Plasma II - Exercises

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1 Multi-choice questions

1.1 If the electron flux to an electrode is not equal to the ion flux, this means that:

( ) the electrode is electrically floating.

( ) there will soon be no gas left between the electrodes.

( ) the ions are much heavier than the electrons.

(x) there is an electric current flowing.

SOLUTION: See Mooc Module 5d @ 5m31s. The current flow to an electrode is the flux of ions minus the flux of electrons to the electrode, multiplied by the magnitude of the electron charge (assuming singly-charged ions). The electrode is said to be electrically floating if the current is zero (effectively infinite impedance to ground).

1.2 The sheath is a dark layer principally because:

( ) the Force is not with it.

( ) T_e is lower than in the plasma bulk, which reduces the gas excitation rate.

( ) the electrode surface absorbs the light in the vicinity of the sheath.

(x) n_e is lower than in the plasma bulk, which reduces the gas excitation rate.

SOLUTION: See Module 5d @ 7m15s. The electron density is strongly reduced as the voltage falls across the sheath towards the wall. Fewer electrons means less excitation of the gas, and hence less light coming from the sheath region, compared with the bulk plasma. The sheath appears dark in contrast with the rest of the plasma.

1.3 The Boltzmann relation can be used to calculate the electron density in the sheath because:

(x) the electron drift velocity is much smaller than the electron thermal velocity.

( ) the ion drift velocity is much smaller than the ion thermal velocity.

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SOLUTION: See Module 5e @ 5m29s. The electrons are effectively in thermal equilibrium everywhere, including in the sheath, because the drift velocity of electrons (and ions) is much less than the electron thermal speed. Therefore, the electron temperature is effectively unchanged, but the electron density is strongly reduced, following the Boltzmann relation as the voltage falls across the sheath towards the wall.

1.4 Negative ions in a plasma can have almost the same mass and temperature as positive ions. Therefore:

( ) They cross the sheath in a similar way to positive ions.

( ) They cross the sheath in a similar way to electrons.

( ) They cross the sheath in a similar way to neutrals.

(x) They cannot cross the sheath and are trapped in the plasma.

SOLUTION: See Module 5a @ 2m33s and Module 5e @ 1m13s. The voltage drop across the sheath is to brake the loss of electrons to the wall. The negative ions are also braked by the sheath voltage drop, but because of their low thermal energy, none of the negative ions can cross the sheath voltage barrier. Negative ions are therefore electrostatically confined in the bulk plasma region.

Exercise 1 - Electron motion and ohmic power in radio-frequency fields

a) In order to find an expression for the electron RF current $\vec{j}$, we must use the momentum balance equation for electrons to find the expression of the electron velocity $\vec{u}$. Writing $\vec{E} = \vec{E}_0 \exp{(j\omega t)}$ and $\vec{u} = \vec{u}_0 \exp{(j\omega t)}$, we obtain:

$$mj\omega\vec{u} = -e\vec{E} - m\nu_{e/n}\vec{u} \tag{1}$$

$$\vec{u} = \frac{-e\vec{E}_o \exp{(j\omega t)}}{m(\nu_{e/n} + j\omega)} = \frac{-e\vec{E}_o \exp{(j\omega t)}}{m(\nu_{e/n}^2 + \omega^2)}(\nu_{e/n} - j\omega) \tag{2}$$

The electron RF current is then:

$$\vec{j} = -ne\vec{u} = \frac{ne^2\vec{E}_o \exp{(j\omega t)}}{m(\nu_m^2 + \omega^2)}(\nu_m - j\omega) \tag{3}$$

and the time-averaged ohmic power per unit volume is:

$$P_{\text{ohm}} = \frac{1}{2}\Re(\vec{j} \cdot \vec{E}^*) = \frac{ne^2E_0^2}{2m}\frac{\nu_m}{(\nu_m^2 + \omega^2)}, \tag{4}$$

which equals zero if there are no collisions.

b) To show that the ohmic power transfer to the plasma, for a given RF frequency ω, reaches a maximum when the angular RF frequency $\omega = \nu_m$ as the pressure is varied (varying consequently the electron-neutral collision frequency ν_m), we have to find the value of ω for which the derivative of the power respect ν_m is zero:

$$\frac{\partial P_{\text{ohm}}}{\partial \nu_m} \propto \frac{\omega^2 - \nu_m^2}{(\omega^2 + \nu_m^2)^2} = 0 \quad \Rightarrow \quad \nu_m = \omega \tag{5}$$

Exercise 2 - Practise the derivation of Bohm's criterion, ion flux, and ion energy to a floating wall

(See Lecture Notes.)

Exercise 3 - Ion energy in RF plasmas

For a constant ion flux, use RF frequency above the ion plasma frequency, say, above 2 MHz. Use an allowed (ISM = Industrial, Scientific and Medical) frequency 13.56 MHz. For symmetric electrodes, you can easily demonstrate that the ion energy is a quarter of the peak-to-peak voltage, i.e. use 400 Vpp for a required ion energy of 100 eV. Note that the difference between plasma and floating potential ($5.2 \cdot T_e \sim 5.2 \cdot 2\,\text{V} \sim 10$ V for argon plasma) is relatively small compared to the RF voltage.