RELATIVITY AND COSMOLOGY II

Problem Set 8 16th April 2024

1. Fraction of free protons

Assuming thermal equilibrium, use Saha equation to show that for a given temperature T there is a *unique* solution for the fraction of free protons in the Universe $x = n_p/n_B$, where n_p and n_B are the number of free protons and of baryons, respectively. Assuming that the photon decoupling took place at $T_d \approx 0.25\,\text{eV}$, estimate the fraction of free protons at that time.

2. Decoupling and concentration

Assume that there existed a gas of spinless neutral particles ϕ of the mass $m_\phi = 100\,\text{eV}$, that was in equilibrium with other species in the early Universe. Let the particle ϕ decouple at the temperature $T_d = 150\,\text{MeV}$.
Note: This is bigger than muon mass, $m_\mu = 105.6\,\text{MeV}$.

1. Use the conservation of entropy to estimate the ratio between the temperature of these particles and the temperature of photons at the moment of freezing of e^+e^- annihilation ($\sim 10\,\text{keV}$).

2. Estimate the number density of these particles today.

3. Determine their cosmological abundance. Is the existence of these particles consistent with cosmological observations?

3. Lee-Weinberg bound

Suppose that there exists a fourth neutrino ν_4 which has a mass m_{ν_4} and interacts with other particles in the same way as other neutrinos do. Using observational constraints from cosmology, find available regions of the mass m_{ν_4}.

1. Write down a general condition which determines the neutrino decoupling temperature T^*. Assume that the main reaction keeping ν_4 in equilibrium is its annihilation and that $n_{\nu_4} = n_{\bar{\nu}_4}$.

2. Use the condition that the density of new neutrinos at the present time should not exceed the observed density of dark matter $\rho_{\nu_4} = 2m_{\nu_4} n_{\nu_4}(T_0) \leq \Omega_{DM}\rho_{\text{cr}} \approx 0.25\rho_{\text{cr}}$. Rewrite it in terms of the decoupling temperature T^*.

3. Consider the limiting case in which the neutrinos are ultrarelativistic at the moment of decoupling. Taking into account that the annihilation rate is given by $\langle \sigma v \rangle \sim G_F^2 T^2$, use the conditions from p. 1 and 2 to determine the constraint for m_{ν_4}.

4. Consider the opposite limiting case in which the neutrinos are nonrelativistic at the moment of decoupling. Taking the annihilation rate to be $\langle \sigma v \rangle \sim G_F^2 m_{\nu_4}^2$, use the same conditions to constrain the value of m_{ν_4}.

Note: $G_F \approx 1.16 \times 10^{-5}\,\text{GeV}^{-2}$ is the Fermi constant. Assume electron, muon and tau neutrinos to be massless.