17 The Higgs boson

The Higgs mechanism and the associated Higgs boson are essential parts of the Standard Model. The Higgs mechanism is the way that the W and Z bosons acquire mass without breaking the local gauge symmetry of the Standard Model. It also gives mass to the fundamental fermions. This chapter describes the Higgs mechanism and the discovery of the Higgs boson at the LHC. The Higgs mechanism is subtle and to gain a full understanding requires the additional theoretical background material covered in the sections on Lagrangians and local gauge invariance in quantum field theory.

17.1 The need for the Higgs boson

The apparent violation of unitarity in the $e^+e^- \to W^+W^-$ cross section was resolved by the introduction of the Z boson. A similar issue arises in the $W^+W^- \to W^+W^-$ scattering process, where the cross section calculated from the Feynman diagrams shown in Figure 17.1 violates unitarity at a centre-of-mass energy of about 1 TeV. The unitarity violating amplitudes originate from $W_LW_L \to W_LW_L$ scattering, where the W bosons are longitudinally polarised. Consequently, unitary violation in WW scattering can be associated with the W bosons being massive, since longitudinal polarisation states do not exist for massless particles. The unitarity violation of the $W_LW_L \to W_LW_L$ cross section can be cancelled by the diagrams involving the exchange of a scalar particle, shown in Figure 17.2. In the Standard Model this scalar is the Higgs boson. This cancellation can work only if the couplings of the scalar particle are related to the electroweak couplings, which naturally occurs in the Higgs mechanism.

The Higgs mechanism is an integral part of the Standard Model. Without it, the Standard Model is not a consistent theory; the underlying gauge symmetry of the electroweak interaction is broken by the masses of the associated gauge bosons. As shown by ‘t Hooft, only theories with local gauge invariance are renormalisable, such that the cancellation of all infinities takes place among only a finite number of interaction terms. Consequently, the breaking of the local gauge invariance of the electroweak theory by the gauge boson masses can not simply be dismissed.

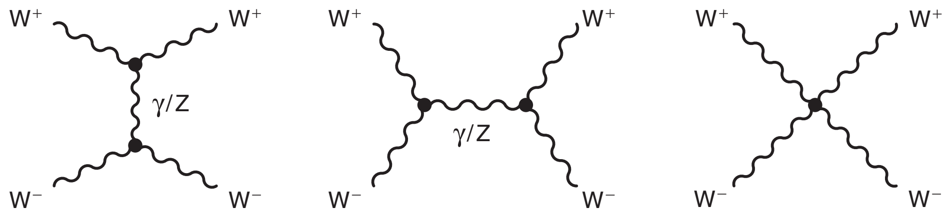


Fig. 17.1 The lowest-order Feynman diagrams for $W^+W^- \to W^+W^-$. The final diagram, corresponds to the quartic coupling of four W bosons.

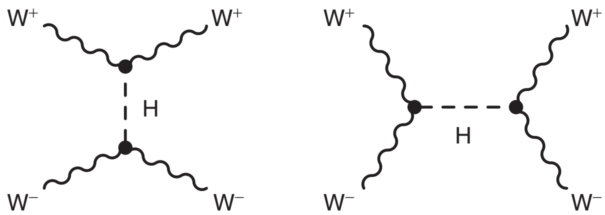


Fig. 17.2 Higgs boson exchange diagrams for $W^+W^- \to W^+W^-$.

The Higgs mechanism generates the masses of the electroweak gauge bosons in a manner that preserves the local gauge invariance of the Standard Model.

17.2 Lagrangians in Quantum Field Theory

The Higgs mechanism is described in terms of the Lagrangian of the Standard Model. In quantum mechanics, single particles are described by wavefunctions that satisfy the appropriate wave equation. In Quantum Field Theory (QFT), particles are described by excitations of a quantum field that satisfies the appropriate quantum mechanical field equations. The dynamics of a quantum field theory can be expressed in terms of the Lagrangian density. Whilst the development of QFT is outside the scope of this book, an understanding of the Lagrangian formalism is necessary for the discussion of the Higgs mechanism. The purpose of this section is to provide a pedagogical introduction to the Lagrangian of the Standard Model, which ultimately contains all of the fundamental particle physics.

17.2.1 Classical fields

In classical dynamics, the motion of a system can be described in terms of forces and the resulting accelerations using Newton's second law, $\mathbf{F} = m\ddot{\mathbf{x}}$. The same equations of motion can be obtained from the Lagrangian L defined as

$$L = T - V, \tag{17.1}$$

where T and V are respectively the kinetic and potential energies of the system. The Lagrangian $L(q_i, \dot{q}_i)$ is a function of a set of generalised coordinates q_i and their time derivatives $\dot{q}_i$ (the possible explicit time dependence of the Lagrangian is not considered here). Once the Lagrangian is specified, the equations of motion are determined by the Euler–Lagrange equations,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0. \tag{17.2}$$

For example, consider a particle moving in one dimension where the Lagrangian is a function of the coordinate x and its time derivative $\dot{x}$, with

$$L = T - V = \tfrac{1}{2}m\dot{x}^2 - V(x).$$

The derivatives of the Lagrangian with respect to x and $\dot{x}$ are

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} \quad \text{and} \quad \frac{\partial L}{\partial x} = -\frac{\partial V}{\partial x},$$

and the Euler–Lagrange equation (17.2) for the coordinate $q_i = x$ is simply

$$m\ddot{x} = -\frac{\partial V(x)}{\partial x}.$$

Since the derivative of the potential gives the force, this is equivalent to $F = m\ddot{x}$ and Newton's second law of motion is recovered.

The Lagrangian treatment of a discrete system of particles, described by n generalised coordinates q_i, can be extended to a continuous system by replacing the Lagrangian of (17.1) with the Lagrangian *density* $\mathcal{L}$,

$$L\left(q_i, \frac{\mathrm{d}q_i}{\mathrm{d}t}\right) \to \mathcal{L}\left(\phi_i, \partial_\mu \phi_i\right).$$

In the Lagrangian density, the generalised coordinates q_i are replaced by the *fields* $\phi_i(t, x, y, z)$, and the time derivatives of the generalised coordinates $\dot{q}_i$ are replaced by the derivatives of the fields with respect to each of the four space-time coordinates,

$$\partial_\mu \phi_i \equiv \frac{\partial \phi_i}{\partial x^\mu}.$$

The fields are continuous functions of the space-time coordinates x^μ and the Lagrangian L itself is given by

$$L = \int \mathcal{L}\, \mathrm{d}^3\mathbf{x}.$$

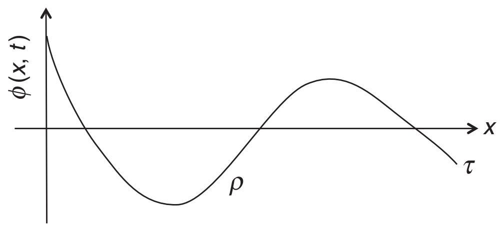


Fig. 17.3 The scalar field $\phi(x, t)$ representing the transverse displacement of a string of mass per unit length ρ under tension τ.

Using the principle of least action,[1] the equivalent of the Euler–Lagrange equation for the fields ϕ_i can be shown to be

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) - \frac{\partial \mathcal{L}}{\partial \phi_i} = 0. \tag{17.3}$$

The field $\phi_i(x^\mu)$ represents a continuous quantity with a value at each point in space-time. It can be a scalar such as temperature $T(\mathbf{x}, t)$, a vector such as the electric field strength $\mathbf{E}(\mathbf{x}, t)$, or a tensor.

To illustrate the application of classical field theory, consider the relatively simple example of a string of mass per unit length ρ under tension τ, as indicated in Figure 17.3. Here the scalar field $\phi(x, t)$ represents the transverse displacement of the string as a function of x and t. The kinetic and potential energies of the string, written in terms of the derivatives of the field, are

$$T = \int \tfrac{1}{2}\rho \left(\frac{\partial \phi}{\partial t} \right)^2 \mathrm{d}x \quad \text{and} \quad V = \int \tfrac{1}{2}\tau \left(\frac{\partial \phi}{\partial x} \right)^2 \mathrm{d}x.$$

Hence the Lagrangian density is

$$\mathcal{L} = \tfrac{1}{2}\rho \left[\left(\frac{\partial \phi}{\partial t} \right)^2 - v^2 \left(\frac{\partial \phi}{\partial x} \right)^2 \right] \equiv \tfrac{1}{2}\rho \left[(\partial_0 \phi)^2 - v^2 (\partial_1 \phi)^2 \right], \tag{17.4}$$

where $v = \sqrt{\tau/\rho}$. Once the Lagrangian density has been specified, the equations of motion follow from the Euler–Lagrange equation of (17.3). For the Lagrangian density of (17.4), the relevant partial derivatives are

$$\frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} = \rho\, \partial_0 \phi \equiv \rho \frac{\partial \phi}{\partial t}, \quad \frac{\partial \mathcal{L}}{\partial(\partial_1 \phi)} = -\rho v^2\, \partial_1 \phi \equiv -\rho v^2 \frac{\partial \phi}{\partial x} \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial \phi} = 0,$$

and the Euler–Lagrange equation gives

$$\rho \partial_0 (\partial_0 \phi) - \rho v^2 \partial_1 (\partial_1 \phi) = 0 \quad \text{or equivalently} \quad \rho \frac{\partial}{\partial t} \left(\frac{\partial \phi}{\partial t} \right) - \rho v^2 \frac{\partial}{\partial x} \left(\frac{\partial \phi}{\partial x} \right) = 0.$$

[1] The derivation can be found in any standard text on classical or Quantum Field Theory.

Therefore the field $\phi(x)$, describing the displacement of the string, satisfies the equation of motion

$$\frac{\partial^2\phi}{\partial t^2} - v^2\frac{\partial^2\phi}{\partial x^2} = 0,$$

which is the usual one-dimensional wave equation with phase velocity given by $v = \sqrt{\tau/\rho}$. Hence, it can be seen that Lagrangian density determines the wave equation for the field.

17.2.2 Relativistic fields

In Quantum Field Theory, the single particle wavefunctions of quantum mechanics are replaced by (multi-particle) excitations of a quantum field, which itself satisfies the appropriate field equation. The field equation is determined by the form of the Lagrangian density, which henceforth will be referred to simply as the Lagrangian. In the above example of a string under tension, it was shown that the Lagrangian,

$$\mathcal{L} = \tfrac{1}{2}\rho\left[(\partial_0\phi)^2 - v^2(\partial_1\phi)^2\right],$$

gives the usual wave equation for the displacement of the string. Similarly the dynamics of the quantum mechanical fields describing spin-0, spin-half and spin-1 particles are determined by the appropriate Lagrangian densities.

Relativistic scalar fields

In QFT, spin-0 scalar particles are described by excitations of the scalar field $\phi(x)$ satisfying the Klein–Gordon equation, first encountered in Section 4.1. The Lagrangian for a free non-interacting scalar field can be identified as

$$\mathcal{L}_S = \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \tfrac{1}{2}m^2\phi^2. \tag{17.5}$$

To see that this Lagrangian corresponds to the Klein–Gordon equation, it is helpful to write (17.5) in full,

$$\mathcal{L}_S = \tfrac{1}{2}\left[(\partial_0\phi)(\partial_0\phi) - (\partial_1\phi)(\partial_1\phi) - (\partial_2\phi)(\partial_2\phi) - (\partial_3\phi)(\partial_3\phi)\right] - \tfrac{1}{2}m^2\phi^2,$$

from which the partial derivatives appearing in the Euler–Lagrange equation are

$$\frac{\partial\mathcal{L}}{\partial\phi} = -m^2\phi, \qquad \frac{\partial\mathcal{L}}{\partial(\partial_0\phi)} = \partial_0\phi \equiv \partial^0\phi \quad \text{and} \quad \frac{\partial\mathcal{L}}{\partial(\partial_k\phi)} = -\partial_k\phi \equiv \partial^k\phi,$$

where $k = 1, 2, 3$. Substituting these partial derivatives into the Euler–Lagrange equation of (17.3) gives

$$\partial_\mu\partial^\mu\phi + m^2\phi = 0,$$

which is the Klein–Gordon equation for a free scalar field $\phi(x)$.

Relativistic spin-half fields

The Lagrangian for the spinor field $\psi(x)$ satisfying the free-particle Dirac equation is

$$\mathcal{L}_D = i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi. \tag{17.6}$$

Here the field $\psi(x)$ is a four-component complex spinor, which can be expressed in terms of eight independent real fields,

$$\psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \Psi_1 + i\Phi_1 \\ \Psi_2 + i\Phi_2 \\ \Psi_3 + i\Phi_3 \\ \Psi_4 + i\Phi_4 \end{pmatrix}.$$

In principle, the Euler–Lagrange equation can be solved in terms of these eight fields. However, the eight independent components of the complex Dirac spinor ψ also can be expressed as linear combinations of ψ and the adjoint spinor $\overline{\psi}$. Hence, the *independent* fields can be taken to be the four components the spinor and the four components of the adjoint spinor. The partial derivatives with respect to one of the components of the adjoint spinor are

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\overline{\psi}_i)} = 0 \quad \text{and} \quad \frac{\partial\mathcal{L}}{\partial\overline{\psi}_i} = i\gamma^\mu\partial_\mu\psi - m\psi,$$

which when substituted into the Euler–Lagrange equation give

$$-\frac{\partial\mathcal{L}}{\partial\overline{\psi}_i} = 0,$$

and consequently, the spinor field ψ satisfies the Dirac equation,

$$i\gamma^\mu(\partial_\mu\psi) - m\psi = 0.$$

Relativistic vector fields

Maxwell's equations for the electromagnetic field $A^\mu = (\phi, \mathbf{A})$ can be expressed in a covariant form (see Appendix D.1) as

$$\partial_\mu F^{\mu\nu} = j^\nu,$$

where $F^{\mu\nu}$ is the field-strength tensor,

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}, \tag{17.7}$$

and $j = (\rho, \mathbf{J})$ is the four-vector current associated with the charge and current densities ρ and $\mathbf{J}$. The corresponding Lagrangian (see Problem 17.4) is

$$\mathcal{L}_{EM} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^{\mu}A_{\mu}.$$

In the absence of sources $j^{\mu} = 0$, and the Lagrangian for the free photon field is

$$\mathcal{L}_{EM} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}. \tag{17.8}$$

Using the form of the field strength tensor of (17.7), this is equivalent to $\mathcal{L}_{EM} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2)$, from which the corresponding Hamiltonian density $\mathcal{H}_{EM} = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2)$ gives the normal expression for the energy density of an electromagnetic field (in Heaviside–Lorentz units with $\epsilon_0 = \mu_0 = 1$). If the photon had mass, the free-particle Lagrangian of (17.8) would be modified to

$$\mathcal{L}_{\mathrm{Proca}} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m_{\gamma}^2 A^{\mu}A_{\mu}, \tag{17.9}$$

which is known as the Proca Lagrangian, from which the field equations for a massive spin-1 particle can be obtained.

17.2.3 Noether's theorem

In the following section, the ideas of local gauge invariance are considered in the context of the symmetries of the Lagrangian. Here a simple example is used to illustrate the connection between a symmetry of the Lagrangian and a conservation law. The Lagrangian for a mass m orbiting in the gravitational potential of a fixed body of mass M is

$$\begin{aligned} L = T - V &= \tfrac{1}{2}m\mathrm{v}^2 + \frac{GMm}{r} \\ &= \tfrac{1}{2}m\dot{r}^2 + \tfrac{1}{2}mr^2\dot{\phi}^2 + \frac{GMm}{r}, \end{aligned}$$

where r and ϕ are the polar coordinates of the mass m in the plane of the orbit. The Lagrangian does not depend on the polar angle ϕ and therefore is invariant under the infinitesimal transformation, $\phi \to \phi' = \phi + \delta\phi$. Since the Lagrangian does not depend on ϕ, the corresponding Euler–Lagrange equation implies

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = 0,$$

and consequently

$$J = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi},$$

is a constant of the motion. The rotational symmetry of the Lagrangian therefore implies the existence of a conserved quantity, which in this example is the angular momentum of the orbiting body m.

In field theory, Noether's theorem relates a symmetry of the Lagrangian to a conserved current. For example, the Lagrangian for the free Dirac field

$$\mathcal{L} = i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\overline{\psi}\psi, \tag{17.10}$$

is unchanged by the global U(1) phase transformation,

$$\psi \to \psi' = e^{i\theta}\psi.$$

In Appendix E it is shown that the corresponding conserved current is the usual four-vector current

$$j^{\mu} = \overline{\psi}\gamma^{\mu}\psi,$$

which automatically satisfies the continuity equation $\partial_{\mu}j^{\mu} = 0$.

17.3 Local gauge invariance

In Section 10.1, the electromagnetic interaction was introduced by requiring the Dirac equation to be invariant under a U(1) *local* phase transformation. The required local gauge symmetry is expressed naturally as the invariance of the Lagrangian under a local phase transformation of the fields,

$$\psi(x) \to \psi'(x) = e^{iq\chi(x)}\psi(x). \tag{17.11}$$

The local nature of the gauge transformation means that the derivatives acting on the field also act on the local phase $\chi(x)$. With this transformation, the Lagrangian for a free spin-half particle of (17.6),

$$\mathcal{L} = i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\overline{\psi}\psi, \tag{17.12}$$

becomes

$$\begin{aligned}\mathcal{L} \to \mathcal{L}' &= ie^{-iq\chi}\overline{\psi}\gamma^{\mu}\left[e^{iq\chi}\partial_{\mu}\psi + iq\left(\partial_{\mu}\chi\right)e^{iq\chi}\psi\right] - me^{-iq\chi}\overline{\psi}e^{iq\chi}\psi \\ &= \mathcal{L} - q\overline{\psi}\gamma^{\mu}\left(\partial_{\mu}\chi\right)\psi. \end{aligned} \tag{17.13}$$

Hence, as it stands, the free-particle Lagrangian for a Dirac field is not invariant under U(1) local phase trasformations. The required gauge invariance can be restored by replacing the derivative ∂_{μ} in (17.12) with the *covariant derivative* D_{μ},

$$\partial_{\mu} \to D_{\mu} = \partial_{\mu} + iqA_{\mu},$$

where A_{μ} is a new field. The desired cancellation of the unwanted $q\overline{\psi}\gamma^{\mu}(\partial_{\mu}\chi)\psi$ term in (17.13) is achieved provided the new field transforms as

$$A_{\mu} \to A'_{\mu} = A_{\mu} - \partial_{\mu}\chi. \tag{17.14}$$

The required U(1) local gauge invariance of the Lagrangian corresponding to the Dirac equation can be achieved only by the introduction of the field A_μ with well-defined gauge transformation properties. Hence the gauge-invariant Lagrangian for a spin-half fermion

$$\mathcal{L} = \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi - q\overline{\psi}\gamma^\mu A_\mu\psi,$$

now contains a term describing the interaction of the fermion with the new field A_μ, which can be identified as the photon. Hence the Lagrangian of QED, describing the fields for the electron (with $q = -e$), the massless photon and the interactions between them can be written as

$$\mathcal{L}_{QED} = \overline{\psi}(i\gamma^\mu\partial_\mu - m_\mathrm{e})\psi + e\overline{\psi}\gamma^\mu\psi A_\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{17.15}$$

The kinetic term for the massless spin-1 field $F_{\mu\nu}F^{\mu\nu}$ is already invariant under U(1) local phase transformations (see Problem 17.3).

The connection to Maxwell's equations can be made apparent by writing the QED Lagrangian of (17.15) in terms of the four-vector current, $j^\mu = -e\overline{\psi}\gamma^\mu\psi$,

$$\mathcal{L} = \overline{\psi}(i\gamma^\mu\partial_\mu - m_\mathrm{e})\psi - j^\mu A_\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

The Euler–Lagrange equation for the derivatives with respect to the photon field A^μ gives (see Problem 17.4)

$$\partial_\mu F^{\mu\nu} = j^\nu,$$

which is the covariant form of Maxwell's equations. Hence the whole of electromagnetism can be derived by requiring a local U(1) gauge symmetry of the Lagrangian for a particle satisfying the Dirac equation.

The weak interaction and QCD are respectively obtained by extending the local gauge principle to require that the Lagrangian is invariant under $\mathrm{SU}(2)_L$ and SU(3) local phase transformations. The prescription for achieving the required gauge invariance is to replace the four-derivative ∂_μ with the covariant derivative D_μ defined in terms of the generators of the group. For example, for the $\mathrm{SU}(2)_L$ symmetry of the weak interaction

$$\partial_\mu \to D_\mu = \partial_\mu + ig_\mathrm{W}\mathbf{T}\cdot\mathbf{W}_\mu(x),$$

where the $\mathbf{T} = \frac{1}{2}\boldsymbol{\sigma}$ are the three generators of SU(2) and $\mathbf{W}(x)$ are the three new gauge fields. The generators of the SU(2) and SU(3) symmetry groups do not commute and the corresponding local gauge theories are termed non-Abelian. In a non-Abelian gauge theory, the transformation properties of the gauge fields are not independent and additional gauge boson self-interaction terms have to be added to the field-strength tensor for it to be gauge invariant. The focus of this chapter is the Higgs mechanism and therefore the more detailed discussion of non-Abelian gauge theories is deferred to Appendix F.

17.4 Particle masses

The local gauge principle provides an elegant description of the interactions in the Standard Model. The success of the Standard Model in describing the experimental data, including the high-precision electroweak measurements, places the local gauge principle on a solid experimental basis. However, the required local gauge invariance of the Standard Model is broken by the terms in the Lagrangian corresponding to particle masses. For example, if the photon were massive, the Lagrangian of QED would contain an additional term $\frac{1}{2}m_\gamma^2 A_\mu A^\mu$,

$$\mathcal{L}_{\mathrm{QED}} \rightarrow \overline{\psi}(i\gamma^\mu\partial_\mu - m_e)\psi + e\overline{\psi}\gamma^\mu A_\mu\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\gamma^2 A_\mu A^\mu.$$

For the U(1) local gauge transformation of (17.11), the photon field transforms as

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu\chi,$$

and the new mass term becomes

$$\tfrac{1}{2}m_\gamma^2 A_\mu A^\mu \rightarrow \tfrac{1}{2}m_\gamma^2\left(A_\mu - \partial_\mu\chi\right)(A^\mu - \partial^\mu\chi) \neq \tfrac{1}{2}m_\gamma^2 A_\mu A^\mu,$$

from which it is clear that the photon mass term is not gauge invariant. Hence the required U(1) local gauge symmetry can only be satisfied if the gauge boson of an interaction is *massless*. This restriction is not limited to the U(1) local gauge symmetry of QED, it also applies to the $SU(2)_L$ and SU(3) gauge symmetries of the weak interaction and QCD. Whilst the local gauge principle provides an elegant route to describing the nature of the observed interactions, it works only for massless gauge bosons. This is not a problem for QED and QCD where the gauge bosons are massless, but it is in apparent contradiction with the observation of the large masses of W and Z bosons.

The problem with particle masses is not restricted to the gauge bosons. Writing the electron spinor field as $\psi = \mathrm{e}$, the electron mass term in QED Lagrangian can be written in terms of the chiral particle states as

$$\begin{aligned} -m_e\bar{\mathrm{e}}\mathrm{e} &= -m_e\bar{\mathrm{e}}\left[\tfrac{1}{2}(1-\gamma^5) + \tfrac{1}{2}(1+\gamma^5)\right]\mathrm{e} \\ &= -m_e\bar{\mathrm{e}}\left[\tfrac{1}{2}(1-\gamma^5)\mathrm{e}_L + \tfrac{1}{2}(1+\gamma^5)\mathrm{e}_R\right] \\ &= -m_e(\bar{\mathrm{e}}_R\mathrm{e}_L + \bar{\mathrm{e}}_L\mathrm{e}_R). \end{aligned} \tag{17.16}$$

In the $SU(2)_L$ gauge transformation of the weak interaction, left-handed particles transform as weak isospin doublets and right-handed particles as singlets, and therefore the mass term of (17.16) breaks the required gauge invariance.

17.5 The Higgs mechanism

In the Standard Model, particles acquire masses through their interactions with the Higgs field. In this section, the Higgs mechanism is developed in three distinct stages. First it is shown how mass terms for a scalar field can arise from a broken symmetry. This mechanism is then extended to show how the mass of a gauge boson can be generated from a broken U(1) local gauge symmetry. Finally, the full Higgs mechanism is developed by breaking the $SU(2)_L \times U(1)_Y$ local gauge symmetry of the electroweak sector of the Standard Model.

17.5.1 Interacting scalar fields

A Lagrangian consists of two parts, a kinetic term involving the derivatives of the fields and a potential term expressed in terms of the fields themselves. For example, in the Lagrangian of QED (17.15), the kinetic terms for the electron and photon are

$$i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi \quad \text{and} \quad -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

The potential term, which represents the interactions between the electron and photon fields, is

$$e\overline{\psi}\gamma^{\mu}\psi A_{\mu}.$$

This can be associated with the normal three-point interaction vertex of QED, shown on the left of Figure 17.4. In general, the nature of the interactions between the fields and the strength of the coupling is determined by the terms in the Lagrangian involving the combinations of the fields, here $\overline{\psi}\psi A$.

Now, consider a scalar field ϕ with the potential

$$V(\phi) = \tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^4. \tag{17.17}$$

The corresponding Lagrangian is given by

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) - V(\phi)\\ &= \tfrac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) - \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4.\end{aligned} \tag{17.18}$$

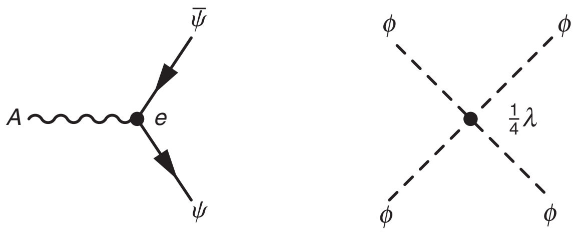


Fig. 17.4 The three-point interaction of QED and the four-point interaction for a scalar field with the potential $\lambda\phi^4/4$.

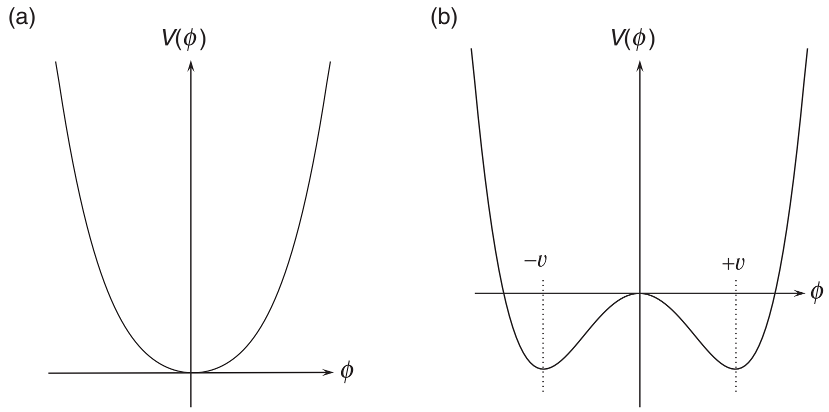


Fig. 17.5 The one-dimensional potential $V(\phi) = \mu^2\phi^2/2 + \lambda\phi^4/4$ for $\lambda > 0$ and the cases where (a) $\mu^2 > 0$ and (b) $\mu^2 < 0$.

The term proportional to $(\partial_\mu\phi)(\partial^\mu\phi)$ can be associated with the kinetic energy of the scalar particle and $\frac{1}{2}\mu^2\phi^2$ can represent the mass of the particle. The ϕ^4 term can be identified as self-interactions of the scalar field, corresponding to the four-point interaction vertex shown in the right-hand plot of Figure 17.4.

The vacuum state is the lowest energy state of the field ϕ and corresponds to the minimum of the potential of (17.17). For the potential to have a finite minimum, λ must be positive. If μ^2 is also chosen to be positive, the resulting potential, shown in Figure 17.5a, has a minimum at $\phi = 0$. In this case, the vacuum state corresponds to the field ϕ being zero and the Lagrangian of (17.18) represents a scalar particle with mass μ and a four-point self-interaction term proportional to ϕ^4. However, whilst λ must be greater than zero for there to be a finite minimum, there is no such restriction for μ^2. If $\mu^2 < 0$, the associated term in the Lagrangian can no longer be interpreted as a mass and the potential of (17.17) has minima at

$$\phi = \pm v = \pm\left|\sqrt{\frac{-\mu^2}{\lambda}}\right|,$$

as shown in Figure 17.5b. For $\mu^2 < 0$, the lowest energy state does not occur at $\phi = 0$ and the field is said to have a non-zero vacuum expectation value v. Since the potential is symmetric, there are two degenerate possible vacuum states. The actual vacuum state of the field either will be $\phi = +v$ or $\phi = -v$. The choice of the vacuum state breaks the symmetry of the Lagrangian, a process known as *spontaneous symmetry breaking*. A familiar example of spontaneous symmetry breaking is a ferromagnet with magnetisation $\mathbf{M}$. The Lagrangian (or Hamiltonian) depends on $\mathbf{M}^2$ and has no preferred direction. However, below the Curie temperature, the spins will be aligned in a particular direction, spontaneously breaking the underlying rotational symmetry of the Lagrangian.

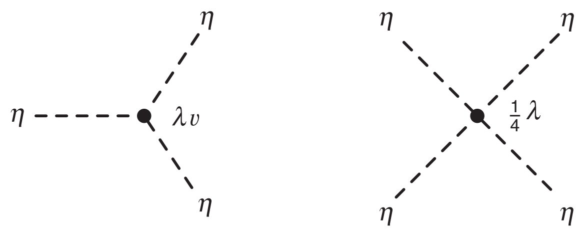


Fig. 17.6 The self-interactions of the field η that lead to Feynman diagrams for the processes $\eta \to \eta\eta$ and $\eta\eta \to \eta\eta$.

If the vacuum state of the scalar field is chosen to be at $\phi = +v$, the excitations of the field, which describe the particle states, can be obtained by considering perturbations of the field ϕ around the vacuum state by writing $\phi(x) = v + \eta(x)$. Since the vacuum expectation value v is a constant, $\partial_\mu \phi = \partial_\mu \eta$ and the Lagrangian of (17.18), expressed in terms of the field η, is

$$\begin{aligned}\mathcal{L}(\eta) &= \tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - V(\eta)\\ &= \tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \tfrac{1}{2}\mu^2(v+\eta)^2 - \tfrac{1}{4}\lambda(v+\eta)^4.\end{aligned}$$

Since the minimum of the potential is given by $\mu^2 = -\lambda v^2$, this expression can be written as

$$\mathcal{L}(\eta) = \tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2 - \lambda v \eta^3 - \tfrac{1}{4}\lambda \eta^4 + \tfrac{1}{4}\lambda v^4. \qquad (17.19)$$

From the comparison with the Lagrangian for a free scalar field of (17.5), it can be seen that the term proportional to η^2 can be interpreted as a mass

$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2},$$

and therefore the Lagrangian of (17.19) describes a massive scalar field. The terms proportional to η^3 and η^4 can be identified as triple and quartic interaction terms, as indicated in Figure 17.6. Finally, the term $\lambda v^4/4$ is just a constant, and has no physical consequences. Hence after spontaneous symmetry breaking, and having expanded the field about the vacuum state, the Lagrangian can be written as

$$\mathcal{L}(\eta) = \tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \tfrac{1}{2}m_\eta^2 \eta^2 - V(\eta), \quad \text{with} \quad V(\eta) = \lambda v \eta^3 + \tfrac{1}{4}\lambda \eta^4. \qquad (17.20)$$

It is important to realise that the Lagrangian of (17.20) is the same as the original Lagrangian of (17.18), but is now expressed as excitations about the minimum at $\phi = +v$. In principle, the same physical predictions can be obtained by using either form. However, in order to use perturbation theory, it is necessary to express the fields as small perturbations about the vacuum state.

17.5.2 Symmetry breaking for a complex scalar field

The idea of spontaneous symmetry breaking, introduced above in the context of a real scalar field, can be applied to the *complex* scalar field,

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2),$$

for which the corresponding Lagrangian is

$$\mathcal{L} = (\partial_\mu\phi)^*(\partial^\mu\phi) - V(\phi) \quad \text{with} \quad V(\phi) = \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2. \tag{17.21}$$

When expressed in terms of the two (real) scalar fields ϕ_1 and ϕ_2 this is just

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi_1)(\partial^\mu\phi_1) + \tfrac{1}{2}(\partial_\mu\phi_2)(\partial^\mu\phi_2) - \tfrac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \tfrac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2. \tag{17.22}$$

As before, for the potential to have a finite minimum, $\lambda > 0$. The Lagrangian of (17.21) is invariant under the transformation $\phi \to \phi' = e^{i\alpha}\phi$, because $\phi'^*\phi' = \phi^*\phi$, and therefore possesses a *global* U(1) symmetry. The shape of the potential depends on the sign of μ^2, as shown in Figure 17.7. When $\mu^2 > 0$, the minimum of the potential occurs when both fields are zero. If $\mu^2 < 0$, the potential has an infinite set of minima defined by

$$\phi_1^2 + \phi_2^2 = \frac{-\mu^2}{\lambda} = v^2,$$

as indicated by the dashed circle in Figure 17.7. The physical vacuum state will correspond to a particular point on this circle, breaking the *global* U(1) symmetry of the Lagrangian. Without loss of generality, the vacuum state can be chosen to

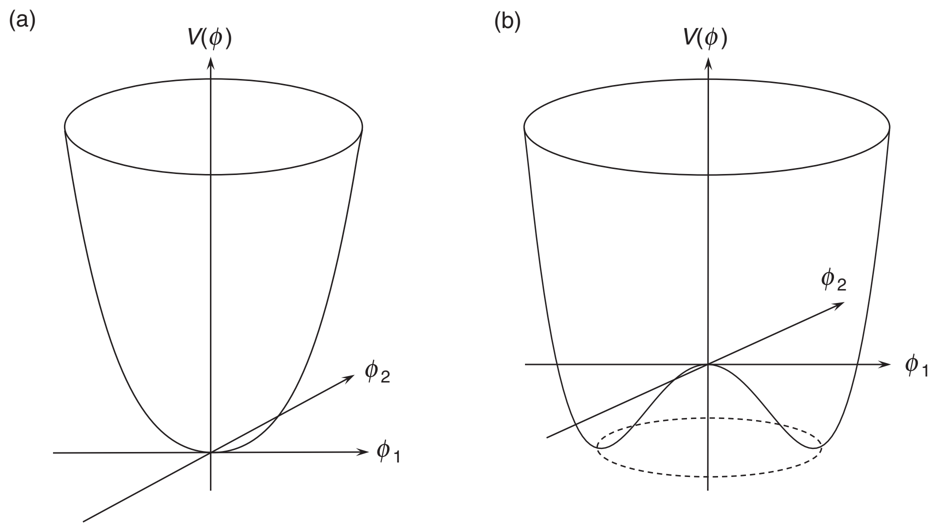


Fig. 17.7 The $V(\phi) = \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2$ potential for a complex scalar field for (a) $\mu^2 > 0$ and (b) $\mu^2 < 0$.

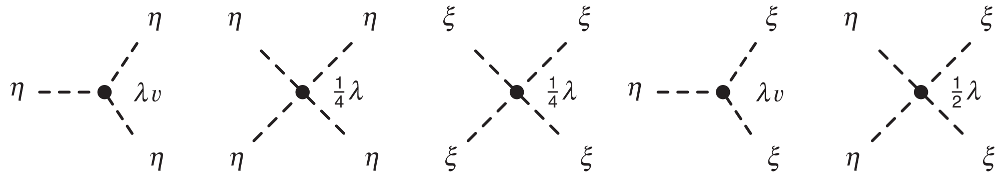


Fig. 17.8 The scalar interactions obtained by breaking the symmetry for a complex scalar field.

be in the real direction, $(\phi_1, \phi_2) = (v, 0)$, and the complex scaler field ϕ can be expanded about the vacuum state by writing $\phi_1(x) = \eta(x) + v$ and $\phi_2(x) = \xi(x)$,

$$\phi = \tfrac{1}{\sqrt{2}}(\eta + v + i\xi).$$

The Lagrangian of (17.22), written in terms of the fields η and ξ, is

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) - V(\eta, \xi),$$

where the potential $V(\eta, \xi)$ is given by

$$V(\eta, \xi) = \mu^2\phi^2 + \lambda\phi^4 \quad \text{with} \quad \phi^2 = \phi\phi^* = \tfrac{1}{2}\left[(v + \eta)^2 + \xi^2\right].$$

The potential can be written in terms of the fields η and ξ using $\mu^2 = -\lambda v^2$,

$$\begin{aligned} V(\eta, \xi) &= \mu^2\phi^2 + \lambda\phi^4 \\ &= -\tfrac{1}{2}\lambda v^2\left\{(v + \eta)^2 + \xi^2\right\} + \tfrac{1}{4}\lambda\left\{(v + \eta)^2 + \xi^2\right\}^2 \\ &= -\tfrac{1}{4}\lambda v^4 + \lambda v^2\eta^2 + \lambda v\eta^3 + \tfrac{1}{4}\lambda\eta^4 + \tfrac{1}{4}\lambda\xi^4 + \lambda v\eta\xi^2 + \tfrac{1}{2}\lambda\eta^2\xi^2. \end{aligned}$$

The term which is quadratic in the field η can be identified as a mass, and the terms with either three or four powers of the fields can be identified as interaction terms. Thus the Lagrangian can be written as

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \tfrac{1}{2}m_\eta^2\eta^2 + \tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) - V_{int}(\eta, \xi), \tag{17.23}$$

with $m_\eta = \sqrt{2\lambda v^2}$ and interactions given by

$$V_{int}(\eta, \xi) = \lambda v\eta^3 + \tfrac{1}{4}\lambda\eta^4 + \tfrac{1}{4}\lambda\xi^4 + \lambda v\eta\xi^2 + \tfrac{1}{2}\lambda\eta^2\xi^2. \tag{17.24}$$

These interaction terms correspond to triple and quartic couplings of the fields η and ξ, as shown in Figure 17.8.

The Lagrangian of (17.23) represents a scalar field η with mass $m_\eta = \sqrt{2\lambda v^2}$ and a *massless* scalar field ξ. The excitations of the massive field η are in the direction where the potential is (to first order) quadratic. In contrast, the particles described by the massless scalar field ξ correspond to excitations in the direction where the potential does not change, as indicated in Figure 17.9. This massless scalar particle is known as a Goldstone boson.

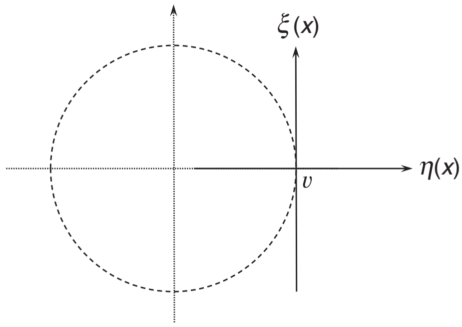


Fig. 17.9 The fields $\eta(x)$ and $\xi(x)$ in terms of the vacuum expectation value at $\phi = (v, 0)$.

17.5.3 The Higgs mechanism

In the Higgs mechanism, the spontaneous symmetry breaking of a complex scalar field with a potential,

$$V(\phi) = \mu^2\phi^2 + \lambda\phi^4, \tag{17.25}$$

is embedded in a theory with a *local* gauge symmetry. In this section, the example of a U(1) local gauge symmetry is used to introduce the main ideas.

Because of the presence of the derivatives in (17.21), the Lagrangian for a complex scalar field ϕ is *not* invariant under the U(1) *local* gauge transformation

$$\phi(x) \to \phi'(x) = e^{ig\chi(x)}\phi(x). \tag{17.26}$$

The required U(1) local gauge invariance can be achieved by replacing the derivatives in the Lagrangian with the corresponding covariant derivatives

$$\partial_\mu \to D_\mu = \partial_\mu + igB_\mu.$$

The resulting Lagrangian,

$$\mathcal{L} = (D_\mu\phi)^*(D^\mu\phi) - V(\phi^2),$$

is gauge invariant (see Problem 17.7) provided the new gauge field B_μ, which appears in the covariant derivative, transforms as

$$B_\mu \to B'_\mu = B_\mu - \partial_\mu\chi(x). \tag{17.27}$$

Just as was the case for Dirac Lagrangian (see Section 17.3), the required local gauge invariance implies the existence a new gauge field with well-defined gauge transformation properties. The combined Lagrangian for the complex scalar field ϕ and the gauge field B is

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - \mu^2\phi^2 - \lambda\phi^4, \tag{17.28}$$

where $F^{\mu\nu}F_{\mu\nu}$ is the kinetic term for the new field with

$$F^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu.$$

The gauge field B is required to be massless, since the mass term $\frac{1}{2}m_B B_\mu B^\mu$ would break the gauge invariance. The term involving the covariant derivatives, when written out in full, is

$$\begin{aligned}(D_\mu\phi)^*(D^\mu\phi) &= (\partial_\mu - igB_\mu)\phi^*(\partial^\mu + igB^\mu)\phi \\ &= (\partial_\mu\phi)^*(\partial^\mu\phi) - igB_\mu\phi^*(\partial^\mu\phi) + ig(\partial_\mu\phi^*)B^\mu\phi + g^2 B_\mu B^\mu\phi^*\phi\end{aligned}$$

and the full expression for the Lagrangian is

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + (\partial_\mu\phi)^*(\partial^\mu\phi) - \mu^2\phi^2 - \lambda\phi^4 \\ &- igB_\mu\phi^*(\partial^\mu\phi) + ig(\partial_\mu\phi^*)B^\mu\phi + g^2 B_\mu B^\mu\phi^*\phi. \end{aligned} \tag{17.29}$$

For the case where the potential for the scalar field of (17.25) has $\mu^2 < 0$, the vacuum state is degenerate and the choice of the physical vacuum state spontaneously breaks the symmetry of the Lagrangian of (17.29). As before, the physical vacuum state is chosen to be $\phi_1 + i\phi_2 = v$, and the complex scalar field ϕ is expanded about the vacuum state by writing

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x)). \tag{17.30}$$

Substituting (17.30) into (17.29) leads to (see Problem 17.6)

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2}_{\text{massive } \eta} + \underbrace{\tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)}_{\text{massless } \xi} - \underbrace{\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2v^2B_\mu B^\mu}_{\text{massive gauge field}} - V_{int} + gvB_\mu(\partial^\mu\xi), \tag{17.31}$$

where $V_{int}(\eta, \xi, B)$ contains the three- and four-point interaction terms of the fields η, ξ and B. As before, the breaking of the symmetry of the Lagrangian produces a massive scalar field η and a massless Goldstone boson ξ. In addition, the previously massless gauge field B has acquired a mass term $\frac{1}{2}g^2v^2B_\mu B^\mu$, achieving the aim of giving a mass to the gauge boson of the local gauge symmetry. Again it should be emphasised that this is exactly the same Lagrangian as (17.28), but with the complex scalar field expanded about the vacuum state at $\phi_1 + i\phi_2 = v$; by expanding the scalar fields about the vacuum where the fields have a non-zero vacuum expectation value, the underlying gauge symmetry of the Lagrangian has been hidden, but has not been removed.

However, there appear to be two problems with (17.31). The original Lagrangian contained four degrees of freedom, one for each of the scalar fields ϕ_1 and ϕ_2, and the two transverse polarisation states for the massless gauge field B. In the Lagrangian of (17.31), the gauge boson has become massive and therefore has the additional longitudinal polarisation state; somehow in the process of spontaneous symmetry breaking an additional degree of freedom appears to have been

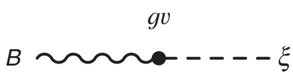


Fig. 17.10 The coupling between the gauge field B and the Goldstone field ξ.

introduced. Furthermore, the $gvB_\mu(\partial^\mu\xi)$ term appears to represent a direct coupling between the Goldstone field ξ and the gauge field B. It would appear that the spin-1 gauge field can transform into a spin-0 scalar field, as indicated in Figure 17.10. This term is somewhat reminiscent of the off-diagonal mass term encountered in the discussion of the neutral kaon system, which coupled the K^0 and $\overline{K}^0$ flavour states, suggesting that the fields appearing in (17.31) are not the physical fields. This coupling to the Goldstone field ultimately will be associated with the longitudinal polarisation state of the massive gauge boson.

The Goldstone field ξ in (17.31) can be eliminated from the Lagrangian by making the appropriate gauge transformation. By writing

$$\tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) + gvB_\mu(\partial^\mu\xi) + \tfrac{1}{2}g^2v^2B_\mu B^\mu = \tfrac{1}{2}g^2v^2\left[B_\mu + \frac{1}{gv}(\partial_\mu\xi)\right]^2,$$

and making the gauge transformation,

$$B_\mu(x) \to B'_\mu(x) = B_\mu(x) + \frac{1}{gv}\partial_\mu\xi(x), \tag{17.32}$$

the Lagrangian of (17.31) becomes

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial^\mu\eta)(\partial_\mu\eta) - \lambda v^2\eta^2}_{\text{massive }\eta} + \underbrace{-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2v^2B^{\mu\prime}B'_\mu}_{\text{massive gauge field}} - V_{int}.$$

Since the original Lagrangian was constructed to be invariant under local U(1) gauge transformations, the physical predications of the theory are unchanged by the gauge transformation of (17.32).

Thus, with the appropriate choice of gauge, the Goldstone field ξ no longer appears in the Lagrangian. This choice of gauge corresponds to taking $\chi(x) = -\xi(x)/gv$ in (17.27). The corresponding gauge transformation of the original complex scalar field $\phi(x)$ is therefore

$$\phi(x) \to \phi'(x) = e^{-ig\frac{\xi(x)}{gv}}\phi(x) = e^{-i\xi(x)/v}\phi(x). \tag{17.33}$$

After symmetry breaking, the complex scalar field was expanded about the physical vacuum by writing $\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$, which to first order in the fields can be expressed as

$$\phi(x) \approx \tfrac{1}{\sqrt{2}}\left[v + \eta(x)\right]e^{i\xi(x)/v}.$$

The effect of the gauge transformation of (17.33) on the original complex scalar field is therefore

$$\phi(x) \to \phi'(x) = \frac{1}{\sqrt{2}} e^{-i\xi(x)/v} \left[v + \eta(x)\right] e^{i\xi(x)/v} = \frac{1}{\sqrt{2}}(v + \eta(x)).$$

Hence, the gauge in which the Goldstone field $\xi(x)$ is eliminated from the Lagrangian, which is known as the *Unitary gauge*, corresponds to choosing the complex scalar field $\phi(x)$ to be entirely real,

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x)) \equiv \frac{1}{\sqrt{2}}(v + h(x)).$$

Here the field $\eta(x)$ has been written as the Higgs field $h(x)$ to emphasise that this is the physical field in the unitary gauge. It is important to remember that the physical predictions of the theory do not depend on the choice of gauge, but in the unitary gauge the fields appearing in the Lagrangian correspond to the physical particles; there are no "mixing" terms like $B_\mu(\partial^\mu \xi)$. The degree of freedom corresponding to the Goldstone field $\xi(x)$ no longer appears in the Lagrangian; it has been replaced by the degree of freedom corresponding to the longitudinal polarisation state of the now massive gauge field B. Sometimes it is said that the Goldstone boson has been "eaten" by the gauge field. Writing $\mu^2 = -\lambda v^2$, and working in the unitary gauge where $\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))$, the Lagrangian of (17.28) can be written

$$\begin{aligned}
\mathcal{L} &= (D_\mu \phi)^*(D^\mu \phi) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \mu^2\phi^2 - \lambda\phi^4 \\
&= \tfrac{1}{2}(\partial_\mu - igB_\mu)(v + h)(\partial^\mu + igB^\mu)(v + h) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}\mu^2(v + h)^2 - \tfrac{1}{4}\lambda(v + h)^4 \\
&= \tfrac{1}{2}(\partial_\mu h)(\partial^\mu h) + \tfrac{1}{2}g^2 B_\mu B^\mu (v + h)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \lambda v^2 h^2 - \lambda v h^3 - \tfrac{1}{4}\lambda h^4 + \tfrac{1}{4}\lambda v^4.
\end{aligned}$$

Gathering up the terms (and ignoring the $\lambda v^4/4$ constant) gives

$$\begin{aligned}
\mathcal{L} = &\underbrace{\tfrac{1}{2}(\partial_\mu h)(\partial^\mu h) - \lambda v^2 h^2}_{\text{massive } h \text{ scalar}} \underbrace{- \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2 v^2 B_\mu B^\mu}_{\text{massive gauge boson}} \\
&+ \underbrace{g^2 v B_\mu B^\mu h + \tfrac{1}{2}g^2 B_\mu B^\mu h^2}_{h,B \text{ interactions}} - \underbrace{\lambda v h^3 - \tfrac{1}{4}\lambda h^4}_{h \text{ self-interactions}} .
\end{aligned} \tag{17.34}$$

This Lagrangian describes a massive scalar Higgs field h and a massive gauge boson B associated with the U(1) local gauge symmetry. It contains interaction terms between the Higgs boson and the gauge boson, and Higgs boson self-interaction terms, indicated in Figure 17.11. The mass of the gauge boson,

$$m_B = g\,v,$$

is related to the strength of the gauge coupling and the vacuum expectation value of the Higgs field. The mass of the Higgs boson is given by

$$m_{\rm H} = \sqrt{2\lambda}\,v.$$

It should be noted that the vacuum expectation value v sets the scale for the masses of both the gauge boson and the Higgs boson.

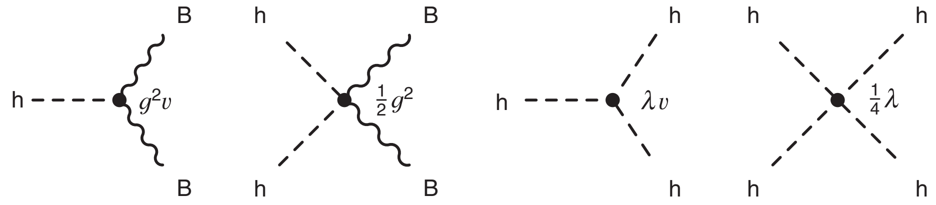


Fig. 17.11 The interaction terms arising from the Higgs mechanism for a U(1) local gauge theory.

17.5.4 The Standard Model Higgs

In the above example, the Higgs mechanism was used to generate a mass for the gauge boson corresponding to a U(1) local gauge symmetry. In the Salam–Weinberg model, the Higgs mechanism is embedded in the $U(1)_Y \times SU(2)_L$ local gauge symmetry of the electroweak sector of the Standard Model. Three Goldstone bosons will be required to provide the longitudinal degrees of freedom of the W^+, W^- and Z bosons. In addition, after symmetry breaking, there will be (at least) one massive scalar particle corresponding to the field excitations in the direction picked out by the choice of the physical vacuum. The simplest Higgs model, which has the necessary four degrees of freedom, consists of two complex scalar fields.

Because the Higgs mechanism is required to generate the masses of the electroweak gauge bosons, one of the scalar fields must be neutral, written as ϕ^0, and the other must be charged such that ϕ^+ and $(\phi^+)^* = \phi^-$ give the longitudinal degrees of freedom of the W^+ and W^-. The minimal Higgs model consists of two complex scalar fields, placed in a weak isospin doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}. \tag{17.35}$$

As usual, the upper and lower components of the doublet differ by one unit of charge. The Lagrangian for this doublet of complex scalar fields is

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - V(\phi), \tag{17.36}$$

with the Higgs potential,

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2.$$

For $\mu^2 < 0$, the potential has an infinite set of degenerate minima satisfying

$$\phi^\dagger \phi = \tfrac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \frac{v^2}{2} = -\frac{\mu^2}{2\lambda}.$$

After symmetry breaking, the neutral photon is required to remain massless, and therefore the minimum of the potential must correspond to a non-zero vacuum expectation value only of the neutral scalar field ϕ^0,

$$\langle 0|\phi|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}.$$

The fields then can be expanded about this minimum by writing

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ v + \eta(x) + i\phi_4(x) \end{pmatrix}.$$

After the spontaneous breaking of the symmetry, there will be a massive scalar and three massless Goldstone bosons, which will ultimately give the longitudinal degrees of freedom of the $W^{\pm}$ and Z bosons. Rather than repeating the derivation given in Section 17.5.3 and “gauging-away” the Goldstone fields, here the Higgs doublet is immediately written in the unitary gauge,

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}.$$

The resulting Lagrangian is known as the Salam–Weinberg model. All that remains is to identify the masses of gauge bosons and the interaction terms.

The mass terms can be identified by writing the Lagrangian of (17.36) such that it respects the $SU(2)_L \times U(1)_Y$ local gauge symmetry of the electroweak model by replacing the derivatives with the appropriate covariant derivatives (discussed further in Appendix F),

$$\partial_\mu \to D_\mu = \partial_\mu + ig_W \mathbf{T}\cdot\mathbf{W}_\mu + ig'\frac{Y}{2}B_\mu, \tag{17.37}$$

where $\mathbf{T} = \frac{1}{2}\boldsymbol{\sigma}$ are the three generators of the SU(2) symmetry. In Chapter 15, the weak hypercharge of the Glashow–Salam–Weinberg (GSW) model was identified as $Y = 2\left(Q - I_W^{(3)}\right)$. Here, the lower component of the Higgs doublet is neutral and has $I_W^{(3)} = -\frac{1}{2}$, and thus the Higgs doublet has hypercharge $Y = 1$. Hence, the effect of the covariant derivative of (17.37) acting on the Higgs doublet ϕ is

$$D_\mu\phi = \tfrac{1}{2}\left[2\partial_\mu + \left(ig_W\boldsymbol{\sigma}\cdot\mathbf{W}_\mu + ig'B_\mu\right)\right]\phi,$$

where D_μ is a 2×2 matrix acting on the two component weak isospin doublet and the identity matrix multiplying the ∂_μ and B_μ terms is implicit in this expression.

The term in the Lagrangian that generates the masses of the gauge bosons is $(D_\mu\phi)^\dagger(D^\mu\phi)$. In the Unitary gauge $D_\mu\phi$ is given by

$$\begin{aligned} D_\mu\phi &= \frac{1}{2\sqrt{2}}\begin{pmatrix} 2\partial_\mu + ig_W W_\mu^{(3)} + ig'B_\mu & ig_W[W_\mu^{(1)} - iW_\mu^{(2)}] \\ ig_W[W_\mu^{(1)} + iW_\mu^{(2)}] & 2\partial_\mu - ig_W W_\mu^{(3)} + ig'B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v + h \end{pmatrix} \\ &= \frac{1}{2\sqrt{2}}\begin{pmatrix} ig_W(W_\mu^{(1)} - iW_\mu^{(2)})(v+h) \\ (2\partial_\mu - ig_W W_\mu^{(3)} + ig'B_\mu)(v+h) \end{pmatrix}. \end{aligned}$$

Taking the Hermitian conjugate gives $(D_\mu\phi)^\dagger$, from which

$$(D_\mu\phi)^\dagger(D^\mu\phi) = \tfrac{1}{2}(\partial_\mu h)(\partial^\mu h) + \tfrac{1}{8}g_W^2(W_\mu^{(1)} + iW_\mu^{(2)})(W^{(1)\mu} - iW^{(2)\mu})(v+h)^2 \\ + \tfrac{1}{8}(g_W W_\mu^{(3)} - g' B_\mu)(g_W W^{(3)\mu} - g' B^\mu)(v+h)^2. \qquad (17.38)$$

The gauge bosons masses are determined by the terms in $(D_\mu\phi)^\dagger(D^\mu\phi)$ that are quadratic in the gauge boson fields, i.e.

$$\tfrac{1}{8}v^2 g_W^2\left(W_\mu^{(1)}W^{(1)\mu} + W_\mu^{(2)}W^{(2)\mu}\right) + \tfrac{1}{8}v^2\left(g_W W_\mu^{(3)} - g' B_\mu\right)\left(g_W W^{(3)\mu} - g' B^\mu\right).$$

In the Lagrangian, the mass terms for the $W^{(1)}$ and $W^{(2)}$ spin-1 fields will appear as

$$\tfrac{1}{2}m_W^2 W_\mu^{(1)}W^{(1)\mu} \quad \text{and} \quad \tfrac{1}{2}m_W^2 W_\mu^{(2)}W^{(2)\mu},$$

and therefore the mass of the W boson is

$$m_W = \tfrac{1}{2}g_W v. \qquad (17.39)$$

The mass of the W boson is therefore determined by the coupling constant of the $SU(2)_L$ gauge interaction g_W and the vacuum expectation value of the Higgs field.

The terms in the Lagrangian of (17.38) which are quadratic in the neutral $W^{(3)}$ and B fields can be written as

$$\tfrac{v^2}{8}\left(g_W W_\mu^{(3)} - g' B_\mu\right)\left(g_W W^{(3)\mu} - g' B^\mu\right) = \tfrac{v^2}{8}\begin{pmatrix} W_\mu^{(3)} & B_\mu \end{pmatrix}\begin{pmatrix} g_W^2 & -g_W g' \\ -g_W g' & g'^2 \end{pmatrix}\begin{pmatrix} W^{(3)\mu} \\ B^\mu \end{pmatrix} \\ = \tfrac{v^2}{8}\begin{pmatrix} W_\mu^{(3)} & B_\mu \end{pmatrix}\mathbf{M}\begin{pmatrix} W^{(3)\mu} \\ B^\mu \end{pmatrix}, \qquad (17.40)$$

where $\mathbf{M}$ is the *non-diagonal* mass matrix. The off-diagonal elements of $\mathbf{M}$ couple together the $W^{(3)}$ and B fields, allowing them to mix. Again this is reminiscent of the non-diagonal mass matrix encountered in the discussion of the neutral kaon system (see Section 14.4.3). The *physical* boson fields, which propagate as independent eigenstates of the free particle Hamiltonian, correspond to the basis in which the mass matrix is diagonal. The masses of the physical gauge bosons are given by the eigenvalues of $\mathbf{M}$, obtained from characteristic equation $\det(\mathbf{M} - \lambda I) = 0$,

$$(g_W^2 - \lambda)(g'^2 - \lambda) - g_W^2 g'^2 = 0,$$

giving

$$\lambda = 0 \quad \text{or} \quad \lambda = g_W^2 + g'^2. \qquad (17.41)$$

Hence, in the diagonal basis the mass matrix of (17.40) is

$$\frac{1}{8}v^2\begin{pmatrix} A_\mu & Z_\mu \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & g_W^2 + g'^2 \end{pmatrix}\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix},$$

where the A_μ and Z_μ are the physical fields corresponding to the eigenvectors of **M**. In the diagonal basis, the term in the Lagrangian representing the masses of the A and Z will be

$$\frac{1}{2}\begin{pmatrix} A_\mu & Z_\mu \end{pmatrix}\begin{pmatrix} m_A^2 & 0 \\ 0 & m_Z^2 \end{pmatrix}\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix},$$

from which the masses of the physical gauge bosons can be identified as

$$m_A = 0 \quad \text{and} \quad m_Z = \tfrac{1}{2}v\sqrt{g_W^2 + g'^2}. \tag{17.42}$$

Hence, in the physical basis there is a massless neutral gauge boson A which can be identified as the photon and a massive neutral gauge boson which can be identified as the Z. The physical fields, which correspond to the normalised eigenvectors of the mass matrix, are

$$A_\mu = \frac{g' W_\mu^{(3)} + g_W B_\mu}{\sqrt{g_W^2 + g'^2}} \quad \text{with} \quad m_A = 0, \tag{17.43}$$

$$Z_\mu = \frac{g_W W_\mu^{(3)} - g' B_\mu}{\sqrt{g_W^2 + g'^2}} \quad \text{with} \quad m_Z = \tfrac{1}{2}v\sqrt{g_W^2 + g'^2}. \tag{17.44}$$

Thus, the physical fields are mixtures of the massless bosons associated with the $U(1)_Y$ and $SU(2)_L$ local gauge symmetries. The combination corresponding to the Z boson, which is associated with the neutral Goldstone boson of the broken symmetry, has acquired mass through the Higgs mechanism and the field corresponding to the photon has remained massless. By writing the ratio of the couplings of the $U(1)_Y$ and $SU(2)_L$ gauge symmetries as

$$\frac{g'}{g_W} = \tan\theta_W, \tag{17.45}$$

the relationship between the physical fields and underlying fields of (17.43) and (17.44) can be written as

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^{(3)},$$
$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^{(3)},$$

which are exactly the relations that were asserted in Section 15.3. Furthermore, by using (17.45), the mass of Z boson in (17.42) can be expressed as

$$m_Z = \frac{1}{2}\frac{g_W}{\cos\theta_W}v.$$

Therefore, when combined with the corresponding expression for the W-boson mass given in (17.39), the Glashow–Salam–Weinberg model predicts

$$\frac{m_{\mathrm{W}}}{m_{\mathrm{Z}}} = \cos\theta_{\mathrm{W}}.$$

The experimental verification of this relation, described in Chapter 16, provides a compelling argument for the reality of the Higgs mechanism.

The GSW model is described by just four parameters, the $SU(2)_L \times U(1)_Y$ gauge couplings g_{W} and g', and the two free parameters of the Higgs potential μ and λ, which are related to the vacuum expectation value of the Higgs field v and the mass of the Higgs boson m_{H} by

$$v^2 = \frac{-\mu^2}{\lambda} \quad \text{and} \quad m_{\mathrm{H}}^2 = 2\lambda v^2.$$

By using the relation $m_{\mathrm{W}} = \frac{1}{2} g_{\mathrm{W}} v$ and the measured values for m_{W} and g_{W}, the vacuum expectation value of the Higgs field is found to be

$$v = 246\,\mathrm{GeV}.$$

The parameter λ in the Higgs potential can be obtained from the mass of the Higgs boson as measured at the LHC (see Section 17.7).

Couplings to the gauge bosons

In the $(D_\mu\phi)^\dagger(D^\mu\phi)$ term in the Lagrangian of (17.38), the gauge boson fields appear in the form of $VV(v+h)^2$, where $V = \mathrm{W}^\pm, \mathrm{Z}$. The VVv^2 terms determine the masses of the gauge bosons and the VVh and $VVhh$ terms give rise to triple and quartic couplings between one or two Higgs bosons and the gauge bosons. From (15.12), the W^+ and W^- fields are the linear combinations

$$\mathrm{W}^\pm = \frac{1}{\sqrt{2}}\left(\mathrm{W}^{(1)} \mp i\mathrm{W}^{(2)}\right).$$

Hence the second term on the RHS (17.38) can be written in terms of the physical W^+ and W^- fields,

$$\tfrac{1}{4} g_{\mathrm{W}}^2 W_\mu^- W^{+\mu}(v+h)^2 = \tfrac{1}{4} g_{\mathrm{W}}^2 v^2 W_\mu^- W^{+\mu} + \tfrac{1}{2} g_{\mathrm{W}}^2 v W_\mu^- W^{+\mu} h + \tfrac{1}{4} g_{\mathrm{W}}^2 W_\mu^- W^{+\mu} hh.$$

Here the first term gives the masses to the W^+ and W^-. The $\mathrm{hW}^+\mathrm{W}^-$ and $\mathrm{hhW}^+\mathrm{W}^-$ terms give rise to the triple and quartic couplings of the Higgs boson to the gauge bosons. The coupling strength at the $\mathrm{hW}^+\mathrm{W}^-$ vertex of Figure 17.12 is therefore

$$g_{\mathrm{HWW}} = \tfrac{1}{2} g_{\mathrm{W}}^2 v \equiv g_{\mathrm{W}} m_{\mathrm{W}}.$$

Hence the coupling of the Higgs boson to the W boson is proportional to the W-boson mass. Likewise, the coupling of the Higgs boson to the Z boson, $g_{\mathrm{HZZ}} = g_{\mathrm{Z}} m_{\mathrm{Z}}$, is proportional to m_{Z}.

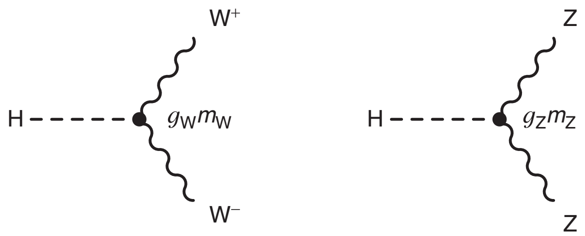


Fig. 17.12 The trilinear couplings of the Higgs boson to the W and Z, where $g_Z = g_W/\cos\theta_W$.

17.5.5 Fermion masses

The Higgs mechanism for the spontaneous symmetry breaking of the $U(1)_Y \times SU(2)_L$ gauge group of the Standard Model generates the masses of the W and Z bosons. Remarkably, it also can be used to generate the masses of the fermions. Because of the different transformation properties of left- and right-handed chiral states, the fermion mass term in the Dirac Lagrangian,

$$-m\overline{\psi}\psi = -m\left(\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R\right),$$

does not respect the $SU(2)_L \times U(1)_Y$ gauge symmetry, and therefore cannot be present in the Lagrangian of the Standard Model.

In the Standard Model, left-handed chiral fermions are placed in SU(2) doublets, here written L, and right-handed fermions are placed in SU(2) singlets, here denoted R. Because the two complex scalar fields of the Higgs mechanism are placed in an SU(2) doublet $\phi(x)$, an infinitesimal SU(2) local gauge transformation has the effect,

$$\phi \to \phi' = (I + ig_W\boldsymbol{\epsilon}(x)\cdot\mathbf{T})\phi.$$

Exactly the same local gauge transformation applies to the left-handed doublet of fermion fields L. Therefore, the effect of the infinitesimal SU(2) gauge transformation on $\overline{L} \equiv L^\dagger\gamma^0$ is

$$\overline{L} \to \overline{L}' = \overline{L}(I - ig_W\boldsymbol{\epsilon}(x)\cdot\mathbf{T}).$$

Consequently, the combination $\overline{L}\phi$ is invariant under the $SU(2)_L$ gauge transformations. When combined with a right-handed singlet, $\overline{L}\phi R$, it is invariant under $SU(2)_L$ *and* $U(1)_Y$ gauge transformations; as is its Hermitian conjugate $(\overline{L}\phi R)^\dagger = \overline{R}\phi^\dagger L$. Hence, a term in the Lagrangian of the form $-g_f(\overline{L}\phi R + \overline{R}\phi^\dagger L)$ satisfies the $SU(2)_L \times U(1)_Y$ gauge symmetry of the Standard Model. For the $SU(2)_L$ doublet containing the electron, this corresponds to

$$\mathcal{L}_e = -g_e\left[\begin{pmatrix}\overline{\nu}_e & \overline{e}\end{pmatrix}_L \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} e_R + \overline{e}_R \begin{pmatrix}\phi^{+*} & \phi^{0*}\end{pmatrix}\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L\right], \tag{17.46}$$

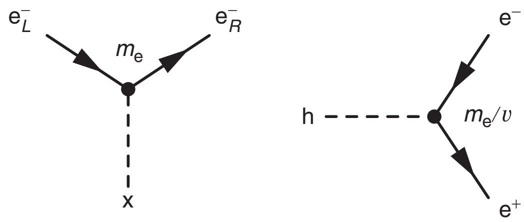


Fig. 17.13 Left: the interaction between a massless chiral electron and the non-zero expectation value of the Higgs field. Right: the interaction vertex for the coupling of the Higgs boson to an electron.

where g_e is a constant known as the Yukawa coupling of the electron to the Higgs field. After spontaneously symmetry breaking, the Higgs doublet in the unitary gauge is

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix},$$

and thus (17.46) becomes

$$\mathcal{L}_e = -\frac{g_e}{\sqrt{2}} v\,(\overline{e}_L e_R + \overline{e}_R e_L) - \frac{g_e}{\sqrt{2}} h\,(\overline{e}_L e_R + \overline{e}_R e_L)\,. \tag{17.47}$$

The first term in (17.47) has exactly the form required for the fermion masses, but has now been introduced in gauge invariant manner. The Yukawa coupling g_e is not predicted by the Higgs mechanism, but can be chosen to be consistent with the observed electron mass,

$$g_e = \sqrt{2}\,\frac{m_e}{v}.$$

In this case, (17.47) becomes

$$\mathcal{L}_e = -m_e \overline{e}e - \frac{m_e}{v}\overline{e}eh. \tag{17.48}$$

The first term in (17.48), which gives the mass of the electron, represents the coupling of electron to the Higgs *field* through its non-zero vacuum expectation value. The second term in (17.48) gives rise to a coupling between the electron and the Higgs *boson* itself. These two terms are illustrated in Figure 17.13, where the fermion masses arise from the coupling of left-handed and right-handed massless chiral fermions though the interaction with the non-zero expectation value of the Higgs field.

Because the non-zero vacuum expectation value occurs in the lower (neutral) component of the Higgs doublet, the combination of fields $\overline{L}\phi R + \overline{R}\phi^\dagger L$ only can generate the masses for the fermion in the lower component of an $SU(2)_L$ doublet. Thus it can be used to generate the masses of the charged leptons and the down-type quarks, but not the up-type quarks or the neutrinos. Putting aside the question of neutrino masses, a mechanism is required to give masses to the up-type quarks.

This can be achieved by constructing the conjugate doublet ϕ_c formed from the four fields in (17.35),

$$\phi_c = -i\sigma_2\phi^* = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} = \tfrac{1}{\sqrt{2}}\begin{pmatrix} -\phi_3 + i\phi_4 \\ \phi_1 - i\phi_2 \end{pmatrix}.$$

Because of the particular properties of SU(2), see Section 9.5, the conjugate doublet ϕ_c transforms in exactly the same way as the doublet ϕ. This is analogous to the representation of up- and down-quarks and anti-up and anti-down in SU(2) isospin symmetry. A gauge invariant mass term for the up-type quarks can be constructed from $\overline{L}\phi_c R + \overline{R}\phi_c^\dagger L$, for example

$$\mathcal{L}_\mathrm{u} = g_\mathrm{u}\begin{pmatrix} \overline{\mathrm{u}} & \overline{\mathrm{d}} \end{pmatrix}_L \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} \mathrm{u}_R + \text{ Hermitian conjugate,}$$

which after symmetry breaking becomes

$$\mathcal{L}_\mathrm{u} = -\tfrac{g_\mathrm{u}}{\sqrt{2}} v\left(\overline{\mathrm{u}}_L\mathrm{u}_R + \overline{\mathrm{u}}_R\mathrm{u}_L\right) - \tfrac{g_\mathrm{u}}{\sqrt{2}} h\left(\overline{\mathrm{u}}_L\mathrm{u}_R + \overline{\mathrm{u}}_R\mathrm{u}_L\right),$$

with the Yukawa coupling $g_\mathrm{u} = \sqrt{2}m_\mathrm{u}/v$, giving

$$\mathcal{L}_\mathrm{u} = -m_\mathrm{u}\overline{\mathrm{u}}\mathrm{u} - \frac{m_\mathrm{u}}{v}\overline{\mathrm{u}}\mathrm{u}h.$$

Hence for all *Dirac* fermions, gauge invariant mass terms can be constructed from either

$$\mathcal{L} = -g_\mathrm{f}\left[\overline{L}\phi R + (\overline{L}\phi R)^\dagger\right] \quad \text{or} \quad \mathcal{L} = g_\mathrm{f}\left[\overline{L}\phi_c R + (\overline{L}\phi_c R)^\dagger\right],$$

giving rise to both the masses of the fermions and the interactions between the Higgs boson and the fermion. The Yukawa couplings of the fermions to the Higgs field are given by

$$g_\mathrm{f} = \sqrt{2}\frac{m_\mathrm{f}}{v},$$

where the vacuum expectation value of the Higgs field is $v = 246\,\mathrm{GeV}$. Interestingly, for the top quark with $m_\mathrm{t} \sim 173.5 \pm 1.0\,\mathrm{GeV}$, the Yukawa coupling is almost exactly unity. Whilst this may be a coincidence, it is perhaps natural that the Yukawa couplings of the fermions are $\mathcal{O}(1)$. If the neutrino masses are also associated with the Higgs mechanism, it is perhaps surprising that they are so small, with Yukawa couplings of $\lesssim 10^{-12}$. This might suggest that the mechanism which generates the neutrino masses differs from that for other fermions. One interesting possibility is the seesaw mechanism described in the addendum to this chapter.

17.6 Properties of the Higgs boson

The Standard Model Higgs boson H is a neutral scalar particle. Its mass is a free parameter of the Standard Model that is given by $m_H = 2\lambda v^2$. The Higgs boson couples to all fermions with a coupling strength proportional to the fermion mass. From (17.48), the Feynman rule for the interaction vertex with a fermion of mass m_f can be identified as

$$-i\frac{m_f}{v} \equiv -i\frac{m_f}{2m_W}g_W. \tag{17.49}$$

The Higgs boson therefore can decay via $H \to f\bar{f}$ for all kinematically allowed decays modes with $m_H > 2m_f$. If it is sufficiently massive, the Higgs boson can also decay via $H \to W^+W^-$ or $H \to ZZ$. The Feynman diagrams and coupling strengths for these lowest-order decay modes are shown in Figure 17.14. In each case, the resulting matrix element is proportional to the mass of the particle coupling to the Higgs boson. The proportionality of the Higgs boson couplings to mass determines the dominant processes through which it is produced and decays; the Higgs boson couples preferentially to the most massive particles that are kinematically accessible.

17.6.1 Higgs decay

In principle, the Higgs boson can decay to all Standard Model particles. However, because of the proportionality of the coupling to the mass of the particles involved, the largest branching ratios are to the more massive particles. For a Higgs boson mass of 125 GeV, the largest branching ratio is to bottom quarks, $BR(H \to b\bar{b}) = 57.8\%$. The corresponding partial decay width $\Gamma(H \to b\bar{b})$ can be calculated from the Feynman rule for the $Hb\bar{b}$ interaction vertex of (17.49) and the spinors for the quark and antiquark. Because the Higgs boson is a scalar particle, no polarisation four-vector is required; it is simply described by a plane wave. Consequently, the matrix element for the Feynman diagram shown in Figure 17.15 is

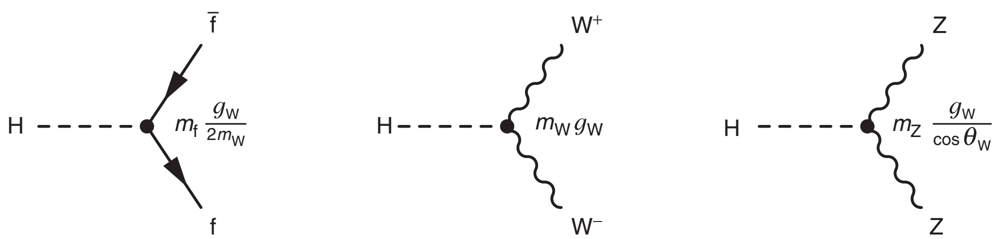


Fig. 17.14 Three lowest-order Feynman diagrams for Higgs decay.

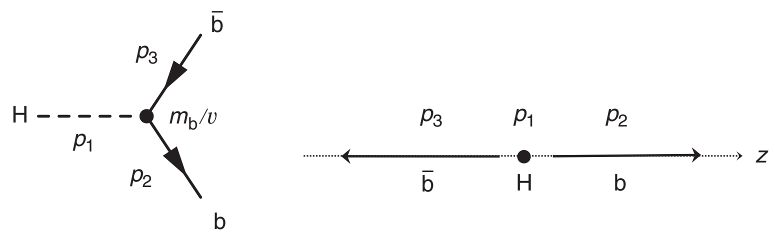


Fig. 17.15 The Feynman diagram for $h \longrightarrow b\bar{b}$ and the four-momenta of the particles in the Higgs rest frame.

$$\mathcal{M} = \frac{m_b}{v}\bar{u}(p_2)v(p_3) = \frac{m_b}{v}u^\dagger\gamma^0 v. \tag{17.50}$$

Without loss of generality, the b-quark momentum can be taken to be in the z-axis. Because $m_H \gg m_b$, the final-state quarks are highly relativistic and therefore have four-momenta $p_2 \approx (E, 0, 0, E)$ and $p_3 \approx (E, 0, 0, -E)$, where $E = m_H/2$. In the ultra-relativistic limit, the spinors for the two possible helicity states for each of the b-quark ($\theta = 0, \phi = 0$) and the $\bar{b}$-antiquark ($\theta = \pi, \phi = \pi$) are

$$u_\uparrow(p_2) = \sqrt{E}\begin{pmatrix}1\\0\\1\\0\end{pmatrix},\; u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix},\; v_\uparrow(p_3) = \sqrt{E}\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}\; v_\downarrow(p_4) = \sqrt{E}\begin{pmatrix}0\\-1\\0\\-1\end{pmatrix}.$$

From the $u^\dagger\gamma^0 v$ form of the matrix element of (17.50), it can be seen immediately that only two of the four possible helicity combinations give non-zero matrix elements, these are

$$\mathcal{M}_{\uparrow\uparrow} = -\mathcal{M}_{\downarrow\downarrow} = \frac{m_b}{v}2E.$$

In both cases, the non-zero matrix elements correspond to spin configurations where the $b\bar{b}$ are produced in a spin-0 state. Because the Higgs is a spin-0 scalar, it decays isotropically and matrix element has no angular dependence. Furthermore, since the Higgs boson exists in a single spin state, the spin-averaged matrix element squared is simply,

$$\langle|\mathcal{M}|^2\rangle = |\mathcal{M}_{\uparrow\uparrow}|^2 + |\mathcal{M}_{\downarrow\downarrow}|^2 = \frac{m_b^2}{v^2}8E^2 = \frac{2m_b^2 m_H^2}{v^2}.$$

The partial decay width, obtained from (3.49), is therefore

$$\Gamma(H \to b\bar{b}) = 3 \times \frac{m_b^2 m_H}{8\pi v^2}, \tag{17.51}$$

where the factor of three accounts for the three possible colours of the $b\bar{b}$ pair. For a Higgs boson mass of 125 GeV, the partial decay width $\Gamma(H \to b\bar{b})$ is $\mathcal{O}(2\,\text{MeV})$.

Table 17.1 The predicted branching ratios of the Higgs boson for $m_H = 125$ GeV.

Decay mode	Branching ratio
$H \to b\bar{b}$	57.8%
$H \to WW^*$	21.6%
$H \to \tau^+\tau^-$	6.4%
$H \to gg$	8.6%
$H \to c\bar{c}$	2.9%
$H \to ZZ^*$	2.7%
$H \to \gamma\gamma$	0.2%

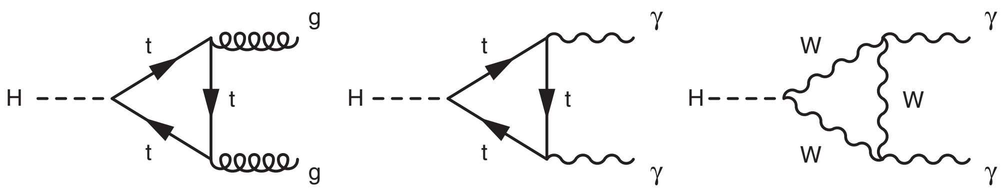


Fig. 17.16 The Feynman diagrams for the decays $H \to gg$ and $H \to \gamma\gamma$.

From (17.51), it can be seen that the partial decay rate to fermions is proportional to the square of the fermion mass, and therefore

$$\Gamma(H \to b\bar{b}) : \Gamma(H \to c\bar{c}) : \Gamma(H \to \tau^+\tau^-) \sim 3m_b^2 : 3m_c^2 : m_\tau^2. \qquad (17.52)$$

It should be noted that quark masses run with q^2 in a similar manner to the running of α_S. Hence the masses appearing in (17.51) are the appropriate values at $q^2 = m_H^2$, where the charm and bottom quark masses are approximately $m_c(m_H^2) \approx 0.6$ GeV and $m_b(m_H^2) \approx 3.0$ GeV.

The branching ratios for a Standard Model Higgs boson with $m_H = 125$ GeV are listed in Table 17.1. Despite the fact that $m_H < 2m_W$, the second largest branching ratio is for the decay $H \to WW^*$, where the star indicates that one of the W bosons is produced off-mass-shell with $q^2 < m_W^2$. From the form of the W-boson propagator of (16.27), the presence of the off-shell W boson will tend to suppress the matrix element. Nevertheless, the large coupling of the Higgs boson to the massive W boson, $g_W m_W$, means that the branching ratio is relatively large. The Higgs boson also can decay to massless particles, $H \to gg$ and $H \to \gamma\gamma$, through loops of virtual top quarks and W bosons, as shown in Figure 17.16. Because the masses of the particles in these loops are large, these decays can compete with the decays to fermions and the off-mass-shell gauge bosons.

17.7 The discovery of the Higgs boson

Prior to the turn-on of the Large Hadron Collider at CERN, the window for a Standard Model Higgs was relatively narrow. The absence of a signal from the direct searches at LEP implied that $m_H > 114\,\text{GeV}$. At the same time, the limits on the size of the quantum loop corrections from the precision electroweak measurements at LEP and the Tevatron suggested that $m_H \lesssim 150\,\text{GeV}$ and that m_H was unlikely to be greater than 200 GeV.

One of the main aims of the LHC was the discovery of the Higgs boson (assuming it existed). The LHC is not only the highest-energy particle collider ever built, it is also the highest-luminosity proton–proton collider to date. During 2010–2011 it operated at a centre-of-mass energy of 7 TeV and during 2012 at 8 TeV. Compelling evidence of the discovery of a new particle compatible with the Standard Model Higgs boson was published by the ATLAS and CMS experiments in the Summer of 2012.

The Higgs boson can be produced at the LHC through a number of different processes, two of which are shown in Figure 17.17. Because the Higgs boson couples preferentially to mass, the largest cross section at the LHC is through gluon–gluon fusion via a loop of virtual top quarks. The cross section for this process can be written in terms of the underlying cross section for gg → H and the gluon PDFs,

$$\sigma(\text{pp} \to \text{h}X) \sim \int_0^1 \int_0^1 g(x_1)g(x_2)\sigma(\text{gg} \to \text{H})\,\text{d}x_1\text{d}x_2.$$

Consequently, the detailed knowledge of the PDFs for the proton is an essential component in the calculation of the expected Higgs boson production rate at the LHC. Fortunately, the proton PDFs are well known and the related uncertainties on the various Higgs production cross sections are less than 10%. Although the gluon–gluon fusion process has the largest cross section, from the experimental

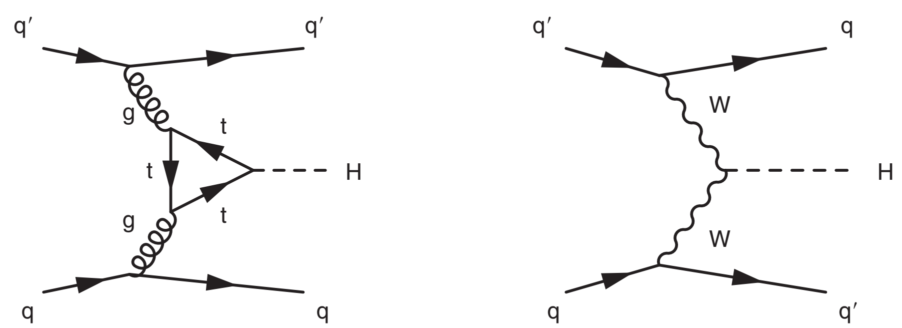


Fig. 17.17 Two of the most important Feynman diagram for Higgs boson production in pp collisions at the LHC. The gluon–gluon fusion process has a significantly higher cross section.

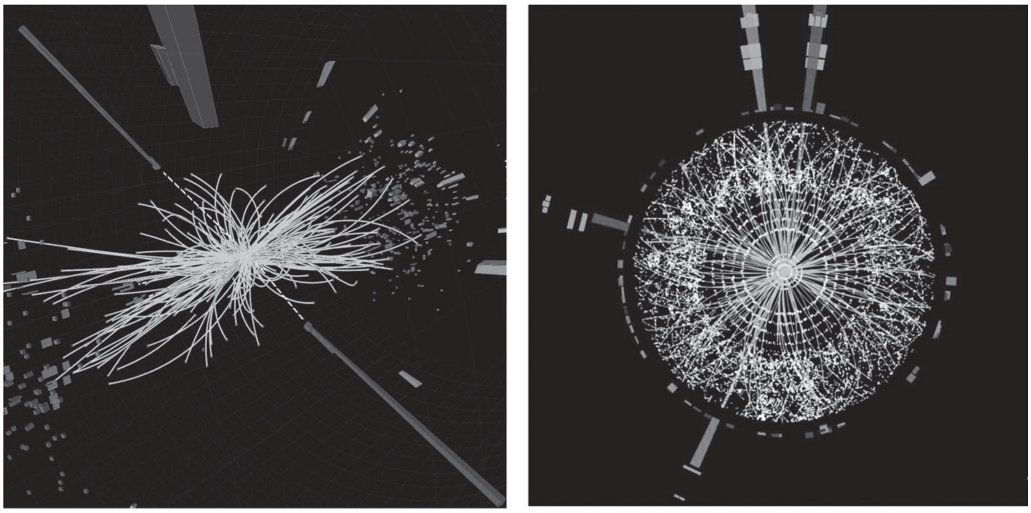


Fig. 17.18 Left: a candidate $H \to \gamma\gamma$ event in the CMS detector. Right: a candidate $H \to ZZ^* \to e^+e^-e^+e^-$ in the ATLAS detector. Reproduced with kind permission from the ATLAS and CMS collaborations, © 2012 CERN.

perspective the vector boson fusion process (shown in Figure 17.17) is also important. This is because it results in more easily identifiable final states consisting of just the decay products of the Higgs boson and two forward jets from the break-up of the colliding protons. In contrast, the gluon–gluon fusion process is accompanied by QCD radiation from the colour field, making the identification of the Higgs boson final states less easy.

In proton–proton collisions at a centre-of-mass energy of ~ 8 TeV, the total production cross section for a Higgs boson with $m_H = 125$ GeV is approximately 20 pb. The first observations of the Higgs boson were based on approximately 20 fb^{-1} of data (ATLAS and CMS combined). This data sample corresponded to a total of approximately $N = \sigma\mathcal{L} = 400\,000$ produced Higgs bosons. Whilst this number might seem large, it is a *tiny* fraction of the total number of interactions recorded at the LHC, most of which involve the QCD production of multi-jet final states. Consequently, it is difficult to distinguish the decays of the Higgs boson producing final states with jets from the large QCD background. For this reason, the most sensitive searches for the Higgs boson at the LHC are in decay channels with distinctive final-state topologies, such as $H \to \gamma\gamma$, $H \to ZZ^* \to \ell^+\ell^-\ell'^+\ell'^-$ (where $\ell = e$ or μ) and $H \to WW^* \to e\nu_e\mu\nu_\mu$. Despite the relatively low branching ratios for these decay modes, the experimental signatures are sufficiently clear for them to be distinguished from the backgrounds from other processes. For example, Figure 17.18 shows a candidate $H \to \gamma\gamma$ event in the CMS detector (left-hand plot). The dashed lines point to the two large energy deposits in the electromagnetic calorimeter from two high-energy photons, which are easily identifiable. Similarly, the right-hand plot of Figure 17.18 shows a candidate $H \to ZZ^* \to e^+e^-e^+e^-$ event in the ATLAS detector. Here the four charged-particle tracks, pointing to four large energy deposits in the electromagnetic calorimeter, are clearly identifiable as high-energy electrons.

17.7.1 Results

The ATLAS and CMS experiments searched for the Higgs boson in several final states, $\gamma\gamma$, ZZ^*, WW^*, $\tau^+\tau^-$ and $b\bar{b}$. In both experiments, the most significant evidence for the Higgs boson was observed in the two most sensitive decay channels, $H \to \gamma\gamma$ and $H \to ZZ^* \to 4\ell$. In both these decay channels, the mass of the Higgs boson candidate can be reconstructed on an event-by-event basis from the invariant mass of its decay products. The left-hand plot of Figure 17.19 shows the distribution of the reconstructed invariant mass of the two photons in candidate $H \to \gamma\gamma$ events in the ATLAS detector. In this plot, each observed event is entered into the histogram with a weight of between zero and one, reflecting the estimated probability of it being compatible with the kinematics of Higgs production and decay. Compared to the expected background, an excess of events is observed at $m_{\gamma\gamma} \approx 126\,\text{GeV}$. The CMS experiment observed a similar excess. The right-hand plot of Figure 17.19 shows the distribution of the invariant masses of the four charged leptons in the CMS $H \to ZZ^* \to 4\ell$ search. The peak at 91 GeV is from Z-boson production. The peak at about 125 GeV can be attributed to the Higgs boson. Whilst the numbers of events are relatively small, the expected background in this region is also small. The ATLAS experiment observed a comparable excess of $H \to ZZ^* \to 4\ell$ candidates at the same mass.

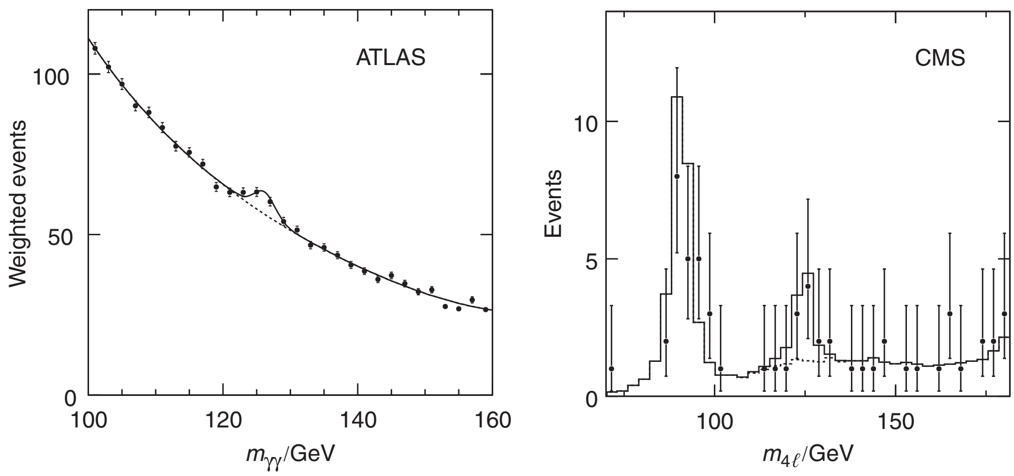


Fig. 17.19 Left: the reconstructed invariant mass distribution of the two photons from candidate $H \to \gamma\gamma$ decays in the ATLAS experiment, adapted from Aad *et al.* (2012). Right: the distribution of the reconstructed invariant masses of the four leptons in candidate $H \to ZZ^* \to 4\ell$ events in the CMS experiment, adapted from Chatrchyan *et al.* (2012). In both plots the solid line shows the expected distribution from background and the observed Higgs signal and the dashed line indicates the expectation from background events alone.

The combined results of the ATLAS and CMS experiments provided statistically compelling evidence for the discovery of a new particle compatible with the expected properties of the Higgs boson. Until it has been demonstrated that the observed particle is a scalar, it is not possible to say conclusively that the Higgs boson has been discovered. However, even at the time of writing, it seems almost certain that the Higgs boson has been discovered; its production cross section is consistent with the Standard Model expectation and its mass is compatible with the indirect determinations from its presence in quantum loop corrections, as inferred from the precision electroweak measurements. Consistent measurements of the mass of the new particle were obtained by the ATLAS ($m = 126.0 \pm 0.6\,\text{GeV}$) and the CMS ($m = 125.3 \pm 0.6\,\text{GeV}$) experiments. On the reasonable assumption that the new particle is the Higgs boson, it can be concluded that

$$m_{\text{H}} \simeq 125.7 \pm 0.5\,\text{GeV}.$$

Since the discovery of the W and Z bosons in the mid 1980s, the search for the Higgs boson has been the highest priority in particle physics. Its discovery finally completed the particle spectrum of the Standard Model.

17.7.2 Outlook

The discovery of the Higgs boson is not the end of the story. The use of a single Higgs doublet in the Standard Model is the most economical choice, but it is not the only possibility. In supersymmetry (see Section 18.2.2), which is a popular extension to the Standard Model, there are (at least) two complex doublets of scalar fields, which give rise to five physical Higgs bosons. Furthermore, it is not clear whether the observed Higgs boson is a fundamental scalar particle or whether it might be composite. In the coming years, the measurements of the spin and branching ratios of the Higgs boson will further test the predictions of the Standard Model. Perhaps more importantly, a detailed understanding of all the properties of the Higgs boson may well open up completely new avenues in our understanding of the Universe and point to what lies beyond the Standard Model.

Summary

The Higgs mechanism is an essential part of the Standard Model. It is based on a doublet of complex scalar fields with the Higgs potential $V(\phi) = \mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$ where $\mu^2 < 0$. As a result, the vacuum state of the Universe is degenerate. The spontaneous breaking of this symmetry, when combined with the underlying $\text{SU}(2)_L \times \text{U}(1)_Y$ gauge symmetry of the electroweak model, provides masses to the W and Z gauge bosons with

$$m_W = m_Z \cos\theta_W = \tfrac{1}{2} g_W\, v,$$

where v is the vacuum expectation value of the Higgs field. The value of

$$v = 246\,\text{GeV},$$

sets the mass scale for the electroweak and Higgs bosons. The interaction between the fermion fields and the non-zero expectation value of the Higgs field, provides a gauge-invariant mechanism for generating the masses of the Standard Model fermions.

In 2012, the discovery of the Higgs boson at the LHC with mass

$$m_H \simeq 126\,\text{GeV},$$

completed the spectrum of Standard Model particles. Following the discovery of the Higgs boson, it is hoped that the studies of its properties will provide clues to physics beyond the Standard Model, which is the main topic of the final chapter of this book.

17.8 *Addendum: Neutrino masses

The right-handed chiral neutrino states ν_R do not participate in any of the interactions of the Standard Model; they do not couple to the gluons or electroweak gauge bosons. Consequently, there is no direct evidence that they exist. However, from the studies of neutrino oscillations it is known that neutrinos do have mass, and therefore there must be a corresponding mass term in the Lagrangian. In the Standard Model, neutrino masses can be introduced in exactly the same way as for the up-type quarks using the conjugate Higgs doublet. In this case, after spontaneous symmetry breaking, the gauge invariant *Dirac mass* term for the neutrino is

$$\mathcal{L}_D = -m_D\,(\overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R).$$

If this is the origin of neutrino masses, then right-handed chiral neutrinos exist. However, the neutrino masses are very much smaller than the masses of the other fermions, suggesting that another mechanism for generating neutrino mass might be present.

Because the *right-handed neutrinos* and *left-handed antineutrinos* transform as singlets under the Standard Model gauge transformations, any additional terms in the Lagrangian formed from these fields alone can be added to the Lagrangian without breaking the gauge invariance of the Standard Model. The left-handed antineutrinos appear in the Lagrangian as the CP conjugate fields defined by

$$\psi^c = \hat{C}\hat{P}\psi = i\gamma^2\gamma^0\psi^*,$$

where the CP conjugate field for the right-handed neutrino, written ν_R^c, corresponds to a left-handed antineutrino. Therefore the Majorana mass term

$$\mathcal{L}_M = -\tfrac{1}{2}M\,(\overline{\nu_R^c}\nu_R + \overline{\nu_R}\nu_R^c),$$

which is formed from right-handed neutrino fields and left-handed antineutrino fields, respects the local gauge invariance of the Standard Model. Consequently, it can be added to the Standard Model Lagrangian. However, there is a price to pay; the Majorana mass term provides a direct coupling between between a particle and an antiparticle. For example, the corresponding Majorana mass term for the electron would allow $e^+ \leftrightarrow e^-$ transitions, violating charge conservation. This problem does not exist for the neutrinos. Furthermore, because neutrinos are neutral, it is possible that they are their own antiparticles, in which case they are referred to as *Majorana neutrinos* as opposed to Dirac neutrinos.

17.8.1 The seesaw mechanism

The most general renormalisable Lagrangian for the neutrino masses includes both the Dirac and Majorana mass terms, indicated in Figure 17.20. Because $\overline{\nu_L}\nu_R$ is equivalent to $\overline{\nu_R^c}\nu_L^c$, the Dirac mass term can be written

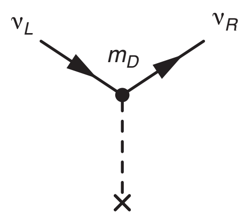
$$\mathcal{L}_D = -\tfrac{1}{2}m_D\,(\overline{\nu_L}\nu_R + \overline{\nu_R^c}\nu_L^c) + \ h.c.,$$

where *h.c.* stands for the corresponding Hermitian conjugate. This term admits the possibility that neutrino masses arise from the spontaneous symmetry breaking of the Higgs mechanism. If in addition, the automatically gauge-invariant Majorana mass term is added by hand, the Lagrangian for the combined Dirac and Majorana masses is

$$\mathcal{L}_{DM} = -\tfrac{1}{2}\left[m_D\,\overline{\nu_L}\nu_R + m_D\,\overline{\nu_R^c}\nu_L^c + M\,\overline{\nu_R^c}\nu_R\right] + \ h.c.$$

or, equivalently,

$$\mathcal{L}_{DM} = -\tfrac{1}{2}\begin{pmatrix}\overline{\nu_L} & \overline{\nu_R^c}\end{pmatrix}\begin{pmatrix}0 & m_D\\ m_D & M\end{pmatrix}\begin{pmatrix}\nu_L^c\\ \nu_R\end{pmatrix} + \ h.c. \qquad (17.53)$$



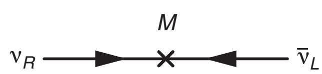


Fig. 17.20 The Dirac and Majorana neutrino mass terms.

The physical states of this system can be obtained from the basis in which the mass matrix is diagonal, analogous to the procedure for identifying the physical states of the neutral kaon system and the neutral gauge bosons of the $U(1)_Y \times SU(2)_L$ gauge symmetry. Hence, with the Lagrangian including Dirac and Majorana mass terms, the masses of the physical neutrino states are the eigenvalues of mass matrix $\mathbf{M}$ in (17.53). These can be found from the characteristic equation $\det(\mathbf{M} - \lambda I) = 0$, which implies $\lambda^2 - M\lambda - m_D^2 = 0$. Hence, in this model, the masses of the physical neutrinos would be

$$m_\pm = \lambda_\pm = \frac{M \pm \sqrt{M^2 + 4m_D^2}}{2} = \frac{M \pm M\sqrt{1 + 4m_D^2/M^2}}{2}.$$

If the Majorana mass M is taken to be much greater than the Dirac mass m_D, then

$$m_\pm \approx \tfrac{1}{2}M \pm \tfrac{1}{2}\left(M + \frac{2m_D^2}{M}\right), \tag{17.54}$$

giving a light neutrino state[2] (ν) and heavy neutrino state (N) with masses

$$|m_\nu| \approx \frac{m_D^2}{M} \quad \text{and} \quad m_N \approx M.$$

In the seesaw mechanism, it is hypothesised that the Dirac mass terms for the neutrinos are of a similar size to the masses of the other fermions, i.e. $\mathcal{O}(1\,\text{GeV})$. The Majorana mass M is then made sufficiently large that the lighter of the two physical neutrino states has a mass $m_\nu \sim 0.01\,\text{eV}$. In this way, the masses of the lighter neutrino states can be made to be very small, even when the Dirac mass term is of the same order of magnitude as the other fermions. For this to work, the Majorana mass must be very large, $M \sim 10^{11}$ GeV.

If a Majorana mass term exists, the seesaw mechanism predicts that for each of the three neutrino generations, there is a very light neutrino with a mass much smaller than the other Standard Model fermions and a very massive neutrino state $m_N \approx M$. The physical neutrino states, which are obtained from the eigenvalues of the mass matrix, are

$$\nu = \cos\theta(\nu_L + \nu_L^c) - \sin\theta(\nu_R + \nu_R^c) \quad \text{and} \quad N = \cos\theta(\nu_R + \nu_R^c) + \sin\theta(\nu_L + \nu_L^c),$$

where $\tan\theta \approx m_D/M$. Since the left-handed chiral components of the light neutrino are multiplied by $\cos\theta$, the effect of introducing the Majorana mass term is to reduce the weak charged-current couplings of the light neutrino states by a factor $\cos\theta$. However, for $M \gg m_D$, the neutrino states are

$$\nu \approx (\nu_L + \nu_L^c) - \frac{m_D}{M}(\nu_R + \nu_R^c) \quad \text{and} \quad N \approx (\nu_R + \nu_R^c) + \frac{m_D}{M}(\nu_L + \nu_L^c),$$

[2] The minus sign for the mass of the light neutrino in (17.54) can be absorbed in to the definition of the fields.

and the couplings of the light neutrinos to the weak charged-current are essentially the same as those of the Standard Model. Since the massive neutrino state is almost entirely right-handed, it would not participate in the weak charged- or neutral-currents.

The seesaw mechanism provides an interesting hypothesis for the smallness of neutrino masses, but it is just a hypothesis. It would be placed on firmer ground if neutrinos were shown to be Majorana particles. One experimental consequence of neutrinos being Majorana particles would be the possibility of observing the phenomenon of neutrinoless double β-decay, which is discussed in the following chapter.

Problems

17.1 By considering the form of the polarisation four-vector for a longitudinally polarised massive gauge bosons, explain why the t-channel neutrino-exchange diagram for $e^+e^- \to W^+W^-$, when taken in isolation, is badly behaved at high centre-of-mass energies.

17.2 The Lagrangian for the Dirac equation is

$$\mathcal{L} = i\overline{\psi}\gamma_\mu\partial^\mu\psi - m\overline{\psi}\psi,$$

Treating the eight fields ψ_i and $\overline{\psi}_i$ as independent, show that the Euler-Lagrange equation for the component ψ_i leads to

$$i\partial_\mu\overline{\psi}\gamma^\mu + m\overline{\psi} = 0.$$

17.3 Verify that the Lagrangian for the free electromagnetic field,

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu},$$

is invariant under the gauge transformation $A_\mu \to A'_\mu = A_\mu - \partial_\mu\chi$.

17.4 The Lagrangian for the electromagnetic field in the presence of a current j^μ is

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^\mu A_\mu.$$

By writing this as

$$\begin{aligned}\mathcal{L} &= -\tfrac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) - j^\mu A_\mu \\ &= -\tfrac{1}{2}(\partial^\mu A^\nu)(\partial_\mu A_\nu) + \tfrac{1}{2}(\partial^\nu A^\mu)(\partial_\mu A_\nu) - j^\mu A_\mu,\end{aligned}$$

show that the Euler–Lagrange equation gives the covariant form of Maxwell's equations,

$$\partial_\mu F^{\mu\nu} = j^\nu.$$

17.5 Explain why the Higgs potential can contain terms with only even powers of the field ϕ.

17.6 Verify that substituting (17.30) into (17.29) leads to

$$\mathcal{L} = \tfrac{1}{2}(\partial^\mu\eta)(\partial_\mu\eta) - \lambda v^2\eta^2 + \tfrac{1}{2}(\partial^\mu\xi)(\partial_\mu\xi), -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2v^2B^\mu B_\mu - V_{int} + gvB_\mu(\partial^\mu\xi).$$

17.7 Show that the Lagrangian for a complex scalar field ϕ,

$$\mathcal{L} = (D_\mu \phi)^*(D^\mu \phi),$$

with the covariant derivative $D_\mu = \partial_\mu + igB_\mu$, is invariant under local U(1) gauge transformations,

$$\phi(x) \to \phi'(x) = e^{ig\chi(x)}\phi(x),$$

provided the gauge field transforms as

$$B_\mu \to B'_\mu = B_\mu - \partial_\mu \chi(x).$$

17.8 From the mass matrix of (17.40) and its eigenvalues (17.41), show that the eigenstates in the diagonal basis are

$$A_\mu = \frac{g' W_\mu^{(3)} + gB_\mu}{\sqrt{g^2 + g' B_\mu}} \quad \text{and} \quad Z_\mu = \frac{gW_\mu^{(3)} - g' B_\mu}{\sqrt{g^2 + g' B_\mu}},$$

where A_μ and Z_μ correspond to the physical fields for the photon and Z.

17.9 By considering the interaction terms in (17.38), show that the HZZ coupling is given by

$$g_{\mathrm{HZZ}} = \frac{g_\mathrm{W}}{\cos\theta_\mathrm{W}} m_\mathrm{Z}.$$

17.10 For a Higgs boson with $m_\mathrm{H} > 2m_\mathrm{W}$, the dominant decay mode is into two on-shell W bosons, $\mathrm{H} \to \mathrm{W^+W^-}$. The matrix element for this decay can be written

$$\mathcal{M} = -g_\mathrm{W} m_\mathrm{W} g_{\mu\nu} \epsilon^\mu(p_2)^* \epsilon^\nu(p_3)^*,$$

where p_2 and p_3 are respectively the four-momenta of the $\mathrm{W^+}$ and $\mathrm{W^-}$.

(a) Taking $\mathbf{p}_2$ to lie in the positive z-direction, consider the nine possible polarisation states of the $\mathrm{W^+W^-}$ and show that the matrix element is non-zero only when both W bosons are left-handed ($\mathcal{M}_{\downarrow\downarrow}$), both W bosons are right-handed ($\mathcal{M}_{\uparrow\uparrow}$), or both are longitudinally polarised ($\mathcal{M}_{LL}$).

(b) Show that

$$\mathcal{M}_{\uparrow\uparrow} = \mathcal{M}_{\downarrow\downarrow} = -g_\mathrm{W} m_\mathrm{W} \quad \text{and} \quad \mathcal{M}_{LL} = \frac{g_\mathrm{W}}{m_\mathrm{W}}\left(\tfrac{1}{2}m_\mathrm{H}^2 - m_\mathrm{W}^2\right).$$

(c) Hence show that

$$\Gamma(\mathrm{H} \to \mathrm{W^+W^-}) = \frac{G_\mathrm{F} m_\mathrm{H}^3}{8\pi\sqrt{2}} \sqrt{1 - 4\lambda^2}\left(1 - 4\lambda^2 + 12\lambda^4\right),$$

where $\lambda = m_\mathrm{W}/m_\mathrm{H}$.

17.11 Assuming a total Higgs production cross section of 20 pb and an integrated luminosity of 10 $\mathrm{fb^{-1}}$, how many $\mathrm{H} \to \gamma\gamma$ and $\mathrm{H} \to \mu^+\mu^-\mu^+\mu^-$ events are expected in each of the ATLAS and CMS experiments.

17.12 Draw the lowest-order Feynman diagrams for the processes $\mathrm{e^+e^-} \to \mathrm{HZ}$ and $\mathrm{e^+e^-} \to \mathrm{H}\nu_\mathrm{e}\overline{\nu}_\mathrm{e}$, which are the main Higgs production mechanism at a future high-energy linear collider.

17.13 In the future, it might be possible to construct a muon collider where the Higgs boson can be produced directly through $\mu^+\mu^- \to \mathrm{H}$. Compare the cross sections for $\mathrm{e^+e^-} \to \mathrm{H} \to \mathrm{b\overline{b}}$, $\mu^+\mu^- \to \mathrm{H} \to \mathrm{b\overline{b}}$ and $\mu^+\mu^- \to \gamma \to \mathrm{b\overline{b}}$ at $\sqrt{s} = m_\mathrm{H}$.