Plasma II - Exercises

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Interferometry on TCV

a) The upper electron density limit is imposed by the electron plasma frequency, which has to be lower than the laser frequency for the e.-m. wave to propagate through the plasma,

$$\omega_{\text{laser}} > \omega_{\text{p,e}} = \sqrt{\frac{n_{\text{e}} e^2}{\epsilon_0 m_{\text{e}}}} \quad .$$

With the laser frequency $\omega_{\text{laser}} = 2\pi c/\lambda_{\text{laser}}$ given, this translates into the density limit,

$$n_{\text{e}} < n_{\text{e,c}} = \left(\frac{2\pi c}{\lambda_{\text{laser}}}\right)^2 \frac{\epsilon_0 m_{\text{e}}}{e^2} \quad . \tag{1}$$

Using $\lambda_{\text{laser}} = 184.3\,\mu\text{m}$, Eq. 1 yields,

$$n_{\text{e}} < n_{\text{e,c}} = 3.29 \times 10^{22} \text{m}^{-3} \quad ,$$

which is well above the densities that can be obtained in TCV.

b) A *fringe* is caused by a phase shift of the *sample beam* by $\Delta\phi = 2\pi$,

$$\Delta\phi = \frac{\omega}{2cn_{\text{e,c}}} \Delta \int n_{\text{e}} dl \quad .$$

It, therefore, corresponds to a change of the line-integrated density,

$$\Delta \int n_{\text{e}} dl = \frac{2cn_{\text{e,c}}\Delta\phi}{\omega}$$

Using $\omega_{\text{laser}} = 2\pi c/\lambda_{\text{laser}}$ and the critical density from a) yields,

$$\Delta \int n_{\text{e}} dl = 1.2 \times 10^{19}\,\text{m}^{-2} \quad .$$

With typical path lengths of the order of one metre and densities of several $10^{19}\,\text{m}^{-3}$ it is desirable to resolve fractions of a fringe.

c) The electric fields of reference and sampling beams at *detector 1* are $\mathbf{E_{RB}} = \mathbf{\hat{E}_{RB}} e^{i\omega t}$ and $\mathbf{E_{SB}} = \mathbf{\hat{E}_{SB}} e^{i(\omega t - \Delta\phi)}$, respectively. The detector measures the average intensity of the e.-m. wave,

$$\begin{aligned} I &= \left|\mathbf{E_{RB}} + \mathbf{E_{SB}}\right|^2 \\ &= \left(\mathbf{\hat{E}_{RB}} e^{i\omega t} + \mathbf{\hat{E}_{SB}} e^{i(\omega t - \Delta\phi)}\right)\left(\mathbf{\hat{E}_{RB}} e^{-i\omega t} + \mathbf{\hat{E}_{SB}} e^{-i(\omega t - \Delta\phi)}\right) \\ &= \mathbf{\hat{E}^2_{RB}} + \mathbf{\hat{E}^2_{SB}} + 2\mathbf{\hat{E}_{RB}}\mathbf{\hat{E}_{SB}} \cos(\Delta\phi) \quad . \end{aligned} \tag{2}$$

Note that Eq. 2 can be rearranged as,

$$\begin{aligned} I &= \left(\mathbf{\hat{E}_{RB}} - \mathbf{\hat{E}_{SB}}\right)^2 + 2\mathbf{\hat{E}_{RB}}\mathbf{\hat{E}_{SB}} \left(1 + \cos(\Delta\phi)\right) \\ &= \left(\mathbf{\hat{E}_{RB}} - \mathbf{\hat{E}_{SB}}\right)^2 + 4\mathbf{\hat{E}_{RB}}\mathbf{\hat{E}_{SB}} \cos^2\left(\frac{\Delta\phi}{2}\right) \quad . \end{aligned}$$

If reference and sampling beams have the same amplitude, the detected intensity varies between twice the intensity of each beam and zero.

d) The detected intensity has extrema for densities that cause a phase shift of multiples of π. A further change in intensity can then be caused by an increase or a decrease of the density alike. *Detector 2* does not resolve this ambiguity. Here, the phase of the beams are offset by π, which simply inverts the interference pattern. Whenever *detector 1* records a maximum, *detector 2* would record a minimum. In order to resolve the ambiguity a second interferometer with a phase difference of $\pi/2$ is needed.

e) In addition to the reference and sampling beams the heterodyne detection scheme also uses a local oscillator, $\mathbf{E_{LO}} = \mathbf{\hat{E}_{LO}} e^{i(\omega t + \Delta\omega t + \phi_{\mathrm{LO}})}$. The intensities of its superposition with SB and RB are,

$$\begin{aligned} I_{\mathrm{plasma}} &= \left|\mathbf{E_{SB}} + \mathbf{E_{LO}}\right|^2 \\ &= \mathbf{\hat{E}^2_{SB}} + \mathbf{\hat{E}^2_{LO}} + 2\mathbf{\hat{E}_{SB}}\mathbf{\hat{E}_{LO}} \cos(\Delta\omega - \Delta\phi + \phi_{\mathrm{LO}}) \end{aligned}$$

and

$$\begin{aligned} I_{\mathrm{reference}} &= \left|\mathbf{E_{RB}} + \mathbf{E_{LO}}\right|^2 \\ &= \mathbf{\hat{E}^2_{RB}} + \mathbf{\hat{E}^2_{LO}} + 2\mathbf{\hat{E}_{RB}}\mathbf{\hat{E}_{LO}} \cos(\Delta\omega + \phi_{\mathrm{LO}}) \quad . \end{aligned}$$

A filter removes the DC part before acquisition. Since plasma and reference signals oscillate with the beat frequency of 100kHz a sufficiently fast acquisition can resolve the oscillations. A comparison of their phases unambiguously reveals the phase shift $\Delta\phi$ at all times.