18 The Standard Model and beyond

The success of the Standard Model of particle physics in describing the wide range of precise experimental measurements is a remarkable achievement. However, the Standard Model is just a model and there are many unanswered questions. This short concluding chapter provides a broad overview of the current state of our understanding of particle physics and describes some of the more important open issues.

18.1 The Standard Model

The ultimate theory of particle physics might consist of a (simple) equation with relatively few free parameters, from which everything else followed. Whilst the Standard Model (SM) is undoubtedly one of the great triumphs of modern physics, it is not this ultimate theory. It is a model constructed from a number of beautiful and profound theoretical ideas put together in a somewhat *ad hoc* fashion in order to reproduce the experimental data. The essential ingredients of the Standard Model, indicated in Figure 18.1, are: the Dirac equation of relativistic quantum mechanics that describes the dynamics of the fermions; Quantum Field Theory that provides a fundamental description of the particles and their interactions; the local gauge principle that determines the exact nature of these interactions; the Higgs mechanism of electroweak symmetry breaking that generates particle masses; and the wide-reaching body of experimental results that guide the way in which the Standard Model is constructed. The recent precision tests of the Standard Model and the discovery of the Higgs boson have firmly established the validity of the Standard Model at energies up to the electroweak scale. Despite this success, there are many unanswered questions.

18.1.1 The parameters of the Standard Model

If neutrinos are normal Dirac fermions, the Standard Model of particle physics has 25 (or 26) free parameters that have to be input by hand. These are: the masses of

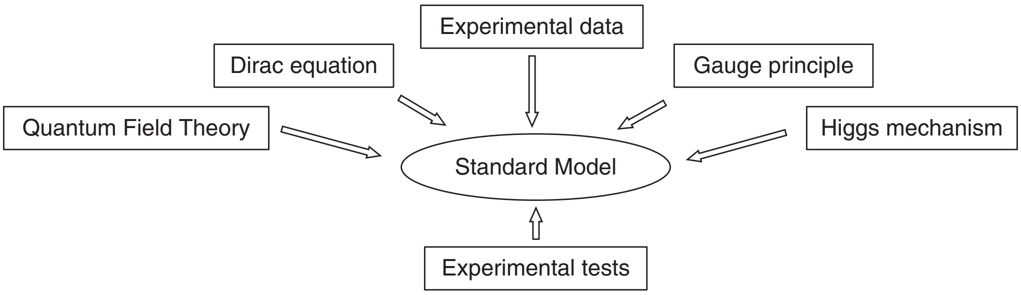

Fig. 18.1 The theoretical and experimental pillars of the Standard Model.

the *twelve* fermions (or perhaps more correctly the twelve Yukawa couplings to the Higgs field),

$$m_{\nu_1},\ m_{\nu_2},\ m_{\nu_3},\ m_{\mathrm{e}},\ m_{\mu},\ m_{\tau},\ m_{\mathrm{d}},\ m_{\mathrm{s}},\ m_{\mathrm{b}},\ m_{\mathrm{u}},\ m_{\mathrm{c}} \text{ and } m_{\mathrm{t}}\,;$$

the *three* coupling constants describing the strengths of the gauge interactions,

$$\alpha,\ G_{\mathrm{F}} \text{ and } \alpha_S\,,$$

or equivalently g', g_{W} and g_S; the *two* parameters describing the Higgs potential, μ and λ, or equivalently its vacuum expectation value and the mass of the Higgs boson,

$$v \text{ and } m_{\mathrm{H}}\,;$$

and the *eight* mixing angles of the PMNS and CKM matrices, which can be parameterised by

$$\theta_{12},\ \theta_{13},\ \theta_{23},\ \delta, \text{ and } \lambda,\ A,\ \rho,\ \eta.$$

In principle, there is one further parameter in the Standard Model; the Lagrangian of QCD can contain a phase that would lead to CP violation in the strong interaction. Experimentally, this strong CP phase is known to be extremely small,

$$\theta_{CP} \simeq 0.$$

and is usually taken to be zero. If θ_{CP} is counted, then the Standard Model has 26 free parameters.

The relatively large number of free parameters is symptomatic of the Standard Model being just that; a model where the parameters are chosen to match the observations, rather than coming from a higher theoretical principle. Putting aside θ_{CP}, of the 25 SM parameters, 14 are associated with the Higgs field, eight with the flavour sector and only three with the gauge interactions. Within each of these three broad areas, patterns emerge between the different parameters, suggesting the

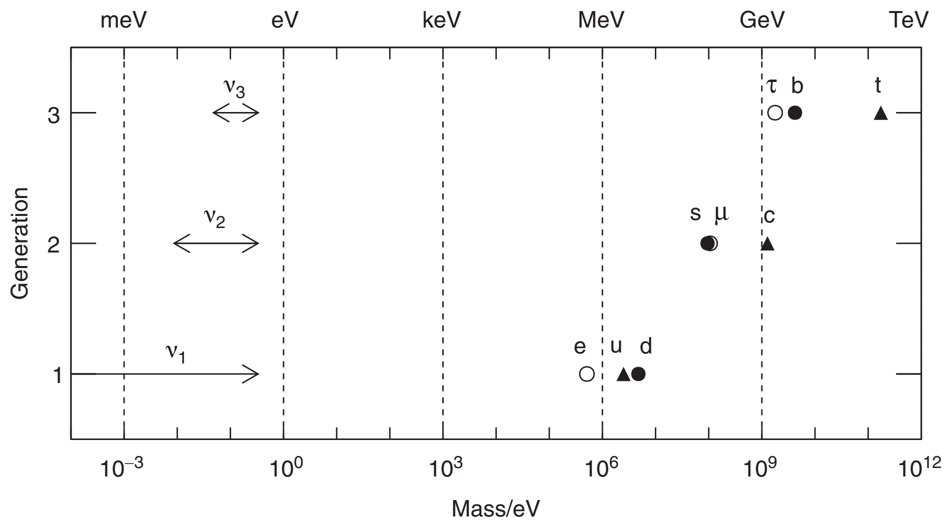

Fig. 18.2 The fermion masses shown by generation. The neutrino masses are displayed as approximate ranges of values assuming the normal hierarchy ($m_1 < m_2 < m_3$) and using the approximate upper limits on the sum of neutrino masses from cosmological constraints.

presence of some, as yet unknown, symmetry principle. For example, Figure 18.2 shows the observed masses of the fermions. With the exception of the neutrinos, the masses within a single generation are similar, and it is unlikely that this happens by chance. Likewise, the coupling constants of the three gauge interactions are of a similar order of magnitude, hinting that they might be different low-energy manifestations of a Grand Unified Theory (GUT) of the forces. These patterns provide hints for, as yet unknown, physics beyond the Standard Model.

18.2 Open questions in particle physics

The Standard Model is not the final theory of particle physics. However, there are many possibilities for the nature of physics beyond the Standard Model, for example, supersymmetry, large-scale extra dimensions, and ultimately perhaps even string theory. Here it is possible to give only a brief overview of a handful of the outstanding issues with the Standard Model and the possible solutions. The chosen topics focus on active areas of current experimental research.

18.2.1 What is dark matter?

The existence of *dark matter* in the Universe provides compelling evidence for physics beyond the Standard Model. Since the mid 1930s, it has been known that a significant fraction of the mass in the Universe is not bound up in the luminous stars that once were thought to comprise most of the mass of the galaxies. The most direct evidence for dark matter comes from the velocity distributions of stars as they orbit the galactic centre. In a spiral galaxy like the Milky Way, the majority of the luminous mass is located in the central bulge. Outside this central region, the tangental velocity of a star of mass m should be given by the usual equation for centripetal acceleration in a gravitational field

$$\frac{mv^2}{r} \approx \frac{Gm}{r^2} M(r),$$

where $M(r)$ is the total mass within a radius r. Assuming that most of this mass is concentrated in the central bulge, the tangental velocities of the stars should decrease as $r^{-1/2}$. This is not consistent with the observed velocity distributions, which decrease only slowly with radius, implying that the distribution of mass in the galaxy is approximately $M(r) \propto r$. From this observation alone, it can be concluded that the mass of a galaxy has a significant non-luminous component, known as dark matter.

Further compelling evidence for the existence of dark matter is provided by a number of cosmological and astrophysical measurements related to the large-scale structure in the Universe and, in particular, the precision measurements of the small fluctuations in the cosmic microwave background (CMB) from the Cosmic Microwave Background Explorer (COBE) and Wilkinson Microwave Anisotropy Probe (WMAP) satellites. These and other observations have provided a firm experimental basis for the ΛCDM cosmological model, which is the standard model of cosmology. In the ΛCDM model, the total energy-matter density Ω of the Universe is consistent with the flat geometry of space-time predicted by inflationary models, with $\Omega = 1$. Within the ΛCDM model, only 5% of energy–matter density of the Universe is in the form of normal baryonic matter, $\Omega_B \simeq 0.05$. A further 23% is in the form of *cold dark matter* (CDM), $\Omega_C \simeq 0.23$, and the majority of the energy–matter density of the Universe is in the form of *dark energy*, $\Omega_\Lambda \simeq 0.72$. In the ΛCDM model, the dark energy is attributed to a non-zero cosmological constant of Einstein's equations of general relativity, $\Lambda \neq 0$, which tends to accelerate the expansion of the Universe.

It is a remarkable fact that our understanding of cosmology has reached the level of precision and sophistication where it now provides constraints on particle physics. Whilst the existence of dark energy does not (yet) impact our understanding of particle physics, the cosmological constraints on dark matter are highly relevant. The particle content of the Universe affects the way in which large-scale structure arises. Because lighter particles, such as neutrinos, remain relativistic throughout the expansion and cooling of the Universe, they affect the evolution of large-scale structure differently than massive particles, which become non-relativistic during the first few years after the Big Bang. On this basis, it is known that the majority of the energy–mass density associated with the non-baryonic dark matter is due to cold (non-relativistic) matter as opposed to hot (relativistic) particles. The cosmological measurements are sufficiently precise to constrain the sum of the neutrino masses to be approximately

$$\sum_{i=1}^{3} m_{\nu_i} \lesssim 1\,\mathrm{eV}.$$

The current experimental evidence indicates that only a small fraction of the cold dark matter is in the form of normal baryons, for example in low-mass brown dwarf stars. The success of the ΛCDM standard model of cosmology, therefore strongly suggests that a significant fraction of the cold dark matter in the Universe may be in the form of a new type of weakly interacting massive particle (WIMP), with a mass in the few GeV−TeV range. Such particles arise naturally in extensions to the Standard Model; for example, in many supersymmetric models the lightest supersymmetric particle is the stable weakly interacting neutralino $\tilde{\chi}_1^0$. Regardless of the precise nature of the dark matter, the direct detection of WIMPs is one of the main goals in particle physics at this time. WIMPs can either be observed through their production at the LHC or through the direct detection of the WIMPs that are believed to pervade our galaxy.

Direct detection of dark matter

The direct detection of the galactic WIMP halo (assuming it exists) is extremely challenging. The WIMPs are predicted to have a Maxwell–Boltzmann velocity distribution with a root-mean-square (rms) velocity in the range 200–250 km s^{-1}, which corresponds to a mean kinetic energy of approximately $\langle T_\chi \rangle \approx 3 \times 10^{-7} m_\chi$, where m_χ is the mass of the WIMP in GeV. WIMPs would interact with normal matter through the elastic scattering with nuclei, $\chi + A \to \chi + A$. Dark matter experiments attempt to detect the recoil of a nucleus after such a scattering process. However, the maximum kinetic energy transferred to a nucleus of mass number A is only

$$T_{\max} \approx \frac{4Am_\chi m_\mathrm{p}}{(m_\chi + Am_\mathrm{p})^2} T_\chi \;\sim\; 1.2 \times 10^{-6} \frac{Am_\chi^2 m_\mathrm{p}}{(m_\chi + Am_\mathrm{p})^2}.$$

Consequently, for WIMP masses greater than 10 GeV, the recoil energies are typically in the range of 1 − 10 keV. By the usual standards of particle physics, this is a very low energy and the possible detection techniques reflect this. There are two main ways of detecting the nuclear recoil. The ionisation produced by the recoiling nucleus can be detected from scintillation light in sodium iodide crystals or liquid noble gas detectors. Alternatively, in cryogenic detectors consisting of very pure silicon or germanium crystals cooled to low temperatures, WIMPs can be detected from the phonons produced by the particle interactions and also from the ionisation produced by the recoiling nucleus.

From the energy–matter density associated with the CDM, the local number density of WIMPs is expected to be about $n \sim 0.3\,/m_\chi[\mathrm{GeV}]\,\mathrm{cm}^{-3}$. This relatively low number density, combined with the low velocities of the WIMPs and the smallness of weak interaction cross sections, means that the expected event rates are very small; typically just a few events per year in the current 10 kg-scale detectors. Furthermore, because the nuclear recoil energies are so low, backgrounds from natural radioactivity have to be controlled carefully.

Despite the occasional tantalising hints for a signal, at the time of writing there has been no confirmed direct detection of dark matter. Nevertheless, for many favoured scenarios (including supersymmetry), the sensitivities of the current experiments are only just beginning to reach that required to observe a possible signal and the results from the experiments in the coming decade are eagerly awaited.

18.2.2 Does supersymmetry exist?

Supersymmetry (SUSY) is a popular extension to the Standard Model. In SUSY each Standard Model particle has a super-partner "sparticle" which differs by half a unit of spin. The super-partner of each chiral fermion is a spin-0 scalar (sfermion) and the super-partners of the spin-1 gauge fields are spin-half gauginos. The partners of the spin-0 Higgs field are a weak isospin doublet of spin-half Higgsinos, $\tilde{\mathrm{H}}^0_{1,2}$ and $\tilde{\mathrm{H}}^\pm$. The physical fields in the minimal supersymmetric model are listed in Table 18.1. The physical chargino and neutralino states are, in general, mixtures of the Higgsinos and gauginos. In many supersymmetric models, the lightest neutralino $\tilde{\chi}^0_1$ is a weakly interacting stable particle, and is a possible WIMP candidate for the dark matter in the Universe.

Table 18.1 The Standard Model particles and their possible super-partners in the minimal supersymmetric model.

Particle		Spin		Super-particle		Spin
Quark	q	$\frac{1}{2}$		Squark	$\tilde{q}_L, \tilde{q}_R$	0
Lepton	$\ell^\pm$	$\frac{1}{2}$		Slepton	$\tilde{\ell}^\pm_L, \tilde{\ell}^\pm_R$	0
Neutrino	ν	$\frac{1}{2}$		Sneutrino	$\tilde{\nu}_L, \tilde{\nu}_R(?)$	0
Gluon	g	1		Gluino	$\tilde{g}$	$\frac{1}{2}$
Photon	γ	1	$\tilde{\gamma}$			
Z boson	Z	1	$\tilde{Z}$	Neutralino	$\tilde{\chi}^0_1, \tilde{\chi}^0_2, \tilde{\chi}^0_3, \tilde{\chi}^0_4$	$\frac{1}{2}$
Higgs	H	0	$\tilde{H}^0_1, \tilde{H}^0_2$; $\tilde{H}^\pm$			
W boson	$W^\pm$	1	$\tilde{W}^\pm$	Chargino	$\tilde{\chi}^\pm_1, \tilde{\chi}^\pm_2$	$\frac{1}{2}$

(Neutralinos: mixtures of $\tilde{\gamma}$, $\tilde{Z}$, $\tilde{H}^0_1$, $\tilde{H}^0_2$; Charginos: mixtures of $\tilde{H}^\pm$, $\tilde{W}^\pm$.)

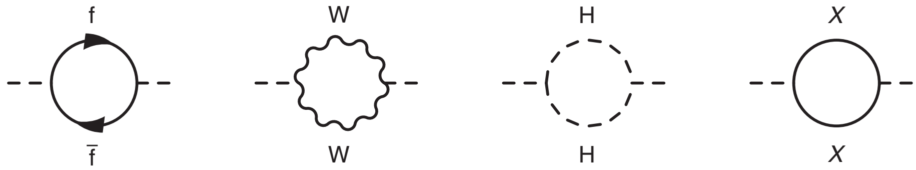

Fig. 18.3 Examples of loop corrections to the Higgs boson self-energy, where X represents a new massive particle.

The possibility of explaining the dark matter in the Universe is not the prime motivation for supersymmetry. Just as quantum loop corrections contributed to the W-boson mass (see Section 16.4), quantum loops in the Higgs boson propagator, such as those indicated in Figure 18.3, contribute to the Higgs boson mass. This in itself is not a problem. However, if the Standard Model is part of theory that is valid up to very high mass scales, such as that of a Grand Unified Theory $\Lambda_{\text{GUT}} \sim 10^{16}$ GeV or the Planck scale $\Lambda_{\text{P}} \sim 10^{19}$ GeV, these corrections become very large. Because of these quantum corrections, which are quadratic in Λ, it is difficult to keep the Higgs mass at the electroweak scale of 10^2 GeV. This is known as the Hierarchy problem. It can be solved by fine-tuning the new contributions to the Higgs mass such that they tend to cancel to a high degree of precision. However, supersymmetry provides a more natural solution to the Hierarchy problem; for every loop of particles there is a corresponding loop of sparticles, which provide a correction with the opposite sign. If the sparticle masses were the same as the particle masses, this cancellation would be exact. If supersymmetry were an exact symmetry of nature, the sparticles would have the same masses as the particles and already would have been discovered. Therefore, if supersymmetry exists, it is a broken symmetry and the mass scale of the SUSY particles is not known *a priori*. Nevertheless, there are theoretical arguments that favour a relatively low mass scale of $\mathcal{O}(1\,\text{TeV})$.

The search for the production of SUSY particles is one of the main focuses of the search for new physics at the LHC. In most SUSY models, sparticles are predicted to decay into final states including the stable lightest supersymmetric particle (LSP), which being neutral escapes detection. For example, at the LHC the signature of squark pair production and subsequent decay, $\tilde{q}\tilde{q} \to qq\tilde{\chi}_1^0\tilde{\chi}_1^0$, is a pair of high-energy jets and a large component of missing transverse momentum from the unobserved neutralinos. At the time of writing, no evidence of the direct production of SUSY particles has been observed at the LHC; the ATLAS and CMS experiments have been able exclude squark and gluino masses below about 1 TeV. The limits on the slepton and gaugino masses are much weaker, since these particles are not produced directly in strong interactions. Whilst there is no current experimental evidence for SUSY, the first operation of the LHC at its full energy of $\sqrt{s} \sim 14$ TeV will provide discovery potential at significantly higher mass scales.

18.2.3 Can the forces be unified?

It has already been noted that the coupling constants of the three forces of the Standard Model have similar strengths. At the electroweak scale of $q^2 = m_Z^2$,

$$\alpha^{-1} : \alpha_W^{-1} : \alpha_S^{-1} \approx 128 : 30 : 9. \qquad (18.1)$$

Furthermore, in Section 10.5 it was shown that the coupling constants of QED and QCD run with energy according to

$$\left[\alpha_i(q^2)\right]^{-1} = \left[\alpha_i(\mu^2)\right]^{-1} + \beta \ln\left(\frac{q^2}{\mu^2}\right),$$

where β depends on the numbers of fermion and boson loops contributing to the gauge boson self-energy. In QED where the photon self-energy arises from fermion loops alone α increases with energy, whereas α_S decreases with energy due to the presence of gluon loops. Because of the weak boson self-interactions, which are a consequence of the SU(2) gauge symmetry, α_W also decreases with increasing energy scale, although not as rapidly as α_S. The running of the different coupling constants therefore tends to bring their values together. It seems plausible that at some high-energy scale, the coupling constants associated with the U(1), SU(2) and SU(3) gauge symmetries converge to a single value. In the mid 1970s, it was suggested by Georgi and Glashow that the observed gauge symmetries of the Standard Model could be accommodated within a larger SU(5) symmetry group. In this Grand Unified Theory (GUT), the coupling constants of the Standard Model are found to converge (although not exactly) at an energy scale of about 10^{15} GeV, as shown in Figure 18.4a.

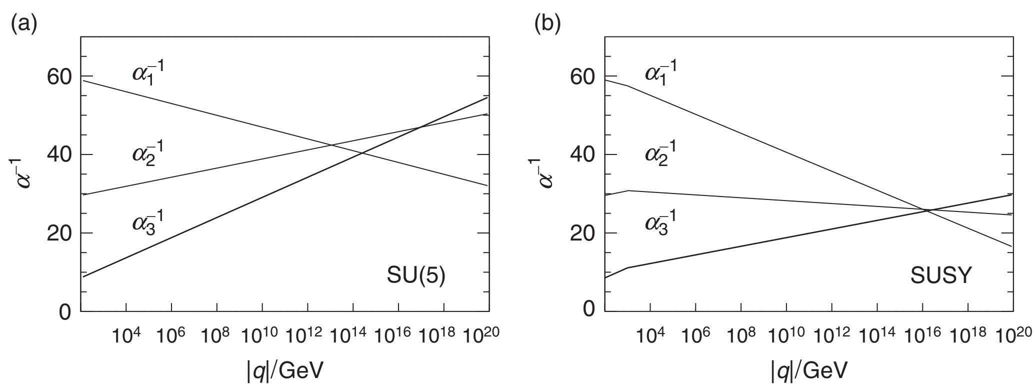

Fig. 18.4 An illustration of the running of the coupling constants in: (a) the SU(5) Grand Unified Theory and (b) a supersymmetric extension of SU(5) with new particles with masses of 1 TeV. It should be noted that, in the SU(5) model, the coupling constant of the U(1) symmetry is not α but $\alpha_1 = 5/3g'$.

The running of the coupling constants shown in Figure 18.4a assumes that only Standard Model particles enter the loops in the gauge boson propagators. If there is physics beyond the Standard Model with new particles at a mass scale Λ, these particles also would contribute to the gauge boson self-energy terms through extra fermionic and bosonic loops, modifying the running of the coupling constants for $q^2 > \Lambda^2$. For example, Figure 18.4b shows how supersymmetric particles at a scale of $\Lambda_{\mathrm{SUSY}} = 1$ TeV would modify the evolution of the U(1), SU(2) and SU(3) couplings within the SU(5) GUT. Remarkably, the coupling constants converge to a single value of $\alpha_{\mathrm{GUT}} \simeq 1/26$ at $|q| \sim 10^{16}$ GeV. In some sense, this convergence is inevitable since two non-parallel lines will always cross, and with the appropriate choice of the mass scale for new physics the three lines can always be made to meet at a single point. Nevertheless, it is interesting that the required mass scale turns out to be only 1 TeV.

It is now known that SU(5) is not the correct gauge group for a GUT; the predicted value for $\sin^2\theta_{\mathrm{W}}$ is incompatible with the measured value. Despite this, the convergence of the coupling constants strongly suggests that the three forces of the Standard Model are the low-energy manifestations of some larger, as yet unknown, unified theory.

18.2.4 What is the nature of the Higgs boson?

The experimental study of the Higgs boson at the LHC is undoubtedly one of the most exciting areas in contemporary particle physics. Within the Standard Model, the Higgs boson is unique; it is the only fundamental scalar in the theory. Establishing the properties of the Higgs boson such as its spin, parity and branching ratios is essential to understand whether the observed particle is the Standard Model Higgs boson or something more exotic.

In the Standard Model, the Higgs mechanism assumes a doublet of complex scalar fields. Whilst this is the simplest choice, it is not unique. For example, supersymmetric extensions to the Standard Model require (at least) two doublets of complex scalar fields. In the two-Higgs doublet model (2HDM), three of the eight scalar fields are the Goldstone bosons that give mass to the W and Z bosons. The remaining five fields correspond to five physical Higgs bosons; two CP-even neutral scalars h and H^0, two charged scalar particles $\mathrm{H}^\pm$, and a CP-odd neutral scalar A^0. In supersymmetry, the neutral Higgs boson (denoted h) must be light and can appear very much like the Standard Model Higgs boson, whereas the $\mathrm{H}^\pm$, A^0 and H^0 can be very massive.

In supersymmetric models, the two Higgs doublets, which have different vacuum expectation values, respectively give the masses to the fermions in the upper and lower components of the weak isospin doublets. In this case, the couplings of the light Higgs boson to the fermions will differ from the Standard Model predictions, although the differences may be quite small. Consequently, the measurements of the branching ratios of the 125 GeV Higgs boson may reveal physics beyond the Standard Model. In the coming years, the study of the Higgs boson at the LHC will form one of the main thrusts of experimental particle physics. On a longer time scale, even more precise studies may be possible at a future e^+e^- linear collider, such as the International Linear Collider (ILC) or the Compact Linear Collider (CLIC).

18.2.5 Flavour and the origin of CP violation

There are a number of fundamental questions related to the flavour sector of the Standard Model. Although it appears that there are exactly three generations of fermions, the Standard Model provides no explanation of *why* this is the case. Likewise, the Standard Model provides no clear explanation of why the CKM matrix is almost diagonal, and in contrast, the PMNS matrix is relatively "flat".

Furthermore, the complex phases in the CKM and PMNS matrix are the only places in the Standard Model where CP violation can be accommodated. Whilst CP violation in the quark sector has been studied in great depth, CP violation in the neutrino sector has yet to be observed. The measurement of the parameter δ in the PMNS matrix will be the focus of the next generation of long-baseline neutrino oscillations experiments.

However, even if CP violation is observed in neutrino oscillations, it seems quite possible that the CP violation in the Standard Model is insufficient to explain the observed matter–antimatter asymmetry of the Universe. One solution to this apparent problem is that (possibly large) CP-violating effects may occur in as yet undiscovered physics beyond the Standard Model. It is possible that such effects will be observed in the coming years, either directly or through loop corrections, in the decays of the vast numbers of b-quarks produced in the LHCb experiment at the LHC and the Belle-II experiment at KEK in Japan.

18.2.6 Are neutrinos Majorana particles?

The masses of the neutrinos are very different from the masses of the other fermions. If neutrinos are normal Dirac particles, this would imply an unnaturally small Yukawa coupling to the Higgs field. Whilst this is possible, the seesaw mechanism, described in Section 17.8.1, provides an attractive explanation for the smallness of the neutrino masses. Although the presence of a Majorana mass term in the Lagrangian would not automatically imply that neutrinos are Majorana particles, this would be a real possibility. In this case, the neutrinos would be their own antiparticles, $\nu \equiv \overline{\nu} \equiv \nu_M$.

Perhaps surprisingly, the observable effects of removing the distinction between neutrinos and antineutrinos are very small. In the Standard Model, the neutrino

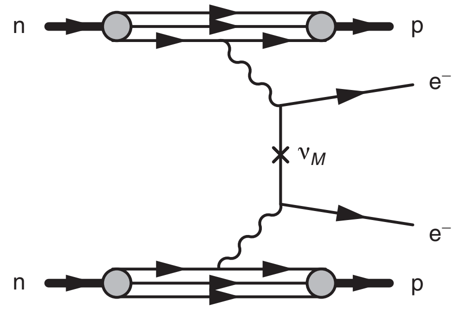

Fig. 18.5 The Feynman diagram for neutrinoless double β-decay.

produced in the decay $\pi^+ \to \mu^+\nu_\mu$ will always produce a μ^- in its subsequent $\nu_\mu n \to \mu^- p$ charged-current interactions (assuming it has not oscillated). Therefore in the Standard Model, the net number of leptons in the Universe, L = N(leptons) − N(antileptons), is constant and lepton number L is said to be conserved. If neutrinos were Majorana particles, the neutrino from the decay $\pi^+ \to \mu^+\nu_M$ could in principle interact as a RH chiral antiparticle $\nu_M p \to \mu^+ n$. The net effect would be a $\Delta L = -2$ change in lepton number and lepton number no longer would be a conserved quantity. However, because of the smallness of neutrino masses, the neutrino helicity states are almost identical to the chiral states and the fraction of lepton-number-violating processes would be suppressed by $O(m_\nu^2/m_\mu^2)$, which is too small to be observable. Consequently, experiments have focussed on the possibility of neutrinoless double β-decay, which can occur *only* if neutrinos are Majorana particles.

Certain even–even nuclei, where the usual $\beta^\pm$-decay or electron-capture processes are energetically forbidden, can decay to a more tightly bound even–even nucleus by the double β-decay process, $(Z, A) \to (Z + 2, A) + 2e^- + 2\overline{\nu}_e$, which can be thought of in terms of two simultaneous single β-decays. Whilst such $2\nu\beta\beta$-decays are rare, with half-lives in the range $\tau_{1/2} \sim 10^{19} - 10^{25}$ years, they have been observed for a number of isotopes. If neutrinos were Majorana particles, the lepton number violating *neutrinoless* double β-decays processes ($0\nu\beta\beta$) can occur through the Feynman diagram shown Figure 18.5. Experimentally $0\nu\beta\beta$ can be distinguished from the more common $2\nu\beta\beta$-decays from the energy spectrum of the electrons. Neutrinoless double β-decays would produce mono-energetic electrons with energy

$$E_e = \tfrac{1}{2}Q = \tfrac{1}{2}\left[M(Z, A) - M(Z + 2, A)\right],$$

where $M(Z, A)$ and $M(Z + 2, A)$ are the masses of the parent and daughter nuclei. In contrast, $2\nu\beta\beta$-decays produce a broad spectrum of electron energies with very few being produced close to the end point of $\frac{1}{2}Q$.

If neutrinos are Majorana particles, the predicted $0\nu\beta\beta$-decay rates are proportional to

$$\Gamma \propto G_F^4 \, |m_{\beta\beta}|^2 \times |\mathcal{M}_{\rm nucl}|^2,$$

where $\mathcal{M}_{\rm nucl}$ is the nuclear matrix element and $m_{\beta\beta}$ is known as the effective Majorana mass. The effective Majorana mass,

$$m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 \, m_{\nu_i}, \tag{18.2}$$

depends on the neutrino masses and the elements of the PMNS matrix. As a result, the predicted decay rates depend on the neutrino mass hierarchy, with the inverted hierarchy typically leading to larger predicted rates.

A number of experiments have searched for $0\nu\beta\beta$-decays in processes such as ${}^{76}_{32}\mathrm{Ge} \to {}^{76}_{34}\mathrm{Se} + e^- + e^-$ and ${}^{136}_{54}\mathrm{Xe} \to {}^{136}_{56}\mathrm{Ba} + e^- + e^-$. To date, there has been no confirmed observation of neutrinoless double β-decay, with the most stringent lifetime limits being set at $\tau_{1/2}^{0\nu\beta\beta} \gtrsim 10^{25}$ years. Nevertheless, the experiments are only just beginning to reach the required level of sensitivity where it might be possible observe $0\nu\beta\beta$-decay, even for the most optimistic values of $m_{\beta\beta}$. In the coming years, a number of larger experiments will start to search for neutrinoless double β-decay. A positive signal would represent a major discover, demonstrating that the neutrinos are fundamentally different from all other particles.

18.3 Closing words

Most of the theoretical concepts in the Standard Model were in place by the end of the 1960s. These ideas gained strong support with the discovery of the W and Z bosons at CERN in the mid 1980s. In the last decade of the twentieth century, the precision studies of the W and Z bosons provided tests of the predictions of the Standard Model at the quantum loop level. The start of the full operation of the LHC in 2010 represented a new stage in the experimental study of particle physics. With the discovery of the Higgs boson in 2012, the full spectrum of the Standard Model particles had been observed. This period of nearly 50 years from the late 1960s to 2012, represented a giant leap forward in our understanding of the Universe at the most fundamental level. I hope this book has helped you appreciate some of the profound theoretical ideas and the beautiful experimental measurements that have made the Standard Model of particle physics one of the central pillars of modern physics.

Despite it success, it should not be forgotten that the Standard Model is not the end of the story; there are just too many loose ends. The coming years will see the high-luminosity operation of the LHC at a centre-of-mass energy close to 14 TeV. In addition, a new generation of experiments will search for signatures for physics beyond the Standard Model. We may be standing at the threshold of new and potentially revolutionary discoveries. Only time will tell whether this will be the direct detection of dark matter, the demonstration that neutrinos are Majorana particles, the discovery of supersymmetry, or quite possibly something completely unexpected. The only certain thing is that interesting times lie ahead of us.