Plasma II - Exercises

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Exercise 1 - Charged particles confinement in the Earth magnetosphere and in a torus.

The combination of the Earth's magnetic field and the solar wind creates an overall magnetic field topology in the Earth's magnetosphere that is very similar to that in a so-called mirror device.

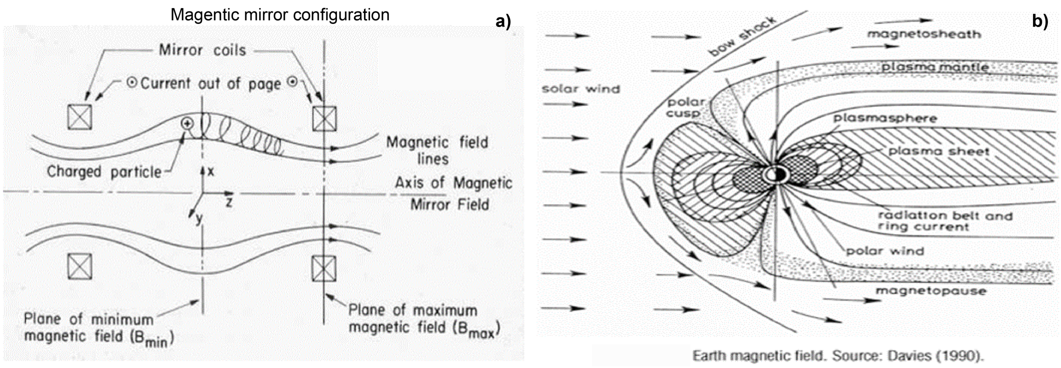

Figure 1: a) Magnetic mirror and b) earth magnetic field.

a) Draw the magnetic field intensity in a mirror device: is this magnetic field uniform along the axis $\hat{e}_z$ the mirror? Can you explain why the magnetic field in a mirror device is similar to the magnetic field topology in the Earth's magnetosphere?

b) Find the equation of motion of charged particles in a mirror device, using conservation of energy and of the magnetic moment $\mu = mv_\perp^2/(2B)$, and derive a criterion for their trapping.

c) Discuss the time evolution of a thermalised plasma in a magnetic mirror. Which characteristic time scale determines the particle confinement time?

d) The discussion of particle drifts in the lecture revealed that we need a combination of a large toroidal field, B_ϕ, and a smaller poloidal field, B_θ, to confine a toroidal plasma. Consider now a field line as it winds around the torus in 3D: is the overall intensity of the magnetic field along this field line a constant for the motion of all charged particles? If not, what is the resulting particle motion (Hint: Use $B_\theta \ll B_\phi$)? Can you find the analogy with the mirror configuration? Consider a Maxwellian isotropic distribution function: what is the fraction of *trapped* particles, i.e. those for which the guiding center motion follows the banana orbit depicted below (ignore the finite *width* of the banana orbit)? Derive the bounce frequency corresponding to the banana orbits.

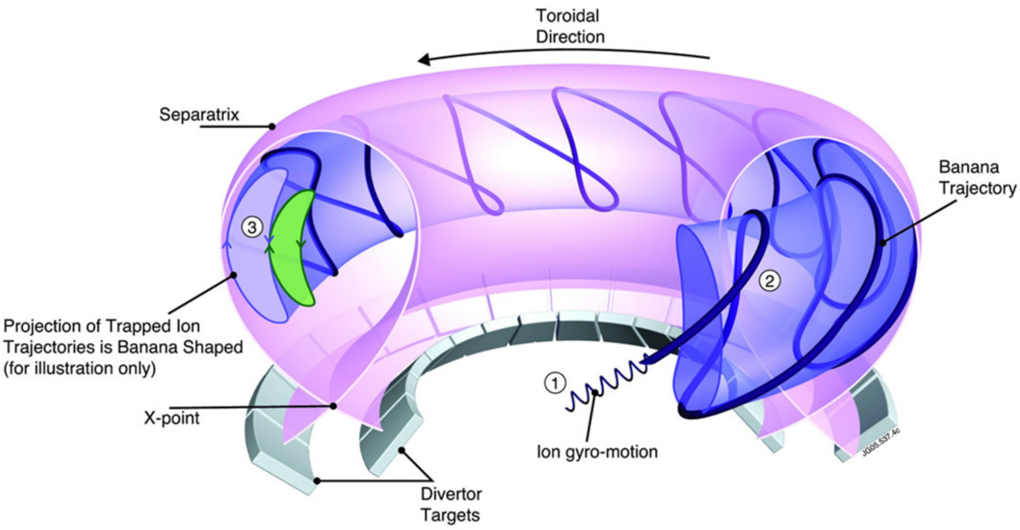

Figure 2: Schematic of banana orbits in a tokamak.

Exercise 2 - Tokamak confinement

Consider the JET tokamak, with major radius, $R_0 = 3\,\text{m}$, minor radius, $a = 1\,\text{m}$, a toroidal magnetic field, $B_{\phi 0} = 3\,\text{T}$, and a pure deuterium plasma. Assume that the profiles of safety factor, q, and elongation, κ, are monotonic with a quadratic dependence on the normalised radial coordinate $\rho = r/a$. The plasma elongation on-axis is $\kappa_0 = \kappa(\rho = 0) = 1.2$ and $\kappa_{\text{edge}} = \kappa(\rho = 1) = 1.7$, the safety factor on-axis is $q_0 = 0.95$ and $q_{\text{edge}} = 6$. Consider an electron density profile $n_e(\rho) = n_{e0}[0.9 * (1 - \rho^2) + 0.1]$ and a temperature profile $T(\rho) = T_0[0.9 * (1 - \rho^2) + 0.1]$, with $n_0 = 3 \times 10^{19}\,\text{m}^{-3}$ and $T_{i0} = T_{e0} = 15\,\text{keV}$ for both the ions and the electrons.

a) Calculate the kinetic (i.e. thermal) energy content of the plasma assuming a circular cross section first (i.e. assuming a constant $\kappa = 1$).

b) Now calculate the kinetic energy of the plasma with an elongated cross section. Note that the JET vessel is also elongated and can accommodate elongated plasmas with the same minor radius a. Think about how the volume element changes when going from a circular to a slightly elliptical shape (assuming that κ is close to unity and does not vary strongly across the profile). Does the integrand change?

c) Calculate the radial profile of the average poloidal magnetic field and the total plasma current I_{P}. Use $q = (rB_{\phi 0}/R_0 B_\theta)\sqrt{(1+\kappa^2)/2}$, which is the safety factor in a straight tokamak (also known as the large-aspect-ratio approximation) with elliptical cross section (using the approximation $L_{\mathrm{ellipse}} \approx 2\pi r\sqrt{(1+\kappa^2)/2}$ for the perimeter of the ellipse with elongation κ and semi-minor axis r). What is the effect of plasma shaping on q_{edge}?