

**Heisenberg coupling in Mott and Charge transfer insulator***Author: Noore Elahi Shaik**Please send any suggestions/corrections to [noore.shaik@epfl.ch](mailto:noore.shaik@epfl.ch)***Abstract**

Apart from the on-site coulomb energy  $U$  another important scale that determines the nature of the insulating phase is the charge transfer energy  $\Delta_{CT}$ . In this exercise we look at how  $\Delta_{CT}$  effects the exchange paths and the final Heisenberg coupling in a charge transfer(CT) insulator.

**Exercise 1: Exchange interaction in CT insulator**

Consider a  $\text{CuO}_2$  plane of a HTS, ( $d^9$  configuration) with hopping energy between  $p$  and  $d$  orbitals given by  $t_{pd}$ . Consider a hole representation, in which transferring one hole from  $d$  orbitals to  $p$  orbital, ( $d^9 p^6 \rightarrow d^{10} p^5$ ), costs an energy of  $\Delta_{CT}$  (charge transfer energy).

a) Using the second order perturbation, (same as problem 2a in Exercise-2) show that the effective hopping energy between two adjacent  $d$  orbitals following the process  $d^9 p^6 d^9 \rightarrow d^{10} p^5 d^9 \rightarrow d^{10} p^6 d^8$  is given by:

$$t_{dd}^{eff} = \frac{t_{pd}^2}{\Delta_{CT}}$$

b) If the energy cost of putting two holes on a Cu site is  $U_{dd}$ , show that the effective Heisenberg exchange for process,

$$d_i^n p^6 d_j^n \rightarrow d_i^n p^5 d_j^{n+1} \rightarrow d_i^{n-1} p^6 d_j^{n+1} \rightarrow d_i^n p^5 d_j^{n+1} \rightarrow d_i^n p^6 d_j^n$$

is given by:

$$J_{dd}^{(1)} = \frac{2t_{pd}^4}{\Delta_{CT}^2 U_{dd}}$$

**Hint:** Use part 'a' to re-write the process as effective hopping between  $d$ - $d$  orbitals and use the Heisenberg coupling formula derived in Exercise-2.

c) If the energy cost of putting two holes on a oxygen atom is  $U_{pp}$ , show that the effective coupling due to the below process

$$d_i^n p^6 d_j^n \rightarrow d_i^n p^5 d_j^{n+1} \rightarrow d_i^{n+1} p^4 d_j^{n+1} \rightarrow d_i^n p^5 d_j^{n+1} \rightarrow d_i^n p^6 d_j^n \quad (0.1)$$

is given by:

$$J_{dd}^{(2)} = \frac{2t_{pd}^4}{\Delta_{CT}^2 (\Delta_{CT} + \frac{U_{pp}}{2})}$$

**Hint:** Use the same method as part 'b'. Note that there are twice the number of exchange paths when compared to the previous case.

d) Which is the dominant exchange for  $\Delta_{CT} \gg U_{dd}$  (Mott-Hubbard) and  $U_{dd} \gg \Delta_{CT}$  (Charge transfer).