Solutions: Stoner Model

Exercise 1: Stoner Criterion

1. Stoner Criterion Derivation

The Stoner criterion is derived by considering the total energy of an electron gas with the exchange interaction. For a system of electrons, the total energy E can be written as:

$$E = E_{\text{kin}} + E_{\text{ex}}$$

where E_{kin} is the kinetic energy and E_{ex} is the exchange energy. With:

$$N^{\pm} = \frac{1}{2}N \pm \frac{1}{2}D(E_F)\delta E$$

and molecular field ansatz:

$$M = \frac{1}{2}g_s\mu_B\Delta N = -\mu_B D(E_F)\delta E$$

For a ferromagnetic state, the exchange energy can be approximated as:

$$E_{\text{ex}} = -\int_0^{B_{ext}} M\, dB = \int_0^{M} M\, dM = -\frac{1}{2}\mu_0\gamma M^2 = -\frac{1}{2}\mu_0\gamma\mu_B^2(D(E_F)\delta E)^2 = -\frac{1}{4}U(2\Delta N)^2$$

With $U = 2\mu_0\gamma\mu_B^2$ and $\Delta N = \frac{1}{2}D(E_F)\delta E$

and the kinetic energy:

$$\delta N\delta E = \frac{N^+ - N^-}{2}\delta E = \frac{1}{2}D(E_F)\delta E^2$$

To determine the stability of the ferromagnetic state, we calculate the change in total energy:

$$\Delta E = \frac{1}{2}D(E_F)\delta E^2[1 - \frac{1}{2}UD(E_F)]$$

This simplifies to:

$$\frac{1}{2}UD(E_F) > 1$$

This is the Stoner criterion for ferromagnetism.

2. Density of States Calculation

Using the given density of states for a free electron gas:

$$D(E_F) = \frac{3n}{2E_F}$$

Substitute this into the Stoner criterion:

$$\frac{1}{2}U\left(\frac{3n}{2E_F}\right) > 1$$

Thus, the Stoner criterion in terms of n, E_F, and U is:

$$U > \frac{4E_F}{3n}$$

Exercise 2: Exchange Interaction and Magnetization

1. Magnetization in the Stoner Model

The magnetization M in terms of the spin polarization ΔN is given by:

$$M = -\frac{1}{2} g_s \mu_B (n_\uparrow - n_\downarrow) = -\frac{1}{2} g_s \mu_B \Delta N = -\mu_B D(E_F) \delta E$$

A material with ferromagnetic exchange in a magnetic field experiences an additional energy term compared to Ex.1:

$$\Delta E = \frac{1}{2} D(E_F) \delta E^2 [1 - \frac{1}{2} U D(E_F)] - M B_{ext} = \frac{M^2}{2\mu_B^2 D(E_F)} [1 - \frac{1}{2} U D(E_F)] - M B_{ext}$$

Find the minimum for a material not experiencing spontaneous magnetization (not fulfilling the Stoner criterion).

$$\frac{\partial \Delta E}{\partial M} = \frac{M}{\mu_B^2 D(E_F)} [1 - \frac{1}{2} U D(E_F)] - B_{ext} \stackrel{!}{=} 0$$

Solve for M:

$$M = \frac{\mu_B^2 D(E_F)}{1 - \frac{1}{2} U D(E_F)} B_{ext}$$

And the susceptibility:

$$\chi = \mu_0 \frac{\partial M}{\partial B_{ext}} = \frac{\mu_B^2 D(E_F)}{1 - \frac{1}{2} U D(E_F)}$$

Writing the expression sing the Pauli susceptibility results in the Stoner enhancement:

$$\chi = \frac{\chi_{Pauli}}{1 - \frac{1}{2} U D(E_F)}$$

Exercise 3: Band Structure Effects

1. Band Structure and Stoner Criterion

The shape of the density of states $D(E)$ near the Fermi level is crucial. If $D(E)$ has a peak near E_F, it increases the likelihood of satisfying the Stoner criterion because $D(E_F)$ would be large. Conversely, a flat $D(E)$ near E_F makes it less likely.

2. Stoner Model and Transition Metals

For transition metals like iron, cobalt, and nickel, their d-band has a high density of states near the Fermi level. This high $D(E_F)$, combined with a significant exchange interaction I, makes these metals good candidates for ferromagnetism as per the Stoner criterion.

Exercise 4: Numerical Problems

1. Numerical Calculation

The given values result in the Stoner criterion being fulfilled for Fe but not for Pd.

Exercise 5: Advanced Concepts

The Stoner model does not account for several factors:

- Spin fluctuations: Thermal and quantum spin fluctuations can reduce the effective magnetization.
- Electron correlations: Beyond the mean-field approximation, electron-electron interactions can modify the magnetic behavior.
- Anisotropies and spin-orbit coupling: These effects can influence the magnetic properties but are not considered in the basic Stoner model.