Useful formulae

- Pauli matrices:
$$\sigma_i = \sigma_i^\dagger, \quad \sigma_i^* = \sigma_i^T \tag{1}$$
$$\epsilon^T = \epsilon^{-1} = -\epsilon \tag{2}$$
$$\epsilon\sigma_i^* = -\sigma_i\epsilon \tag{3}$$
$$U^\dagger\sigma_i U = \sigma_i R_{ij}, \qquad U = \exp\left[i\theta_i\frac{\sigma_i}{2}\right], \quad R = \exp\left[i\theta_i J_i\right], \tag{4}$$
where $[J_i]_{jk} = -i\epsilon_{ijk}$ are the $SO(3)$ generators.

- Levi-civita symbol in N dimension: given a generic $N \times N$ matrix M,
$$\epsilon_{i_1 i_2 \ldots i_N} M_{i_1 j_1} M_{i_2 j_2} \ldots M_{i_N j_N} = \det(M)\, \epsilon_{j_1 j_2 \ldots j_N}. \tag{5}$$

- Levi-civita contraction:
$$\epsilon^{ijk}\epsilon^{ilm} = \delta^{jl}\delta^{km} - \delta^{jm}\delta^{kl}, \tag{6}$$

- Gamma matrices:
$$\sigma^\mu = (\mathbb{1}_{2\times2}, \vec{\sigma}), \qquad \bar{\sigma}^\mu = (\mathbb{1}_{2\times2}, -\vec{\sigma}), \tag{7}$$
$$\epsilon(\bar{\sigma}^\mu)^* = \sigma^\mu\epsilon \tag{8}$$
$$\epsilon(\sigma^\mu)^* = \bar{\sigma}^\mu\epsilon \tag{9}$$
$$\gamma^\mu = \begin{pmatrix} 0_{2\times2} & \sigma^\mu \\ \bar{\sigma}^\mu & 0_{2\times2} \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} -\mathbb{1}_{2\times2} & 0_{2\times2} \\ 0_{2\times2} & \mathbb{1}_{2\times2} \end{pmatrix}. \tag{10}$$

- Distributive properties of commutators:
$$\begin{aligned} [A, BC] &= \{A, B\}C - B\{A, C\}, \\ [A, BC] &= B[A, C] + [A, B]C, \end{aligned} \tag{11}$$

- Definitions of $\sqrt{p\cdot\bar{\sigma}}, \sqrt{p\cdot\sigma}$
$$\sqrt{p\cdot\bar{\sigma}} = \frac{1}{2}\left(\sqrt{\omega_{\mathbf{p}} + m} + \frac{\mathbf{p}\cdot\sigma}{\sqrt{\omega_{\mathbf{p}} + m}}\right) \tag{12}$$
$$\sqrt{p\cdot\sigma} = \frac{1}{2}\left(\sqrt{\omega_{\mathbf{p}} + m} - \frac{\mathbf{p}\cdot\sigma}{\sqrt{\omega_{\mathbf{p}} + m}}\right) \tag{13}$$