Plasma II - Exercises

Dr. H. Reimerdes, Dr. E. Tonello - SPC/EPFL

Problem Set 6 - March 28, 2025

Exercise 1 - Need for additional heating power

Consider an ITER plasma with minor radius $a = 1.5\,\mathrm{m}$, major radius $R_0 = 6\,\mathrm{m}$, magnetic field $B_0 = 6\,\mathrm{T}$ and plasma current $I_p = 10\,\mathrm{MA}$ and assume flat density and temperature profiles with $n = 0.7 \cdot 10^{20}\,\mathrm{m}^{-3}$ and $T_e = T_i = 15\,\mathrm{keV}$, respectively.

a) What is the resistance of the plasma? Use the Spitzer resistivity $\eta_{||} = 1.65 \cdot 10^{-9}$ $\ln\Lambda/(T_e|_{\mathrm{keV}})^{3/2}[\Omega\ \mathrm{m}]$, where $T_e|_{\mathrm{keV}}$ is the electron temperature in keV, and assume a Coulomb logarithm $\ln\Lambda = \ln(\Lambda_D/r_0) = 17$.

b) What is the ohmic power dissipated in this plasma?

c) What is the power lost by the plasma? Assume $\tau_E = 2.5\,\mathrm{s}$.

d) What is the fusion power? Assume $\langle\sigma v\rangle|_{T=15\mathrm{keV}} = 2 \cdot 10^{-22}\ \mathrm{m}^3\ \mathrm{s}^{-1}$.

e) Are the ohmic and fusion power enough to keep the plasma temperature at 15 keV? If not, how much additional heating power is still needed to equilibrate the power losses in order to keep a steady-state operation?

f) Calculate the physical fusion gain factor $\mathrm{Q_{fus}}$.

g) Verify the assumption for τ_E in (c) by comparing its value with the empirical scaling[1]

$$\tau_E = 0.1\ (I_p|_{\mathrm{MA}})^{0.93}\ (B_0|_{\mathrm{T}})^{0.15}\ (n|_{10^{20}\mathrm{m}^{-3}})^{0.41}\ \ (P_{\mathrm{in}}|_{\mathrm{MW}})^{-0.69}\ (R_0|_{\mathrm{m}})^{1.97}[\mathrm{s}]\ , \qquad (1)$$

where $I_p|_{\mathrm{MA}}$ is the value of the plasma current in MA, $n|_{10^{20}\mathrm{m}^{-3}}$ the plasma density in $10^{20}\mathrm{m}^{-3}$, $P_{\mathrm{in}}|_{\mathrm{MW}}$ the total heating power in MW, etc.

[1] The scaling is a simplified version of the IPB98(y,2) scaling and is commonly used to predict the energy confinement in the high confinement mode, which is the operating regime foreseen for ITER.

Exercise 2 - Control of plasma burn

In principle, burning plasmas can undergo a thermal runaway, which involves an uncontrolled transition to ignited conditions, with an ever increasing temperature. Analyse this possibility in ITER using a simple 0-D model, based on a power balance. Neglect Bremsstrahlung and consider a situation in which the density is fixed.

a) Does a burning ITER plasma risk undergoing a thermal instability?

b) What measure can you think of to control the burn and prevent instability?

Hints:

1. Consider for $\langle \sigma v \rangle_{DT}$ the approximation $\langle \sigma v \rangle_{DT} \approx 1.1 \times 10^{-24} T^2|_{\text{keV}} \quad (\text{m}^3/\text{s})$
2. Consider the following ELMy H-mode scaling law

$$\tau_E \propto \tau_B \rho_*^{-0.7} \beta^{-0.9} \nu_*^{-0.01} \tag{2}$$

with the *Bohm-time* $\tau_B \approx \frac{a^2 B_T}{T_e}$, the normalised gyro-radius $\rho_* \approx \frac{\sqrt{T_e}}{a B_T}$, the normalised collisionality $\nu_* \approx \frac{n_e a}{T_e^2}$ and the normalised plasma pressure $\beta \approx \frac{n_e T_e}{B_T^2/(2\mu_0)}$.