Quantum Field Theory

Formulaire for the QFT II Exam

Spinors

$$(\not{p}-m)u^s(p) = \bar{u}^s(p)(\not{p}-m)$$
$$(\not{p}+m)v^s(p) = \bar{v}^s(p)(\not{p}+m) = 0$$

$$\sum_s u^s(p)\bar{u}^s(p) = \not{p}+m, \qquad \sum_s v^s(p)\bar{v}^s(p) = \not{p}-m$$

$$P_L = \frac{1-\gamma^5}{2}, \qquad P_R = \frac{1+\gamma^5}{2}$$

Photons

$$\sum_{\text{polarizations}} \epsilon^*_\mu(k)\epsilon_\nu(k) \longrightarrow -\eta_{\mu\nu}$$

This substitution works only when this sum is contracted with a gauge-invariant amplitude.

Massive Vectors

$$\sum_{\text{polarizations}} \epsilon^*_\mu(k)\epsilon_\nu(k) = -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}$$

Pauli Matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$\sigma^\mu = (1,\vec{\sigma}), \qquad \bar{\sigma}^\mu = (1,-\vec{\sigma})$$
$$[\sigma^i,\sigma^j] = 2i\varepsilon^{ijk}\sigma^k, \qquad \{\sigma^i,\sigma^j\} = 2\delta^{ij}$$

Gamma Matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\{\gamma^\mu,\gamma^\nu\} = 2\eta^{\mu\nu}, \qquad \{\gamma^\mu,\gamma^5\} = 0$$

$$\begin{aligned}
\mathrm{tr}(1) &= 4 \\
\mathrm{tr}(\text{odd \# of } \gamma\text{'s except } \gamma^5) &= 0 \\
\mathrm{tr}(\gamma^\mu\gamma^\nu) &= 4\eta^{\mu\nu} \\
\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= 4(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}) \\
\mathrm{tr}(\gamma^5) &= 0 \\
\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^5) &= 0 \\
\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5) &= -4i\epsilon^{\mu\nu\rho\sigma}
\end{aligned}$$

$$\begin{aligned}
\gamma^\mu\gamma_\mu &= 4 \\
\gamma^\mu\gamma^\nu\gamma_\mu &= -2\gamma^\nu \\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= 4\eta^{\nu\rho} \\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= -2\gamma^\sigma\gamma^\rho\gamma^\nu
\end{aligned}$$

$$\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\cdots) = \mathrm{tr}(\cdots\gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\mu)$$

$SU(2)$ Generators

Fundamental representation: $T_a = \frac{\sigma^a}{2}$.

Adjoint representation: $[T_a]_{bc} = -i\varepsilon_{abc}$.

Cross-Sections and Decay Rates

$$d\sigma = \frac{1}{2E_{\mathcal{A}}2E_{\mathcal{B}}|v_{\mathcal{A}}-v_{\mathcal{B}}|}\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)|\mathcal{M}(p_{\mathcal{A}},p_{\mathcal{B}}\to\{p_f\})|^2(2\pi)^4\delta^{(4)}\big(p_{\mathcal{A}}+p_{\mathcal{B}}-\sum_f p_f\big)$$

$$d\Gamma = \frac{1}{2m_{\mathcal{A}}}\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)|\mathcal{M}(m_{\mathcal{A}}\to\{p_f\})|^2(2\pi)^4\delta^{(4)}\big(p_{\mathcal{A}}-\sum_f p_f\big)$$

Phase space when there are only two final particles:

$$\int\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)(2\pi)^4\delta^{(4)}\big(\sum_i p_i-\sum_f p_f\big) = \int\frac{d\Omega_{CM}}{4\pi}\frac{1}{8\pi}\left(\frac{2|\vec{p}|}{E_{CM}}\right)$$

where $|\vec{p}|$ is the magntude of the spacial momentum of either final particle in the center of mass frame, E_{CM} is the total energy in that frame, and $d\Omega_{CM}$ is a solid angle in that frame.

3-Body Phase Space and Decay Rate

$$\begin{aligned}d\Gamma_{A\to BCD} &= \frac{1}{2E_A}|\mathcal{M}_{A\to BCD}|^2 d\Phi_3\\ &= \frac{1}{2M}|\mathcal{M}_{A\to BCD}|^2\frac{d^3p_B}{(2\pi)^3 2E_B}\frac{d^3p_C}{(2\pi)^3 2E_C}\frac{d^3p_D}{(2\pi)^3 2E_D}(2\pi)^4\delta^4\left(P_A-P_B-P_C-P_D\right).\end{aligned}$$

Assuming $|\mathcal{M}_{A\to BCD}|^2$ only depends on the invariants s,t,u ($\equiv$ either ABCD are all scalars or their spins have been averaged) the integral reduces to

$$d\Gamma_{A\to BCD} = \frac{1}{2M}|\mathcal{M}_{A\to BCD}|^2\frac{dE_B dE_C}{4(2\pi)^3}.$$