Plasma II - Exercises

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Solutions to problem set 7 - April 4, 2025

1 Multi-choice questions

1.1 A low temperature plasma is principally heated by:

( ) the ions which gain energy from the electric field and transfer it to the electrons.

( ) the electrons which gain energy from the magnetic field and transfer it to the gas.

(x) the electrons which gain energy from the electric field and transfer it to the gas.

( ) the ions which gain energy from the magnetic field and transfer it to the gas.

SOLUTION: See Mooc Module 5a @ 6m35s. Electrons accelerate faster than ions in electric fields because of their small mass. The plasma current is carried mostly by electrons, especially in time-varying electric fields. Electron collisions then transfer energy to the gas. Magnetic fields do not transfer energy to charged particles, because the magnetic force is perpendicular to the particle velocity.

1.2 Neutral species can be transported to the walls and electrodes by:

( ) drift in the electric field of the plasma (Bohm's criterion).

( ) collisions with electrons.

( ) orbiting the magnetic field.

(x) diffusion in a density/concentration gradient of that neutral species (Fick's law).

SOLUTION: See Module 5a @ 4m53s. Neutrals diffuse in density gradients, but are not influenced by electric or magnetic fields.

1.3 Townsend's first coefficient is called alpha, and his second coefficient is called gamma, because:

( ) Townsend did not know the Greek alphabet.

(x) Townsend originally thought that ion collisions also ionised the gas (his beta coefficient).

( ) Townsend intended to include an effect due to beta radiation.

( ) the coefficients were not named by Townsend, but by somebody else with the same name.

SOLUTION: In his original theory, Townsend included the possibility of ion collisional ionisation, with coefficient beta, as well as electron collisional excitation with coefficient alpha.

1.4 A condition for breakdown can be described as follows:

( ) All electrons must reach the anode.

(x) Every electron leaving the cathode must create another to replace it.

( ) All ions must reach the cathode.

( ) Every ion must cause a secondary emission event.

SOLUTION: See Module 5b @ 10m03s. When every electron leaving the cathode creates at least one other electron leaving the cathode by the combined effects of ionisation and secondary emission, then the discharge is said to be self-sustaining, and breakdown has occurred. Except for losses to the walls, all electrons reach the anode, and all ions reach the cathode for all conditions, with or without breakdown. Even at breakdown, it is not necessary that every ion causes a secondary emission event.

1.5 If the external source of ionisation is removed when the voltage is less than the breakdown voltage,

(x) the current will fall to zero.

( ) the electron ionisation rate will increase to compensate.

( ) the ions will ionise the gas as well as the electrons.

( ) the current will remain steady but not increase.

SOLUTION: See Module 5b @ 9m10s. Below the breakdown voltage, the rate of electrons created at the cathode is smaller than the loss rate of electrons to the anode. Therefore, if the external source of ionisation is removed, the current will fall to zero.

1.6 For vacuum breakdown, the optical emission spectrum is different from the gas breakdown because:

( ) the gas atoms are strongly perturbed by the electric field (Stark shift).

(x) the atoms being excited are not the same as the gas atoms.

( ) the gas ions radiate at different wavelengths from the gas atoms.

( ) there is no gas to absorb the emitted light (optically thin).

SOLUTION: See Module 5c @ 8m25s to 9m25s. The vacuum discharge occurs essentially in the metal vapour of the electrodes, whose atoms are different from the gaseous atoms for gas breakdown (Paschen breakdown) at higher pressure.

1.7 For breakdown experiments using the same gas but different electrode materials,

( ) the measured Paschen curve is always the same.

(x) Townsend's first coefficient alpha is always the same.

( ) Townsend's second coefficient gamma is always the same.

( ) the theoretical Paschen curve is always the same.

SOLUTION: See Module 5c @ 6m02s. Townsend's first coefficient alpha depends only on gas properties, whereas the second coefficient depends on the electrode material and on the type and energy of gas ion striking it. Therefore, only alpha remains the same if the electrode material is changed.

Exercise 1 - Recombination, momentum and energy transfer in binary collisions

a) Since we require the maximum energy transfer, we analyse the situation with a head-on elastic collision.

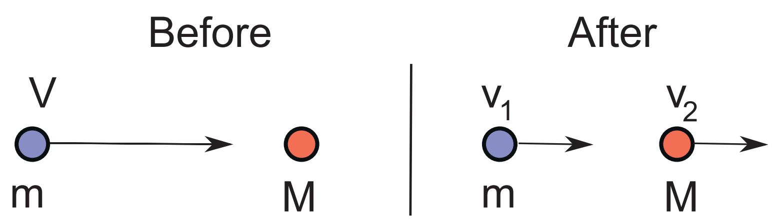

Figure 1: Sketch of an elastic head-on collision.

Let's consider the conservation of momentum and energy, since we deal with an elastic collision:

$$\begin{cases} \text{Conservation of momentum} & mV = mv_1 + Mv_2 \\ \text{Conservation of energy} & \frac{1}{2}mV^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 \end{cases} \tag{1}$$

From the first equation we get $v_1 = (mV - Mv_2)/m$, which inserted in the second one leads to:

$$v_2 = \frac{2mV}{M+m} \tag{2}$$

The maximum fraction of energy transferred to M is then:

$$\delta_{\text{max}} = \frac{\text{Energy transferred to M}}{\text{Initial energy of m}} = \frac{\frac{1}{2}Mv_2^2}{\frac{1}{2}mV^2} = \frac{4mM}{(M+m)^2} \tag{3}$$

We can now analyse the two possible situations:

- For an electron striking an atom head-on:

$$\delta_{\text{max}} \simeq \frac{4m}{M} << 1 \qquad \text{(1/18400 for Argon)} \tag{4}$$

  This implies that electrons are inefficient in heating the gas ($T_e >> T_i, T_{\text{gas}}$).

- For an ion striking an atom head-on ($m = M$):

$$\delta_{\text{max}} = 1 \tag{5}$$

  So ions and atoms thermalise efficiently ($T_i \approx T_{\text{gas}}$).

b) We should now consider a general impact angle θ.

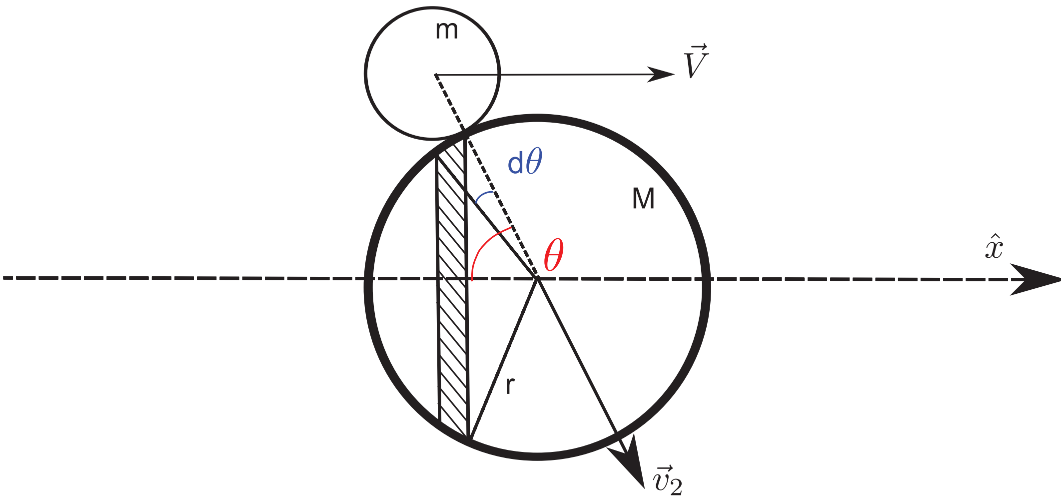

Figure 2: Sketch of the collision with a general angle θ.

To compute the energy transfer fraction corresponding to the angle θ, we can proceed in the same way of point a), keeping in mind that now in the equation of the momentum conservation along the line of centres, i.e. along the direction of v_2, we have the projection of the initial velocity of the incoming particle, $V\cos\theta$. It then follows:

$$\delta(\theta) = \frac{4mM}{(m+M)^2}\cos^2\theta \tag{6}$$

To compute the average of this quantity, we should first calculate the probability $p(\theta)$ of a collision with a certain angle θ (fraction of collisions corresponding to an angle θ). This can be obtained as:

$$p(\theta) = \frac{dA}{A} = \frac{2\pi r\sin\theta\cos\theta(rd\theta)}{\pi r^2} = 2\sin\theta\cos\theta d\theta \tag{7}$$

where $dA = 2\pi r\sin\theta\cos\theta(rd\theta)$ is the impact area between the angle θ and $\theta + d\theta$ projected along V, while $A = \pi r^2$ is the total available impact area. Then the average energy transfer fraction is:

$$\bar{\delta} = \int_0^{\pi/2}\delta(\theta)p(\theta) = \frac{4mM}{(m+M)^2}\int_0^{\pi/2} 2\sin\theta\cos^3\theta = \delta_{\max}2\left(-\frac{\cos^4\theta}{4}\right)\bigg|_0^{\pi/2} = \frac{\delta_{\max}}{2} \tag{8}$$

We see that for an electron striking an argon atom ($m \ll M$):

$$\bar{\delta} \simeq \frac{2m}{M} = \frac{1}{9200} \ll 1 \tag{9}$$

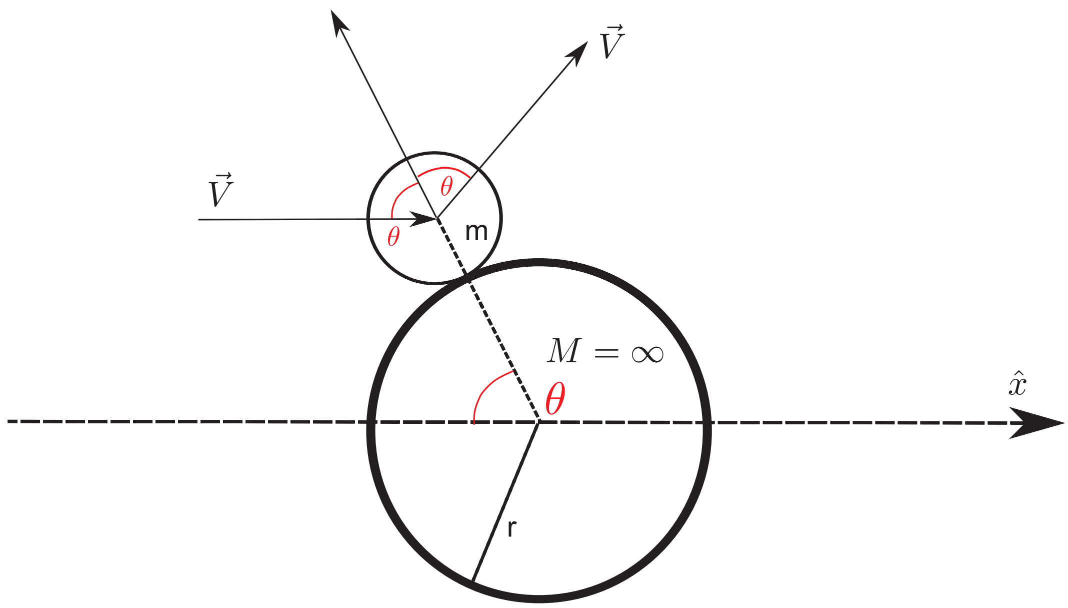

Figure 3: Sketch of a collision with an infinite mass target particle.

The average momentum in the original direction for the situation of a target particle of infinite mass can be obtained in a similar way, first calculating the projected momentum along the impact direction for a general angle θ, and then averaging.
It can be easily seen as the projected momentum along the impact direction ($\hat{x}$) is:

$$q_x(\theta) = -mV\cos(2\theta) \tag{10}$$

The average is then:

$$\bar{q} = \int_0^{\pi/2} q_x(\theta)p(\theta) = -mV \int_0^{\pi/2} \cos(2\theta)\sin(2\theta)d\theta = -mV \left[\frac{\sin^2(2\theta)}{4}\right]\Bigg|_0^{\pi/2} = 0 \quad (11)$$

This means that on average, all the forward momentum is lost in a collision with an infinite heavy scattering centre.

c) We consider a head-on inelastic collision as before, in order to maximise the energy transfer. The sketch is the same as the one seen at point a), except that now the kinetic energy is not conserved. This can be seen as a resulting excited atom with internal energy ΔU after the collision. If we write the conservation equations:

$$\begin{cases} \text{Conservation of momentum} & mV = mv_1 + Mv_2 \\ \text{Conservation of energy} & \frac{1}{2}mV^2 = \frac{1}{2}mv_1^2 + (\frac{1}{2}Mv_2^2 + \Delta U) \end{cases} \quad (12)$$

We can already see that we have 2 conditions but 3 unknowns. To get the third equation, we eliminate v_1 obtaining:

$$0 = -2mMVv_2 + M^2v_2^2 + mMv_2^2 + 2m\Delta U \quad (13)$$

and then maximize ΔU respect v_2 ($\partial\Delta U/\partial v_2 = 0$), which gives:

$$v_2 = \frac{mV}{m+M} \quad \Rightarrow \quad \Delta U_{\text{max}} = \frac{1}{2}mV^2\left(\frac{M}{M+m}\right) \quad (14)$$

To show that an electron can transfer almost all of its energy in an inelastic collision, we estimate the energy transfer fraction:

$$\delta_{\text{max}}^{\Delta U} = \frac{\Delta U_{\text{max}}}{\frac{1}{2}mV^2} = \frac{M}{m+M} \underbrace{\approx}_{e^- \to Ar} 1 \quad (15)$$

The kinetic energy transfer fraction is:

$$\delta_{\text{max}} = \frac{\frac{1}{2}Mv_2^2}{\frac{1}{2}mV^2} = \frac{Mm}{(m+M)^2} \underbrace{\approx}_{e^- \to Ar} \frac{m}{M} \ll 1 \quad (16)$$

This means that high energy electrons can strongly modify the chemical bonds and atomic/molecular internal energy without heating the atom/molecule.

d) Let's consider the situation where two bodies collide and associate into one body:

The equations of momentum and energy are:

$$\begin{cases} \text{Conservation of momentum} & mV = (m+M)v \\ \text{Conservation of energy} & \frac{1}{2}mV^2 = \frac{1}{2}(m+M)v^2 + \Delta U_{\text{ass}} \end{cases} \quad (17)$$

corresponding to 2 equations with 2 unknowns.
From the momentum equation we get $v = mV/(m+M)$, which can be used in the energy equation, leading to:

$$\Delta U_{\text{ass}} = \frac{1}{2}mV^2\left(\frac{M}{M+m}\right) > 0 \quad (18)$$

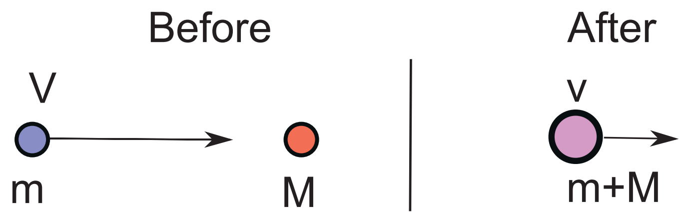

Figure 4: Sketch of the association event between two bodies.

This implies that elastic ($\Delta U_{\text{ass}} = 0$) and exothermic ($\Delta U_{\text{ass}} < 0$) association is impossible. As a consequence, electron-ion recombination is impossible by simple association, resulting in a long lifetime in low pressure plasmas. Association of H atoms is impossible as well ($H + H \rightarrow H_2$), implying a long life time and mean free path in such a way that they can reach the walls (for $p < 1$ mbar, H atom recombination mean free path is of the order of km).

We now briefly summarise the important points of this exercise:

- The average fraction of energy transfer for elastic collisions of electrons (mass m) with atoms (mass M) is $\delta = 2m/M \ll 1$. Electrons do not heat the gas efficiently. On the contrary, ions thermalise efficiently with atoms (collisions with neutrals), and the atoms thermalise efficiently with the walls. Conclusion: the electrons can reach high temperatures, whereas all other particles remain near ambient temperatures.
- An electron can transfer almost all of its energy in an inelastic collision, in contrast to elastic collisions. High temperature plasma chemistry is possible (2 eV $\simeq$ 23000 K) with low temperature substrates (glass, plastic).
- It is more difficult to break down molecular gases because electrons lose their energy in low-energy inelastic rovibrational collisions before reaching ionisation energy.
- Electrons and positive ions don't easily recombine to re-form neutral atoms because it is impossible to conserve energy and momentum without a three-body collision (or photon emission). The excited complex therefore re-dissociates after a short time, whereas 3-body collisions are rare at low pressures. On the other hand, charge recombination (neutralisation) is very efficient at surfaces (which absorb momentum and energy, like a huge 'third body'). A surface is therefore a sink ('black hole') for ions and electrons. Also, H atoms recombine at surfaces (useful chemistry), but have long mean free paths (km) in low pressure plasma sources. This is very convenient for sources of atomic hydrogen because the atomic flux reaches the surface without recombining to H_2 in the volume.

Exercise 2 - Comparison of reaction rates in gas and in plasma

a) Calculate the electron temperature in K of an electron temperature of 1 eV:

   $e \cdot 1$ eV $= kT$ [K], where e is the electron charge, and k is Boltzmann's constant. Therefore T [K] $= e/k = 11'600$ K for 1 eV.

b) A plasma medicine technique requires a flux of oxygen molecular ions (ionisation energy 12 eV) in air. Using the Arrenhius rate (ignore the temperature dependence of the pre-exponential factor), compare the rate of ionisation by gas heating at 1000 K with the ionisation rate by electrons at temperature 2 eV. If the degree of ionisation due to the plasma in the air is as low as 10^{-8}, which is the most efficient method of oxygen ion production - the hot air, or the weak plasma?

   The ionisation rate by plasma is proportional to $\exp(-12/2) = \exp(-6)$. For hot gas, the rate is proportional to $\exp(-12 \cdot 11600/1000) = \exp(-139)$. Since the electron density is 10^8 times lower than the gas density, the ratio of plasma rate to gas rate is $10^{-8} \cdot \exp(-6)/\exp(-139) \sim 10^{50}$. The plasma is, therefore, much more efficient than gas heating for creating ionised species, even for a very weakly ionised plasma.

Exercise 3 - Derive Paschen's law from Townsend's breakdown criterion

See slide 41 of the lecture notes.

Exercise 4 - Paschen's law applied to modern satellite solar panels

We have seen that the breakdown voltage is given by,

$$V_B = \frac{B(pd)}{\ln[C(pd)]} \qquad \text{with} \qquad \mathrm{C} = \frac{\mathrm{A}}{\ln\left(1+\frac{1}{\gamma}\right)} \quad , \tag{19}$$

where A and B are the constants of the Townsend's first ionisation coefficient α. From this expression it's clear as the required voltage to have a breakdown goes to infinity for $C(pd)_\infty = 1$ (because $\ln 1 = 0$), corresponding to,

$$(pd)_\infty = \frac{1}{C} = \frac{1}{A} \ln\left(1 + \frac{1}{\gamma}\right) . \tag{20}$$

By differentiating V_B, Eq. (19), with respect to (pd), we can also see that the minimum of the breakdown voltage is $V_B|_{\min} = B(pd)_{\min}$, corresponding to $\ln[C(pd)|_{\min}] = 1$, implying,

$$C(pd)|_{\min} = e \quad \Rightarrow \quad (pd)|_{\min} = \frac{e}{C} = e(pd)|_{\infty} \ . \tag{21}$$

We can now obtain the minimum breakdown voltage in air, first computing,

$$(pd)_{\infty} = \frac{1}{A} \ln\left(1 + \frac{1}{\gamma}\right) = \frac{1}{15 \text{ cm Torr}} \ln(1 + 100) \simeq 0.308 \text{ cm Torr} \tag{22}$$

then using expression (21):

$$(pd)|_{\min} = e(pd)_{\infty} \simeq 0.837 \text{ cm Torr} \ . \tag{23}$$

We finally obtain,

$$V_B|_{\min} = B(pd)_{\min} = 365 \text{ V cm}^{-1} \text{ Torr}^{-1} \times 0.837 \text{ cm Torr} \simeq 300 \text{ V} \ . \tag{24}$$

The corresponding pressure for parallel iron electrodes 1 cm apart is,

$$p = \frac{(pd)|_{\min}}{d} = \frac{0.837 \text{ cm Torr}}{1\text{cm}} \simeq 0.84 \text{ Torr} \ . \tag{25}$$

If we consider the minimum voltage value that we have found, we conclude that a slip-ring, with solar panels generating a potential of 400V, could break down. However, we should consider that the parameters of the slip-ring could vary from the ones considered here (d and p), as well as the geometry of the plates which is not parallel.