Ψ_D?. Particularize for the case $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$.