Plasma II - Exercises

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Multiple-choice question

Reconnection converts

( ) kinetic energy into magnetic energy.

( ) magnetic energy into kinetic energy.

( ) magnetic energy into kinetic energy and heat.

Design of an experiment to study the kink instability

You work in a laboratory with little funding and want to build an experiment to study the kink instability of flux ropes. You have a cylindrical vacuum chamber (radius $r = 0.5\,\mathrm{m}$, length $L = 2\,\mathrm{m}$) with a $1\,\mathrm{cm}$ radius central copper conductor along its axis at your disposal. You can also recycle two flat circular electrodes of $a = 5\,\mathrm{cm}$ radius and a couple of power supplies that can drive $500\,V \times 5\,\mathrm{kA}$. Imagine how you could use this hardware to create a circular flux rope and then drive it kink-unstable.

a) Draw a sketch of the experimental set up.

b) A plasma group in your lab had previously used these two electrodes placed at a distance of $10\,\mathrm{cm}$ to measure the Paschen curve for H_2. Performing a scan in voltage for several values of gas pressure, finding the minimum breakdown voltage of $U_B = 273\,\mathrm{V}$ for a gas pressure of $0.15\,\mathrm{mbar}$.
What is the connection length (distance between the two electrodes along the magnetic field line connecting them) you should work with, knowing that you want to use the lowest voltage to obtain the breakdown with $n_m = 10^{-3}$ mol of H_2?[1] Assume an ideal gas at room temperature.

[1] Reminder: The molar gas constant is $R = 8.31\mathrm{J}/(\mathrm{mol}\cdot\mathrm{K})$.

c) Knowing that the threshold for the flux rope to become unstable is $q_a = 1$, where q_a is the safety factor at the edge of the plasma column, write the expression for the *Kruskal-Shafranov* critical current for a given (*external*) field along the plasma column B_θ.

d) What is the range of current of the central axial conductor you can work with, fulfilling the conditions of having an unstable kink mode and a magnetised plasma?
Remember that a plasma is magnetised when the temporal and spatial (perpendicular respect to the magnetic field) characteristic scales of the system are bigger, respectively, then the inverse of the electron cyclotron frequency ω_{ce} and the electron cyclotron radius ρ_{ce}. The spatial scale length of the system can be assumed to be $L_\perp = a$, while for the temporal one the time T a particle moving at the ion sound speed c_s takes to move from one electrode to the other can be considered.
Assume that the plasma current inside the kink corresponds to the ion saturation current collected by the anode, and that 50% of the gas is ionised with an average temperature of 10 eV. Moreover, for technical reasons, the collecting surfaces of the two electrodes have to lay on the same plane.

e) How much does the heating of the axial conductor constrain the duration of your experiment? Use a reasonable value of current.
Remember that the copper has a density of $d_{\text{copper}} = 8.96 \times 10^3\,\text{kg}\,\text{m}^{-3}$, specific heat $c_{\text{s-copper}} = 385\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ and assume a temperature independent resistivity of $\rho_{\text{copper}} = 1.5 \times 10^{-8}\,\Omega\,\text{m}$.

f) What is the required frame-rate of a fast camera you would use to study the development of the unstable kink mode?