RELATIVITY AND COSMOLOGY II

Problem Set 10 30th April 2024

1. Sphalerons in the Standard Model and beyond

In the Standard Model of particle physics there exist[1] sphaleron transitions that do not conserve the baryon (B) and lepton (L) numbers, and have a rate $\Gamma_{sph} \sim \alpha_W^5 T^4$ for $T \geq 100\,\text{GeV}$. These processes have the following selection rules

$$\Delta B = 3\Delta L_e = 3\Delta L_\mu = 3\Delta L_\tau \ ,$$

which imply that there are three conserved combinations of baryon and lepton numbers that can be chosen as

$$L_e - \frac{1}{3}B, \quad L_\mu - \frac{1}{3}B, \quad L_\tau - \frac{1}{3}B \ .$$

1. Consider a process that converts 3 baryons (9 quarks) to 3 antileptons. Assuming only sphalerons contribute to this process, show that in thermal equilibrium

$$B = \frac{4}{13}(B - L) \ ,$$

   thus sphaleron processes generate non-zero B from non-zero $B - L$.
   Hint: *use the result you derived in Problem set 7 exercise 3.*

2. Let X be a beyond-the-standard-model super-heavy boson (*leptoquark*) that can interact with both quarks and leptons. The decay channels of X are

$X \to ud$	with Γ_{ud} decay rate
$X \to \bar{u}\bar{l}$	with $\Gamma_{\bar{u}\bar{l}}$ decay rate
$X \to \bar{d}\nu$	with $\Gamma_{\bar{d}\nu}$ decay rate

   where l is charged lepton, u and d up and down type quarks and ν a neutrino.[2] Show that processes involving X don't conserve $B-L$. What's the net $B-L$ result from the decay of such leptoquark?

3. Consider early universe containing equal number n_X of leptoquarks X and anti-leptoquarks $\bar{X}$ (with respective decay rates of $\tilde{\Gamma}_{\bar{u}\bar{d}}, \tilde{\Gamma}_{ul}, \tilde{\Gamma}_{d\bar{\nu}}$). Show that, if this process violates CP symmetry, the decay of such particles produce net $B-L$ charge.

[1] This is a non-perturbative process and cannot be represented via any Feynman diagram.

[2] This may seem similar to Y boson decay of SU(5) GUT. However, Y decays into $\bar{d}\bar{\nu}$, not $\bar{d}\nu$ and $B - L$ is exactly conserved in SU(5) model.

2. Rotation curves of galaxies

Consider a rotation of small objects around an isolated galaxy. Assume for simplicity that the galaxy is spherically symmetric. How does their linear velocity depend on the distance to the galaxy center, according to Newton's law? Consider two cases:

1. The density of the galaxy is concentrated around its center ($\rho = \text{const.}$ for $r \leq r_{\text{core}}$; $\rho = 0$ for $r > r_{\text{core}}$);
2. The density of the galaxy decreases according to the power law $\rho \propto r^{-\alpha}$, $\alpha > 0$. What is the value of α that fits flat rotation curves at intermediate distances $r \gtrsim r_{\text{core}}$?

3. Potential well

Consider the potential $V(x)$ such that $V(x) = 0$ in a cube of size R, and $V(x) = \infty$ otherwise. Find the number of energy levels E with momentum less than p_{max} for a quantum particle of mass m in this potential. How many neutrinos of three types can be put in these levels?
Compare the result obtained with the approximate method of the course.

4. Primordial Black Holes

Primordial black holes are also considered to be a candidate of dark matter: they are non-relativistic and effectively collisionless. Estimate the minimum mass of primordial black holes such that they can survive till present to be considered dark matter candidate.

Note: Primordial black holes have Hawking temperature $T = \frac{1}{8\pi GM}$, where M is the mass of the black hole and G the gravitational constant.