Plasma II - Exercises

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1 Ambipolar diffusion

Consider a weakly ionised hydrogen plasma with $n_e = 10^{17}\,\mathrm{m}^{-3}$, $T_e = 10\,\mathrm{eV}$, $T_i = 1\,\mathrm{eV}$ and ionisation fraction 5%. Calculate the ambipolar diffusion coefficient,

$$D_{\mathrm{amb}} = \frac{\mu_i D_e - \mu_e D_i}{\mu_i - \mu_e} \quad ,$$

where $D_s = k_{\mathrm{B}} T_s/(m_s \nu_{\mathrm{coll}}^{s/n})$ is the collisional diffusion coefficient, $\mu_s = q_s D_s/(k_{\mathrm{B}}\, T_s)$ is the mobility, and $\nu_{\mathrm{coll}}^{s/n}$ is the collision frequency of species "s" (electrons and ions) .
In this case, particles are scattered by collisions with the neutrals, which occur approximately at a rate,

$$\nu_{\mathrm{coll}}^{s/n} \simeq n_0\, \sigma_0 \sqrt{\frac{k_{\mathrm{B}}\, T_s}{m_s}} \quad .$$

Here n_0 is the neutral density, $\sigma_0 \approx 5 \times 10^{-19}\,\mathrm{m}^2$ is a typical cross section and k_{B} is Boltzmann's constant.

2 Diffusion processes in a tokamak

Consider a deuterium plasma in JET with a toroidal magnetic field $B_{\phi,0} = 3\,\mathrm{T}$, plasma current $I_{\mathrm{P}} = 3.5\,\mathrm{MA}$, electron density $n_e = 10^{20}\,\mathrm{m}^{-3}$ and temperature $T_e = T_i = 10\,\mathrm{keV}$. Assume $\ln\Lambda = 17$ and remember that JET has a minor radius $a = 1\,\mathrm{m}$ and a major radius $R_0 = 3\,\mathrm{m}$.

a) Calculate the electron $D_\perp$ for classical and neoclassical banana, plateau and Pfirsch-Schlüter diffusion at the plasma border $r = a$.
Remember that,

- The electron classical perpendicular diffusion coefficient is,

$$D_{e,\perp}^{\mathrm{class}} = \rho_{Le}^2 \nu_p^{e/i} \quad \text{with} \quad \nu_p^{e/i} = \frac{1}{3}\sqrt{\frac{2}{\pi}} n_i \frac{Z^2 e^4 \ln\Lambda}{4\pi\epsilon_0^2 m_e^{1/2}} \frac{1}{(k_{\mathrm{B}} T_e)^{3/2}} \quad .$$

- The electron neoclassical banana diffusion coefficient is,

$$D_{e,\perp}^{\text{Ban}} = \frac{\sqrt{2}q^2}{\varepsilon^{3/2}} D_{e,\perp}^{\text{class}} \quad ,$$

  where q is the safety factor and ε is the inverse of the aspect ratio. Use the approximation $q(a) \approx a\, B_\phi(a)/(R_0\, B_\theta(a))$.

- The electron neoclassical plateau and Pfirsch-Schlüter diffusion coefficients are, respectively,

$$D_{e,\perp}^{\text{Pl}} = \frac{q v_{\text{th}}}{R} \rho_{Le}^2 \quad \text{and} \quad D_{e,\perp}^{\text{PS}} = q^2 D_{e,\perp}^{\text{class}} \quad .$$

Comment on the ordering of the values you have found and on ambipolarity.

b) Is a flat radial density profile in a plasma discharge with a central particle source (e.g. provided by neutral beam heating discussed in L6) compatible with a classical or neoclassical diffusive cross-field transport? Prove your answer.

c) Calculate the particle loss dN/dt from the central region of the plasma, i.e. the number of ion/electron pairs leaving the region $|r| \leq a/2$ per second, and estimate the approximate particle confinement time assuming a parabolic density profile.

d) Calculate parallel and perpendicular heat conductivity for ions and electrons,

$$\kappa_{s,\parallel} = \begin{cases} \text{electrons} & 3.2\ n_e T_e/(m_e \nu_E^{e/e}) \\ \text{ions} & 3.9\ n_i T_i/(m_i \nu_E^{i/i}) \end{cases} \qquad \kappa_{s,\perp} = \begin{cases} \text{electrons} & 4.7\ n_e T_e \nu_E^{e/e}/(m_e \Omega_e^2) \\ \text{ions} & 2\ n_i T_i \nu_E^{i/i}/(m_i \Omega_i^2) \end{cases}$$

e) Is a parabolic radial temperature profile compatible with a classical/neoclassical diffusion process? Prove your answer.

f) Calculate the loss of thermal energy , i.e. dE/dt, from the central region of the plasma, i.e. the heat flux leaving the region $|r| \leq a/2$ per second, and estimate the approximate energy confinement time assuming a parabolic temperature profile.