Plasma II - Exercises

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Multiple-choice question

Why are sunspots cooler than their surroundings?

( ) The large magnetic field of sun spots suppresses convection and, thereby, the primary heat transport mechanisms from deeper regions towards the surface.

( ) The typical flaring of flux tubes caused by radial gradients in the magnetic field causes transported heat to spread over a greater surface area.

( ) The lower temperature is only an artefact of high magnetic field affecting our temperature measurements.

Why does a dynamic dynamo require a sufficiently high magnetic Reynolds number R_m?

( ) Because MHD theory is not valid at low magnetic Reynolds number.

( ) A low magnetic Reynolds number indicates high resistive dissipation, which primarily leads to a conversion of mechanical energy (flow) into heat rather than magnetic energy.

( ) Plasmas with low Reynolds number rotate in the wrong direction for magnetic field amplification to take place.

Exercise 1 - Magnetic buoyancy in the Sun

Calculate the rising speed of a magnetic flux tube (length L and minor radius a) emerging from the convection region of the Sun into the photosphere. Assume for simplicity an ideal gas, a drag coefficient of 1 and a flux tube with:

- a magnetic field intensity much greater than the one outside the tube;
- the same temperature of the surrounding environment;
- a direction perpendicular to the solar gravity;
- a constant rising velocity.

Reminder: The drag force is $F_D = \frac{1}{2}\rho v^2 C_D A$, where C_D is a dimensionless drag coefficient and A the cross sectional area.

Exercise 2 - Kink instability of a flux rope / screw pinch

Consider a screw pinch of length L and radius a as sketched in the following figure. Note that this picture could also be used to model a linear flux rope in the Sun.

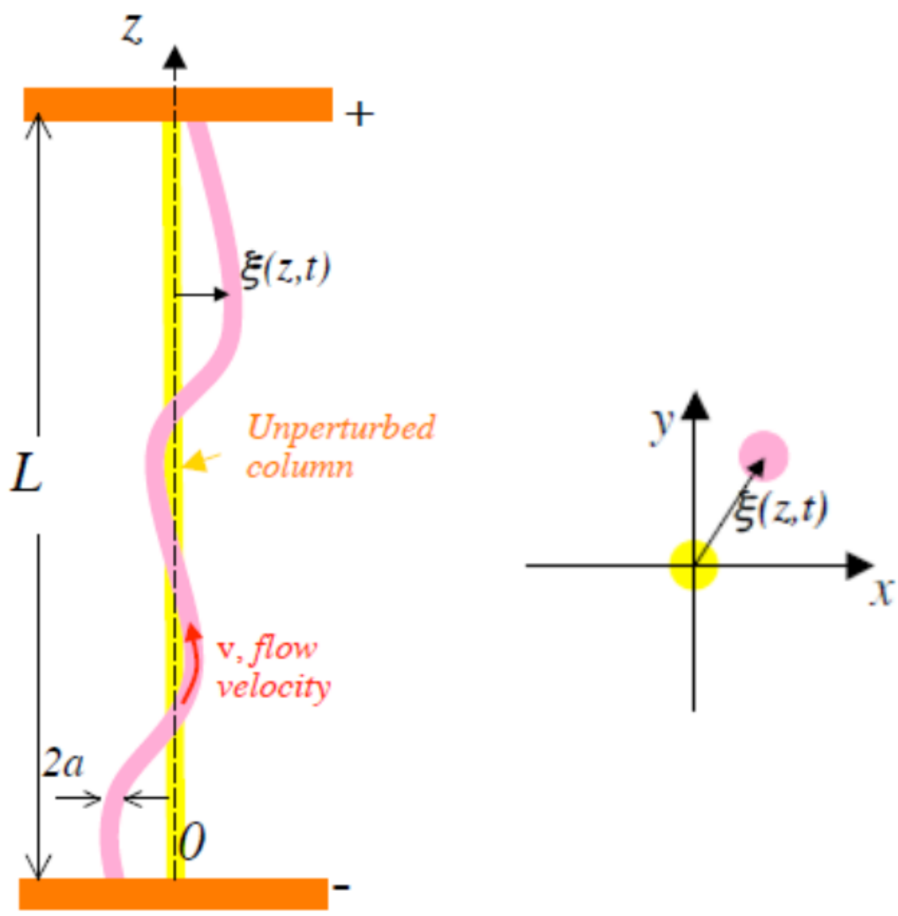

Figure 1: Schematic of a kink instability in a screw pinch.

Starting from the equation of motion obtained from the energy principle

$$\frac{\partial^2 \eta}{\partial t^2} = v_A^2 \left(\frac{\partial^2 \eta}{\partial z^2} + i k_0 \frac{\partial \eta}{\partial z} \right) \qquad \text{with} \qquad \eta = \xi_x + i \xi_y \quad k_0 = \frac{B_\theta}{a B_z} \tag{1}$$

where η is the plasma displacement in the x-y plane and B_θ computed at the edge of the plasma column, calculate the growth rate γ of the most unstable kink mode ($n = 1$) as a function of the plasma current I, the length of the plasma column L, the Alfven velocity v_A and the Kruskal-Shafranov critical current $I_{K-S} = 4\pi^2 a^2 B_z/(\mu_0 L)$. Consider as boundary conditions $\eta(z = 0) = \eta(z = L) = 0$, also known as line-tied BC (since the flux rope / screw pinch foot points are not allowed to move).