Plasma II - Exercises

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Multi-choice questions

What process sets the lower limit of the mass of main sequence stars to approximately a tenth of the mass of our sun?

( ) There is no lower limit, but since the luminosity depends strongly on the mass (power of 3-4), light stars radiate too little power to be detected.

( ) Since main sequence stars are gravitationally confined clouds of hydrogen, their temperature increases with their mass and lighter clouds of hydrogen are not sufficiently hot to initiate fusion reactions.

( ) There is a lower mass limit for gas to be gravitationally confined.

( ) Light stars have already burnt all their fuel and extinguished a long time ago.

What happens when energy is added to a gravitational confined system such as a star, i.e. through increased nuclear heating?

( ) The star heats up, potentially increasing nuclear reaction rates.

( ) Nothing, since the temperature depends primarily on the mass of the system.

( ) The star cools down, as an increase of the system's energy increases the radius and hence cools down the star.

Exercise 1 - On the role of gravitational energy for the lifetime of our Sun

Test the hypothesis that the radiated energy of the Sun primarily originates from its gravitational energy, rather than nuclear reactions. Prove that the hypothesis is wrong by calculating the lifetime of the Sun, neglecting nuclear energy, following these steps:

a) From the balance of pressure and gravitational forces (hydrostatic equilibrium), demonstrate the general form of the *virial theorem* which gives the relation between the product of the average pressure in the sphere of radius R by the volume of the sphere ($\langle P \rangle$ V), and the gravitational energy E_g.

   Hint: Assume that the pressure drops to zero at the edge of the Sun.

b) Considering the plasma in the Sun as an ideal gas, calculate the lifetime of the Sun remembering that the luminosity is given by $L = -dE/dt$ and its measured value is $L = 3.84 \times 10^{26}$ W.

   Hint: gravitational constant $G = 6.67 \times 10^{-11}$N m^2 kg^{-2}, solar mass $M_R \simeq 2 \times 10^{30}$kg, solar radius $R \simeq 7 \times 10^8$m.

Estimate now the energy contribution coming from nuclear reactions, assuming that 10% of the total number of protons of the Sun are available for the reaction $p+p+p+p \rightarrow He$.

Exercise 2 - Convective speed at the Sun photosphere

Estimate the convective speed at the Sun photosphere using the mixing-length approach, where the plasma is viewed as a fluid of blobs that move up or down, travelling a distance of the order of the pressure scale length H before they dissolve releasing their energy to the environment. Assuming an ideal gas, follow the steps listed below:

a) Calculate the pressure scale length H assuming a pressure equilibrium between the blob and the surrounding environment, which can be considered in hydrostatic equilibrium.

b) Determine the blob convective velocity v_{conv} as a function of the temperature difference (ΔT_{conv}) between the blob and its surroundings when it dissolves.

c) Calculate an upper limit of ΔT_{conv} as a function of the luminosity L and the density ρ, expressing the energy per unit volume transferred by the blob in terms of the enthalpy change $n\frac{\gamma}{\gamma-1}k_{\text{B}}\Delta T_{\text{conv}}$.

d) Estimate H, ΔT_{conv} and v_{conv} in the convection region ($r \simeq 0.7R$) for an ideal monotonic gas of density $\rho = 200$ kg m^{-3} and temperature $T \simeq 10^6$K.