Determination of the CKM Matrix elements

- In total, there are 9 matrix elements to determine:

$$V_{\text{CKM}} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM Matrix parameterization

- The CKM matrix is unitary: can be parameterized by 3 mixing angles and the CP-violating phase
- A standard choice is the same as was shown for the PMNS matrix:

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- Since experimentally $s_{13} \ll s_{23} \ll s_{12} \ll 1$, the matrix hierarchy is exhibited using the Wolfenstein parameterization (written in terms of λ, A, ρ, η):

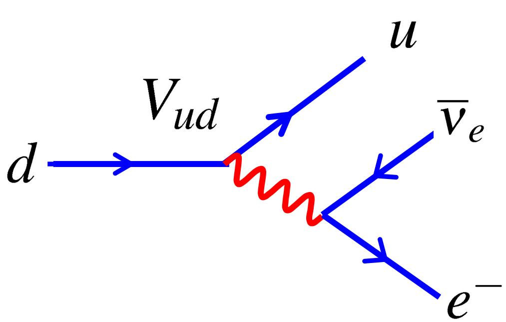
$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Determination of the CKM Matrix elements

- The experimental determination of the CKM matrix elements comes mainly from measurements of leptonic decays (the leptonic part is well understood).
- It is easy to produce/observe meson decays, however theoretical uncertainties associated with the decays of bound states often limits the precision
- Contrast this with the measurements of the PMNS matrix, where there are few theoretical uncertainties and the experimental difficulties in dealing with neutrinos limits the precision.

1 **$|V_{ud}|$** **from nuclear beta decay**

$$\begin{pmatrix} \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



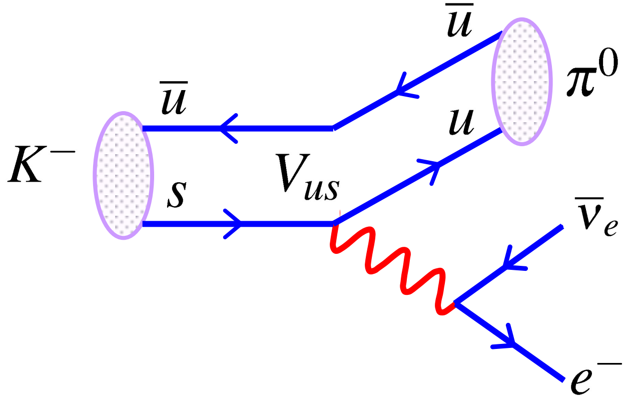
Super-allowed $0^+ \rightarrow 0^+$ beta decays are relatively free from theoretical uncertainties

$$\Gamma \propto |V_{ud}|^2$$

$$|V_{ud}| = 0.97370 \pm 0.00014 \qquad (\approx \cos\theta_c)$$

2 **$|V_{us}|$** **from semi-leptonic kaon decays**

$$\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



$$\Gamma \propto |V_{us}|^2$$

$$|V_{us}| = 0.2245 \pm 0.0008 \qquad (\approx \sin\theta_c)$$

3 **$|V_{cd}|$** **from neutrino scattering; (semi-)leptonic D decays**

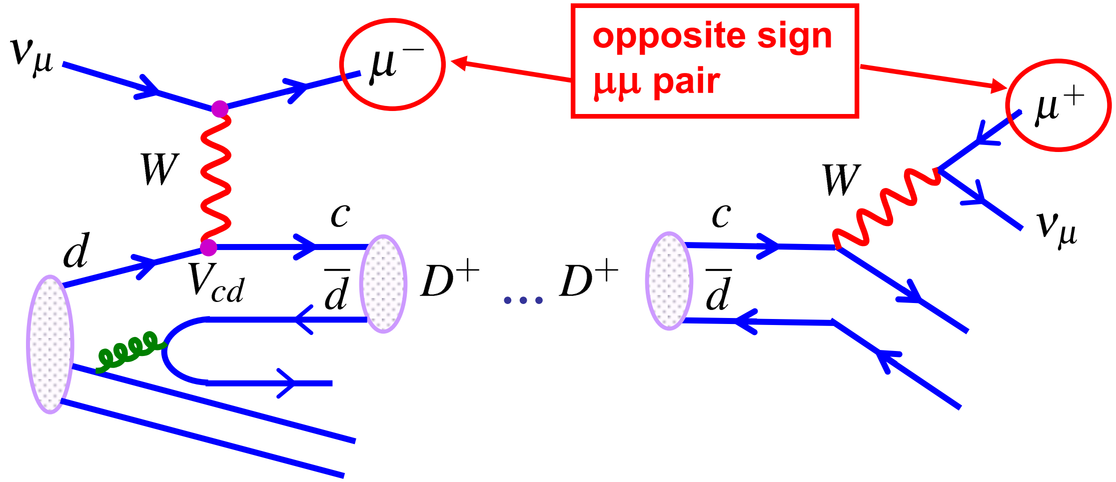
$$\nu_\mu + N \to \mu^+\mu^- X$$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

Look for opposite charge di-muon events in ν_μ scattering from production and decay of a $D^+(c\overline{d})$ meson

$$\text{Rate} \propto |V_{cd}|^2 \text{Br}(D^+ \to X\mu^+\nu_\mu)$$

Measured in various collider experiments

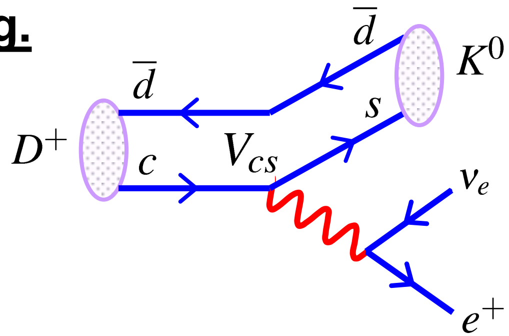


$$\Rightarrow \quad |V_{cd}| = 0.221 \pm 0.004$$

4 $|V_{cs}|$ from semi-leptonic charmed meson decays

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



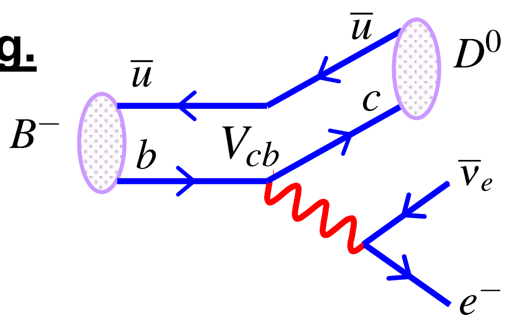
$$\Gamma \propto |V_{cs}|^2$$

$$|V_{cs}| = 0.987 \pm 0.011$$

5 $|V_{cb}|$ from semi-leptonic B hadron decays

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



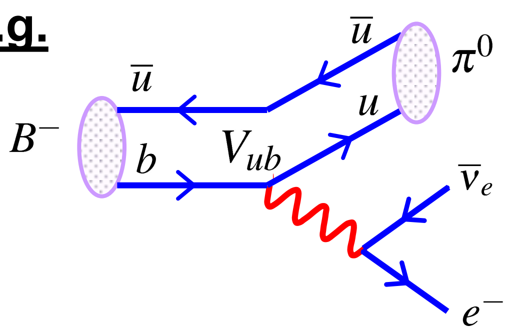
$$\Gamma \propto |V_{cb}|^2$$

$$|V_{cb}| = (41.0 \pm 1.4) \times 10^{-3}$$

6 $|V_{ub}|$ from semi-leptonic B hadron decays

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.

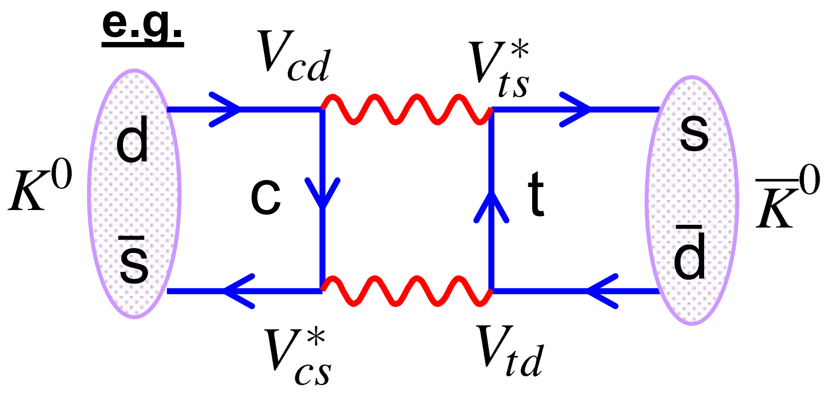


$$\Gamma \propto |V_{ub}|^2$$

$$|V_{ub}| = (3.82 \pm 0.24) \times 10^{-3}$$

7/8 $|V_{td}|$ and $|V_{ts}|$ — from K or B oscillations measurements

e.g.



- E.g. ratio $|V_{td}/V_{ts}|$ determined from $\Delta m_d/\Delta m_s$ (mass difference in B^0 and in B_s systems)
- In ratios many uncertainties cancel out
- Precision limited by theoretical uncertainties

$$|V_{td}/V_{ts}| = 0.205 \pm 0.001 \pm 0.006$$

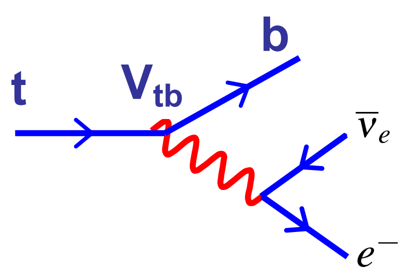
experimental error — theory uncertainty

- Compare with uncertainties here:

$$|V_{td}| = (8.0 \pm 0.3) \times 10^{-3}\,, \qquad |V_{ts}| = (38.8 \pm 1.1) \times 10^{-3}$$

9 $|V_{tb}|$ — from from top quark decays

e.g.



- Use ratio of BF: $B(t\rightarrow Wb)/B(t\rightarrow Wq) \propto |V_{tb}|^2$
- Also use single top quark production

$$|V_{tb}| = 1.013 \pm 0.030$$

- Experimental uncertainties dominate

Particle – Anti-Particle Mixing

- The wave-function for a single particle with lifetime $\tau = 1/\Gamma$ evolves with time as:

$$\psi(t) = Ne^{-\Gamma t/2}e^{-iMt}$$

which gives the appropriate exponential decay of

$$\langle\psi(t)|\psi(t)\rangle = \langle\psi(0)|\psi(0)\rangle e^{-t/\tau}$$

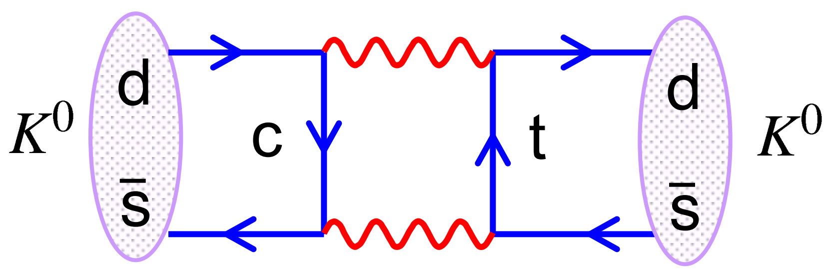
- The wave-function satisfies the time-dependent wave equation:

$$\hat{H}|\psi(t)\rangle = \boxed{(M - \tfrac{1}{2}i\Gamma)}|\psi(t)\rangle = i\frac{\partial}{\partial t}|\psi(t)\rangle \qquad \text{(A1)}$$

- For a bound state such as a K^0 the mass term includes the "mass" from the weak interaction "potential" $\hat{H}_{\text{weak}}$

$$M = m_{K^0} + \langle K^0|\hat{H}_{\text{weak}}|K^0\rangle + \sum_j \frac{|\langle K^0|\hat{H}_{\text{weak}}|j\rangle|^2}{m_{K^0} - E_j}$$

Sum over intermediate states j



The third term is the 2nd order term in the perturbation expansion corresponding to box diagrams resulting in $K^0 \to K^0$

• The total decay rate is the sum over all possible decays $K^0 \rightarrow f$

$$\Gamma = 2\pi \sum_f |\langle f|\hat{H}_{weak}|K^0\rangle|^2 \rho_F$$

Density of final states

★ Because there are also diagrams which allow $K^0 \leftrightarrow \overline{K}^0$ mixing need to consider the time evolution of a mixed state

$$\psi(t) = a(t)K^0 + b(t)\overline{K}^0 \qquad \text{(A2)}$$

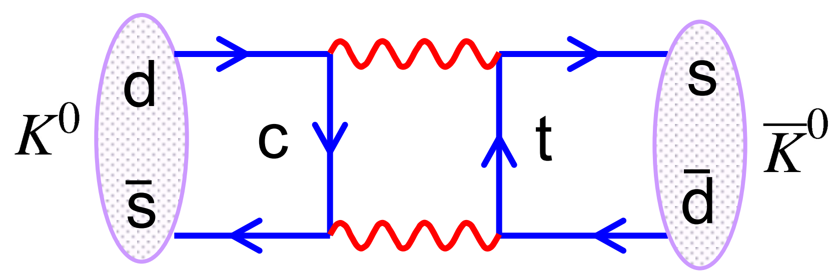
★ The time dependent wave-equation of (A1) becomes

$$\begin{pmatrix} M_{11} - \frac{1}{2}i\Gamma_{11} & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{21} - \frac{1}{2}i\Gamma_{21} & M_{22} - \frac{1}{2}i\Gamma_{22} \end{pmatrix} \begin{pmatrix} |K^0(t)\rangle \\ |\overline{K}^0(t)\rangle \end{pmatrix} = i\frac{\partial}{\partial t} \begin{pmatrix} |K^0(t)\rangle \\ |\overline{K}^0(t)\rangle \end{pmatrix} \qquad \text{(A3)}$$

the diagonal terms are as before, and the off-diagonal terms are due to mixing.

$$M_{11} = m_{K^0} + \langle K^0|\hat{H}_{\text{weak}}|K^0\rangle + \sum_n \frac{|\langle K^0|\hat{H}_{\text{weak}}|K^0\rangle|^2}{m_{K^0} - E_n}$$

$$M_{12} = \sum_j \frac{\langle K^0|\hat{H}_{\text{weak}}|j\rangle^* \langle j|\hat{H}_{\text{weak}}|\overline{K}^0\rangle}{m_{K^0} - E_j}$$



- The off-diagonal decay terms include the effects of interference between decays to a common final state

$$\Gamma_{12} = 2\pi \sum_f \langle f|\hat{H}_{weak}|K^0\rangle^* \langle f|\hat{H}_{weak}|\overline{K}^0\rangle \rho_F$$

- In terms of the time dependent coefficients for the kaon states, (A3) becomes

$$\left[\mathbf{M} - i\tfrac{1}{2}\Gamma\right] \begin{pmatrix} a \\ b \end{pmatrix} = i\frac{\partial}{\partial t} \begin{pmatrix} a \\ b \end{pmatrix}$$

where the Hamiltonian can be written:

$$\mathbf{H} = \mathbf{M} - i\tfrac{1}{2}\Gamma = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \tfrac{1}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

- Both the mass and decay matrices represent observable quantities and are Hermitian

$$M_{11} = M_{11}^*, \quad M_{22} = M_{22}^*, \quad M_{12} = M_{21}^*$$

$$\Gamma_{11} = \Gamma_{11}^*, \quad \Gamma_{22} = \Gamma_{22}^*, \quad \Gamma_{12} = \Gamma_{21}^*$$

- Furthermore, if CPT is conserved then the masses and decay rates of the $\overline{K}^0$ and K^0 are identical:

$$M_{11} = M_{22} = M; \qquad \Gamma_{11} = \Gamma_{22} = \Gamma$$

•Hence the time evolution of the system can be written:

$$\begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M - \frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} a \\ b \end{pmatrix} \qquad \text{(A4)}$$

•To solve the coupled differential equations for a(t) and b(t), first find the eigenstates of the Hamiltonian (the K_L and K_S) and then transform into this basis. The eigenvalue equation is:

$$\begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M - \frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad \text{(A5)}$$

•Which has non-trivial solutions for

$$|\mathbf{H} - \lambda I| = 0$$

$$\Rightarrow \quad (M - \tfrac{1}{2}i\Gamma - \lambda)^2 - (M_{12}^* - \tfrac{1}{2}i\Gamma_{12}^*)(M_{12} - \tfrac{1}{2}i\Gamma_{12}) = 0$$

with eigenvalues

$$\lambda = M - \tfrac{1}{2}i\Gamma \pm \sqrt{(M_{12}^* - \tfrac{1}{2}i\Gamma_{12}^*)(M_{12} - \tfrac{1}{2}i\Gamma_{12})}$$

•The eigenstates can be obtained by substituting back into (A5)

$$(M - \tfrac{1}{2}i\Gamma)x_1 + (M_{12} - \tfrac{1}{2}i\Gamma_{12}) = (M - \tfrac{1}{2}i\Gamma \pm \sqrt{(M_{12}^* - \tfrac{1}{2}i\Gamma_{12}^*)(M_{12} - \tfrac{1}{2}i\Gamma_{12})})x_1$$

$$\Rightarrow \qquad \frac{x_2}{x_1} = \pm\sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

★ **Define**

$$\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

★ **Hence the normalised eigenstates are**

$$|K_\pm\rangle = \frac{1}{\sqrt{1+|\eta|^2}}\begin{pmatrix}1\\ \pm\eta\end{pmatrix} = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle \pm \eta|\overline{K}^0\rangle)$$

★ **Note, in the limit where M_{12}, Γ_{12} are real, the eigenstates correspond to the CP eigenstates K_1 and K_2. Hence we can identify the general eigenstates as as the long and short lived neutral kaons:**

$$|K_L\rangle = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle + \eta|\overline{K}^0\rangle) \qquad |K_S\rangle = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle - \eta|\overline{K}^0\rangle)$$

★ **Substituting these states back into (A2):**

$$
\begin{aligned}
|\psi(t)\rangle &= a(t)|K^0\rangle + b(t)|\overline{K}^0\rangle \\
&= \sqrt{1+|\eta|^2}\left[\frac{a(t)}{2}(K_L+K_S) + \frac{b(t)}{2\eta}(K_L-K_S)\right] \\
&= \sqrt{1+|\eta|^2}\left[\left(\frac{a(t)}{2}+\frac{b(t)}{2\eta}\right)K_L + \left(\frac{a(t)}{2}-\frac{b(t)}{2\eta}\right)K_S\right] \\
&= \frac{\sqrt{1+|\eta|^2}}{2}\left[a_L(t)K_L + a_S(t)K_S\right]
\end{aligned}
$$

with

$$a_L(t) \equiv a(t) + \frac{b(t)}{\eta} \qquad\qquad a_S(t) \equiv a(t) - \frac{b(t)}{\eta}$$

★ **Now consider the time evolution of** $a_L(t)$

$$i\frac{\partial a_L}{\partial t} = i\frac{\partial a}{\partial t} + \frac{i}{\eta}\frac{\partial b}{\partial t}$$

★ **Which can be evaluated using (A4) for the time evolution of a(t) and b(t):**

$$
\begin{aligned}
i\frac{\partial a_L}{\partial t} &= \left[(M-\tfrac{1}{2}i\Gamma_{12})a+(M_{12}-\tfrac{1}{2}i\Gamma_{12})b\right]+\frac{1}{\eta}\left[(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})a+(M-\tfrac{1}{2}i\Gamma)b\right] \\
&= (M-\tfrac{1}{2}i\Gamma)\left(a+\frac{b}{\eta}\right)+(M_{12}-\tfrac{1}{2}i\Gamma_{12})b+\frac{1}{\eta}(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})a \\
&= (M-\tfrac{1}{2}i\Gamma)a_L+(M_{12}-\tfrac{1}{2}i\Gamma_{12})b+\left(\sqrt{(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right)a \\
&= (M-\tfrac{1}{2}i\Gamma)a_L+\left(\sqrt{(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right)\left(a+\frac{b}{\eta}\right) \\
&= (M-\tfrac{1}{2}i\Gamma)a_L+\left(\sqrt{(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right)a_L \\
&= (m_L-\tfrac{1}{2}i\Gamma_L)a_L
\end{aligned}
$$

★ **Hence:**

$$
\boxed{i\frac{\partial a_L}{\partial t} = (m_L-\tfrac{1}{2}i\Gamma_L)a_L}
$$

with $$m_L = M+\Re\left\{\sqrt{(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right\}$$

and $$\Gamma_L = \Gamma-2\Im\left\{\sqrt{(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right\}$$

★ **Following the same procedure obtain:**

$$i\frac{\partial a_S}{\partial t} = (m_S - \tfrac{1}{2}i\Gamma_S)a_S$$

with $$m_S = M - \Re\left\{\sqrt{(M_{12}^* - \tfrac{1}{2}i\Gamma_{12}^*)(M_{12} - \tfrac{1}{2}i\Gamma_{12})}\right\}$$

and $$\Gamma_S = \Gamma + 2\Im\left\{\sqrt{(M_{12}^* - \tfrac{1}{2}i\Gamma_{12}^*)(M_{12} - \tfrac{1}{2}i\Gamma_{12})}\right\}$$

★ **In matrix notation we have**

$$\begin{pmatrix} M_L - \frac{1}{2}i\Gamma_L & 0 \\ 0 & M_S - \frac{1}{2}i\Gamma_S \end{pmatrix}\begin{pmatrix} a_L \\ a_S \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} a_L \\ a_S \end{pmatrix}$$

★ **Solving we obtain**

$$a_L(t) \propto e^{-im_L t - \Gamma_L t/2} \qquad a_S(t) \propto e^{-im_S t - \Gamma_S t/2}$$

★ **Hence in terms of the K_L and K_S basis the states propagate as independent particles with definite masses and lifetimes (the mass eigenstates). The time evolution of the neutral kaon system can be written**

$$|\psi(t)\rangle = A_L e^{-im_L t - \Gamma_L t/2}|K_L\rangle + A_S e^{-im_S t - \Gamma_S t/2}|K_S\rangle$$

where A_L and A_S are constants

CP Violation : $\pi\pi$ decays

★ Consider the development of the $K^0 - \overline{K}^0$ system **now** including CP violation

★ Repeat previous derivation using

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_1\rangle + \varepsilon|K_2\rangle\right] \qquad |K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]$$

• Writing the CP eigenstates in terms of $K^0, \overline{K}^0$

$$|K_L\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K_0\rangle + (1-\varepsilon)|\overline{K}^0\rangle\right]$$

$$|K_S\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K_0\rangle - (1-\varepsilon)|\overline{K}^0\rangle\right]$$

• Inverting these expressions obtain

$$|K^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1+\varepsilon}\left(|K_L\rangle + |K_S\rangle\right) \qquad |\overline{K}^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1-\varepsilon}\left(|K_L\rangle - |K_S\rangle\right)$$

• Hence a state that was produced as a K^0 evolves with time as:

$$|\psi(t)\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1+\varepsilon}\left(\theta_L(t)|K_L\rangle + \theta_S(t)|K_S\rangle\right)$$

where as before $\theta_S(t) = e^{-(im_S+\frac{\Gamma_S}{2})t}$ and $\theta_L(t) = e^{-(im_L+\frac{\Gamma_L}{2})t}$

- If we are considering the decay rate to $\pi\pi$ need to express the wave-function in terms of the CP eigenstates (remember we are neglecting CP violation in the decay)

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}[(|K_2\rangle + \varepsilon|K_1\rangle)\theta_L(t) + (|K_1\rangle + \varepsilon|K_2\rangle)\theta_S(t)]$$

$$= \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}[(\theta_S + \varepsilon\theta_L)|K_1\rangle + (\theta_L + \varepsilon\theta_S)|K_2\rangle]$$

CP Eigenstates

- Two pion decays occur with CP = +1 and therefore arise from decay of the CP = +1 kaon eigenstate, i.e. K_1

$$\Gamma(K^0_{t=0} \to \pi\pi) \propto |\langle K_1|\psi(t)\rangle|^2 = \frac{1}{2}\left|\frac{1}{1+\varepsilon}\right|^2 |\theta_S + \varepsilon\theta_L|^2$$

- Since $|\varepsilon| \ll 1$

$$\left|\frac{1}{1+\varepsilon}\right|^2 = \frac{1}{(1+\varepsilon^*)(1+\varepsilon)} \approx \frac{1}{1+2\Re\{\varepsilon\}} \approx 1 - 2\Re\{\varepsilon\}$$

- Now evaluate the $|\theta_S + \varepsilon\theta_L|^2$ term again using

$$|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\Re(z_1 z_2^*)$$

$$
\begin{aligned}
|\theta_S + \varepsilon\theta_L|^2 &= |e^{-im_S t - \frac{\Gamma_S}{2}t} + \varepsilon e^{-im_L t - \frac{\Gamma_L}{2}t}|^2 \\
&= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2\Re\{e^{-im_S t - \frac{\Gamma_S}{2}t}.\varepsilon^* e^{+im_L t - \frac{\Gamma_L}{2}t}\}
\end{aligned}
$$

- **Writing** $\varepsilon = |\varepsilon| e^{i\phi}$

$$
\begin{aligned}
|\theta_S + \varepsilon\theta_L|^2 &= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-(\Gamma_S+\Gamma_L)t/2} \Re\{e^{i(m_L - m_S)t - \phi}\} \\
&= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-(\Gamma_S+\Gamma_L)t/2} \cos(\Delta m.t - \phi)
\end{aligned}
$$

- **Putting this together we obtain:**

$$
\Gamma(K^0_{t=0} \to \pi\pi) = \frac{1}{2}(1 - 2\Re\{\varepsilon\}) N_{\pi\pi} \left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-(\Gamma_S+\Gamma_L)t/2} \cos(\Delta m.t - \phi) \right]
$$

Short lifetime component $K_S \to \pi\pi$

CP violating long lifetime component $K_L \to \pi\pi$

Interference term

- **In exactly the same manner obtain for a beam which was produced as** $\overline{K}^0$

$$
\Gamma(\overline{K}^0_{t=0} \to \pi\pi) = \frac{1}{2}(1 + 2\Re\{\varepsilon\}) N_{\pi\pi} \left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} - 2|\varepsilon| e^{-(\Gamma_S+\Gamma_L)t/2} \cos(\Delta m.t - \phi) \right]
$$

Interference term changes sign

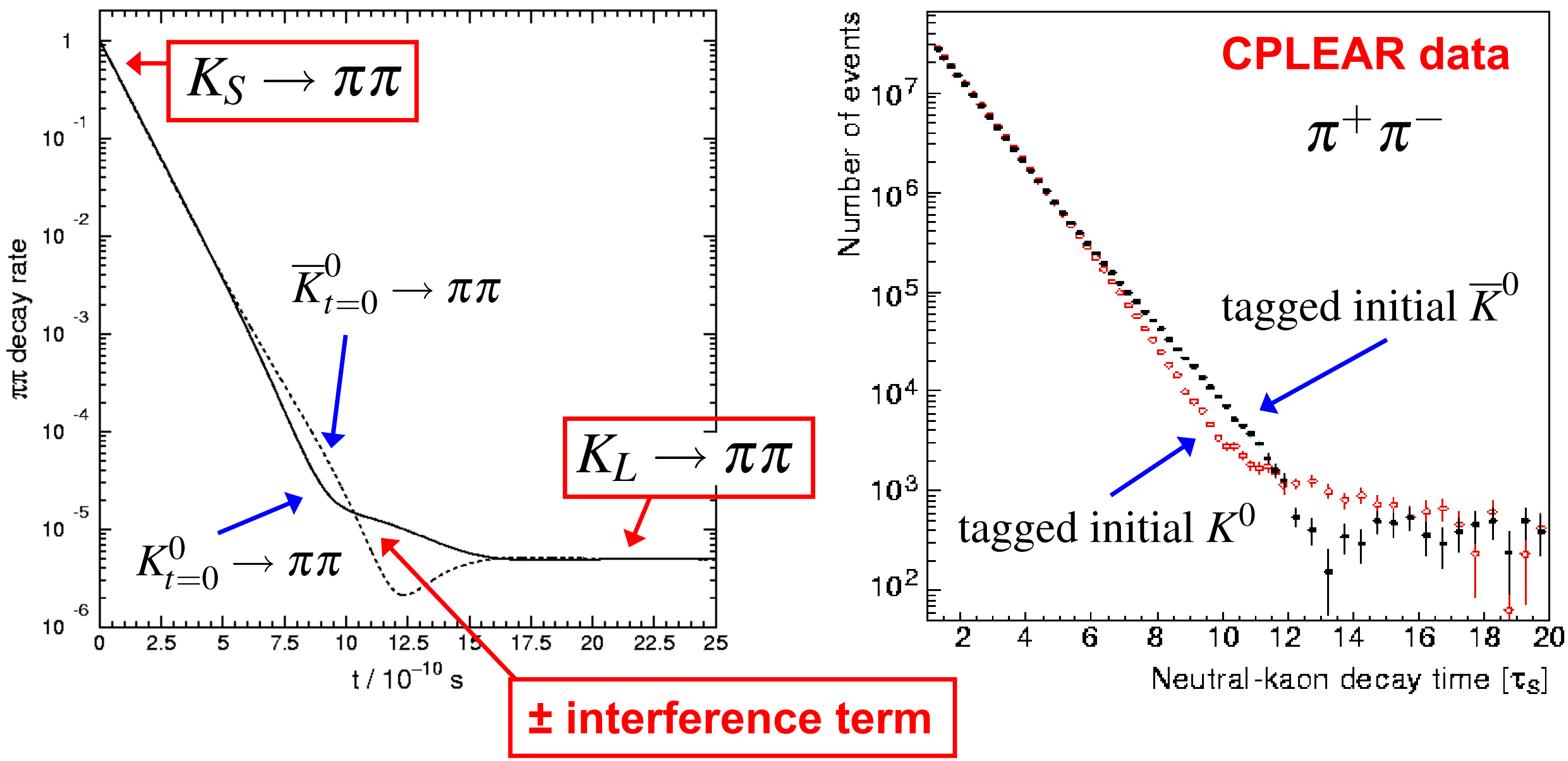
★ **At large proper times only the long lifetime component remains :**

$$\Gamma(K^0_{t=0} \to \pi\pi) \quad \to \quad \frac{1}{2}(1 - 2\Re\{\varepsilon\})N_{\pi\pi}.|\varepsilon|^2 e^{-\Gamma_L t}$$

i.e. CP violating $K_L \to \pi\pi$ **decays**

★ **Since CPLEAR can identify whether a** K^0 **or** $\overline{K}^0$ **was produced, able to measure** $\Gamma(K^0_{t=0} \to \pi\pi)$ **and** $\Gamma(\overline{K}^0_{t=0} \to \pi\pi)$

Prediction with CP violation



★The CPLEAR data shown previously can be used to measure $\varepsilon = |\varepsilon| e^{i\phi}$

- Define the asymmetry:

$$A_{+-} = \frac{\Gamma(\overline{K}^0_{t=0} \to \pi\pi) - \Gamma(K^0_{t=0} \to \pi\pi)}{\Gamma(\overline{K}^0_{t=0} \to \pi\pi) + \Gamma(K^0_{t=0} \to \pi\pi)}$$

- Using previous expressions:

$$A_{+-} = \frac{4\Re\{\varepsilon\}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}\right] - 4|\varepsilon| e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t - \phi)}{2\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}\right] - \underbrace{8\Re\{\varepsilon\}|\varepsilon| e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t - \phi)}}$$

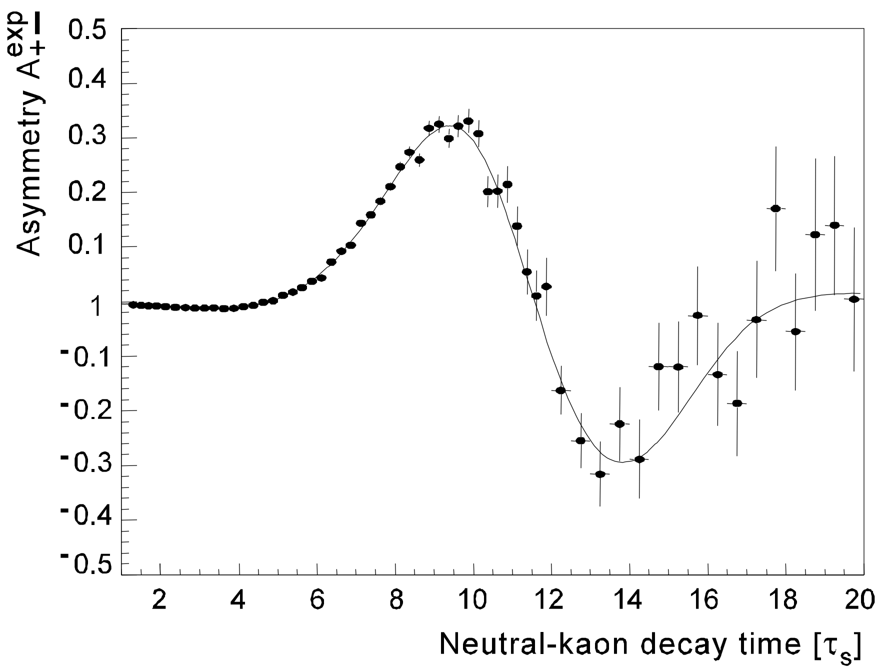
$\propto |\varepsilon|\Re\{\varepsilon\}$ i.e. two small quantities and can safely be neglected

$$A_{+-} \approx \frac{2\Re\{\varepsilon\}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}\right] - 2|\varepsilon| e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t - \phi)}{e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}}$$

$$= 2\Re\{\varepsilon\} - \frac{2|\varepsilon| e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t - \phi)}{e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}}$$

$$= 2\Re\{\varepsilon\} - \frac{2|\varepsilon| e^{(\Gamma_S-\Gamma_L)t/2}\cos(\Delta m.t - \phi)}{1 + |\varepsilon|^2 e^{(\Gamma_S-\Gamma_L)t}}$$

A.Apostolakis et al., Eur. Phys. J. C18 (2000) 41



Best fit to the data:

$$|\varepsilon| = (2.264 \pm 0.035) \times 10^{-3}$$
$$\phi = (43.19 \pm 0.73)^\circ$$

CP Violation via Mixing

★ A full description of the SM origin of CP violation in the kaon system is beyond the level of this course, nevertheless, the relation to the box diagrams is illustrated below

★ The K-long and K-short wave-functions depend on η

$$|K_L\rangle = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle + \eta|\overline{K}^0\rangle)$$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle - \eta|\overline{K}^0\rangle)$$

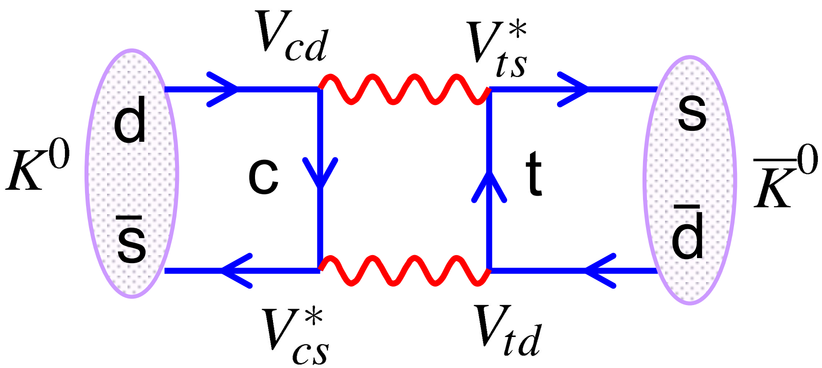
with $$\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

★ If $M_{12}^* = M_{12}; \quad \Gamma_{12}^* = \Gamma_{12}$ then the K-long and K-short correspond to the CP eigenstates K_1 and K_2

- CP violation is therefore associated with imaginary off-diagonal mass and decay elements for the neutral kaon system
- Experimentally, CP violation is small and $\eta \approx 1$
- Define: $\varepsilon = \frac{1-\eta}{1+\eta}$ $\Rightarrow$ $\eta = \frac{1-\varepsilon}{1+\varepsilon}$

•Consider the mixing term M_{12} which arises from the sum over all possible intermediate states in the mixing box diagrams

e.g.



$$M_{12} = A_{ct} V_{cd} V_{cs}^* V_{ts}^* V_{td} + \ldots$$

•Therefore it can be seen that, in the Standard Model, CP violation is associated with the imaginary components of the CKM matrix

•It can be shown that mixing leads to CP violation with

$$|\varepsilon| \propto \Im\{M_{12}\}$$

•The differences in masses of the mass eigenstates can be shown to be:

$$\Delta m_K = m_{K_L} - m_{K_S} \approx \sum_{q,q'} \frac{G_{\mathrm{F}}^2}{3\pi^2} f_K^2 m_K |V_{qd} V_{qs}^* V_{q'd} V_{q's}^*| m_q m_{q'}$$

where q and q' are the quarks in the loops and f_K is a constant

•In terms of the small parameter ε

$$|K_L\rangle = \frac{1}{2\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\overline{K}^0\rangle\right]$$

$$|K_S\rangle = \frac{1}{2\sqrt{1+|\varepsilon|^2}}\left[(1-\varepsilon)|K^0\rangle + (1+\varepsilon)|\overline{K}^0\rangle\right]$$

★ If epsilon is non-zero we have CP violation in the neutral kaon system

Writing $\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}} = \sqrt{\frac{z^*}{z}}$ and $z = ae^{i\phi}$

gives $\eta = e^{-i\phi}$

★ From which we can find an expression for ε

$$\varepsilon.\varepsilon^* = \frac{1-e^{-i\phi}}{1+e^{-i\phi}}.\frac{1-e^{+i\phi}}{1+e^{i\phi}} = \frac{2-\cos\phi}{2+\cos\phi} = \tan^2\frac{\phi}{2}$$

$$|\varepsilon| = |\tan\tfrac{\phi}{2}|$$

★ Experimentally we know ε is small, hence ϕ is small

$$|\varepsilon| \approx \frac{1}{2}\phi = \frac{1}{2}\arg z \approx \frac{1}{2}\frac{\Im\{M_{12} - \frac{1}{2}i\Gamma_{12}\}}{|M_{12} - \frac{1}{2}i\Gamma_{12}|}$$

Time Reversal Violation

- Previously obtained expressions for strangeness oscillations in the absence of CP violation, e.g.

$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

- This analysis can be extended to include the effects of CP violation to give the following rates:

$$\Gamma(K^0_{t=0} \to K^0) \propto \tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma(\overline{K}^0_{t=0} \to \overline{K}^0) \propto \tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma(\overline{K}^0_{t=0} \to K^0) \propto \tfrac{1}{4}(1+4\Re\{\varepsilon\})\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) \propto \tfrac{1}{4}(1-4\Re\{\varepsilon\})\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

★ Including the effects of CP violation find that

$$\Gamma(\overline{K}^0_{t=0} \to K^0) \neq \Gamma(K^0_{t=0} \to \overline{K}^0)$$

Violation of time reversal symmetry !

★ No surprise, as CPT is conserved, CP violation implies T violation

B-physics

- The oscillation of neutral mesons have been observed for heavy mesons as well:

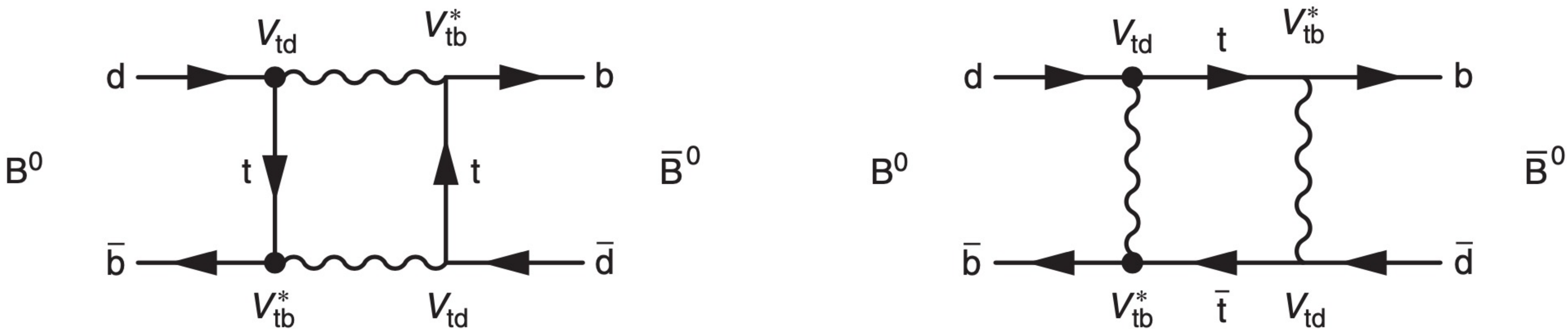
$$B^0(\bar{b}d) \leftrightarrow \overline{B}^0(b\bar{d}), \quad B^0_s(\bar{b}s) \leftrightarrow \overline{B}^0_s(b\bar{s}) \quad \text{and} \quad D^0(\bar{c}u) \leftrightarrow \overline{D}^0(c\bar{u})$$

- The studies of B^0 mesons by BaBar, Belle and LHCb experiments, and of B_s mesons by the LHCb provided crucial information on the CKM matrix and CP violation
- Mathematical description of B-oscillations is the same as for kaons
- Major difference:
  - as B-mesons are heavy they have many decay modes (few hundreds)
  - Typically the decays for B0 and B0-bar are different
  - Hence the interference between the decays is small
  - Consequence: oscillations can be described by a single angle β

$$|B_L\rangle = \frac{1}{\sqrt{2}}\left[|B^0\rangle + e^{-i2\beta}|\overline{B}^0\rangle\right] \quad \text{and} \quad |B_H\rangle = \frac{1}{\sqrt{2}}\left[|B^0\rangle - e^{-i2\beta}|\overline{B}^0\rangle\right]$$

B_L and B_H are a lighter and a heavier states with almost identical lifetimes

B^0 mixing



- In kaon mixing, the contributions from all quarks to a box diagram were similar
- Here, since $|V_{tb}| \gg |V_{ts}| > |V_{td}|$, only box diagrams with top quark matter
- Off-diagonal mass element $M_{12} \propto (V_{td}V_{tb}^*)^2$
- Mass difference $\Delta m_d = m(B_H) - m(B_L) = 2|M_{12}| \propto |(V_{td}V_{tb}^*)^2|$
- Since $|V_{tb}| \approx 1$, Δm_d allows to measure V_{td}

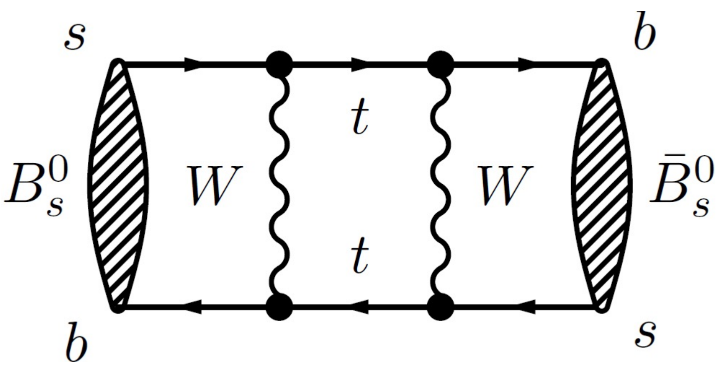
$$\Delta m_{\mathrm{d}} = (0.507 \pm 0.005)\,\mathrm{ps}^{-1} \equiv (3.34 \pm 0.03) \times 10^{-13}\,\mathrm{GeV}$$

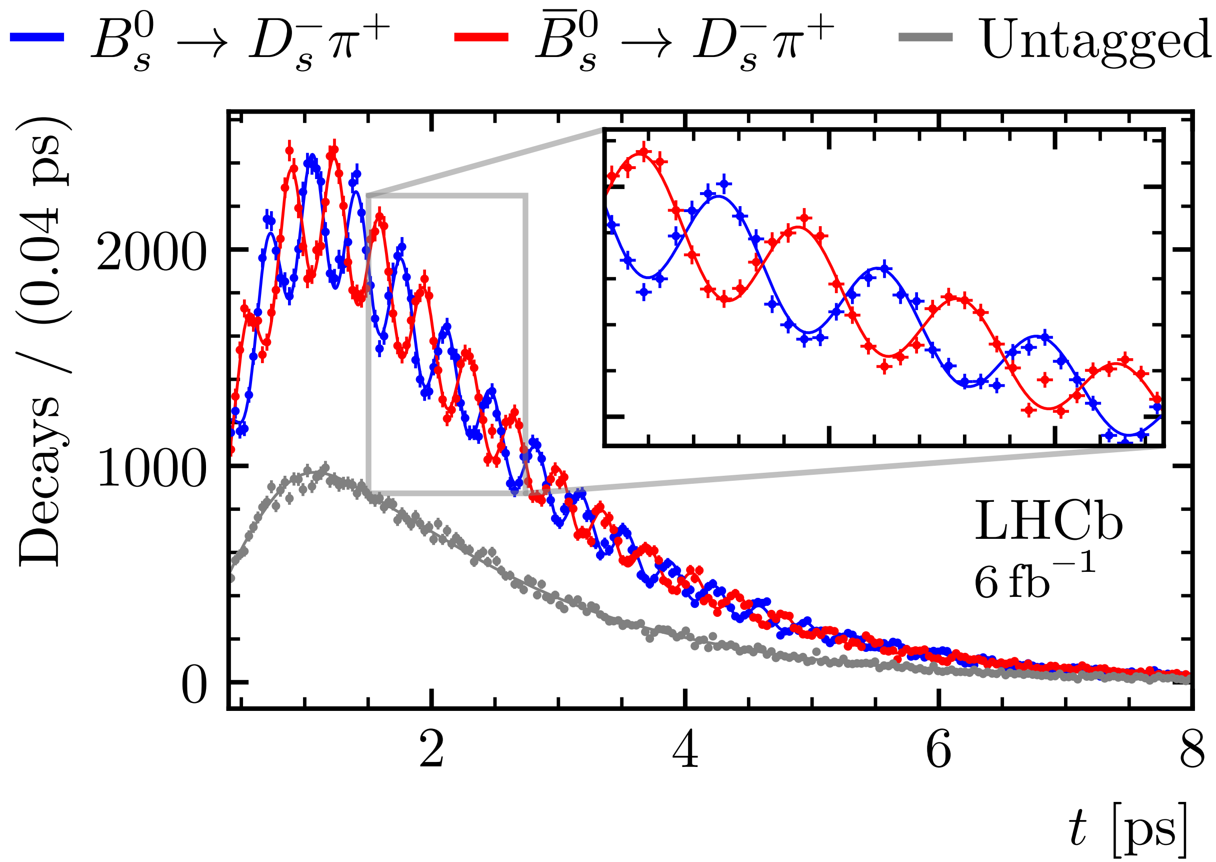
- Similarly, V_{ts} is derived from Δm_s in B_s oscillations

$$\Delta m_s = 17.7656 \pm 0.0057\,\mathrm{ps}^{-1}$$

B_s mixing

- The latest LHCb result: https://arxiv.org/abs/2104.04421





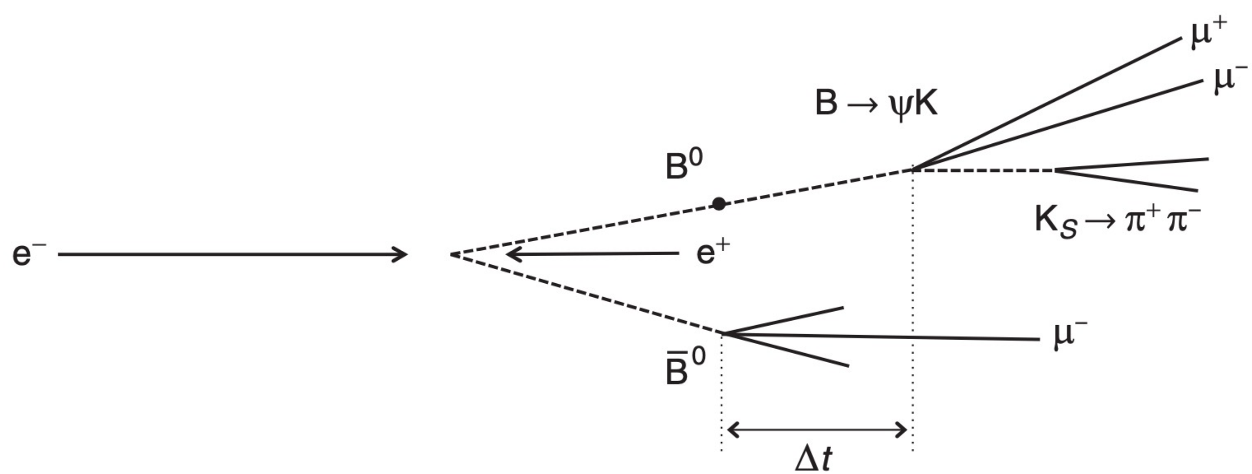
$$\Delta m_s = 17.7656 \pm 0.0057 \text{ps}^{-1}$$

CP violation in B-meson system

- CP violation is observed as three effects:
  - Direct CP violation in decay: $\Gamma(A \rightarrow X) \neq \Gamma(\text{A-bar} \rightarrow \text{X-bar})$, as parameterised by ε' in the neutral kaon system;
  - CP violation in the mixing of neutral mesons as parameterised by ε in the kaon system;
  - CP violation in the interference between decays to a common final state f with and without mixing, for example $B^0 \rightarrow f$ and $B^0 \rightarrow B^0\text{-bar} \rightarrow f$.
- In the SM, the CP violation in B^0 mixing is small
- CP-violating effects in the interference between decays $B^0 \rightarrow f$ and $B^0 \rightarrow B^0\text{-bar} \rightarrow f$ can be relatively large and have been studied extensively by the BaBar, Belle and LHCb experiments

CP violation in the interference: $B^0 \to J/\psi K_S$

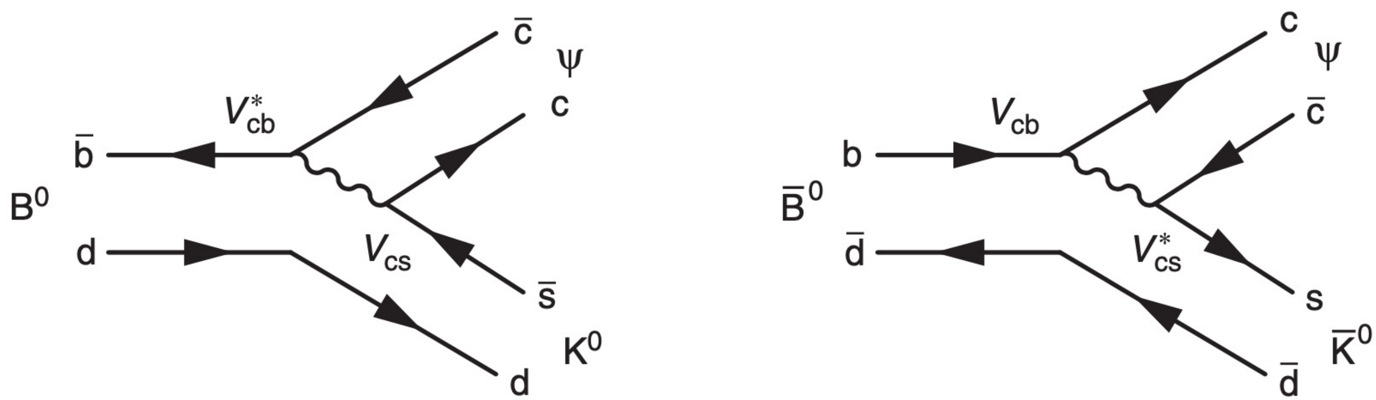
- J/ψ: $J^P = 1^-$, CP = +1
- K_S: $J^P = 0^-$, CP = +1
- B^0 are spin-0 ⇒ $B^0 \to J/\psi K_S$ has L=1 orbital angular momentum
- $CP(J/\psi K_S) = CP(J/\psi)CP(K_S)(-1)^L = (+1)(+1)(-1) = -1$
- Similarly, $B^0 \to J/\psi K_L$ occurs in the CP-even state



- Muon charge from B^0-bar $\to D^+ \mu^- \nu_\mu$-bar tags a flavour of B^0-bar, and a second B^0 at the moment t=0
- Decay to $J/\psi K_S$ happens either directly or after mixing

CP violation in the interference: $B^0 \to J/\psi K_S$

- **Decay proceeds in two stages:**
  - **First, B^0 decays to flavour eigenstate: $B^0 \to J/\psi K^0$ and B^0-bar$\to J/\psi K^0$-bar**
  - **Then the neutral kaons system evolves as a linear combination of physical K_S and K_L states**

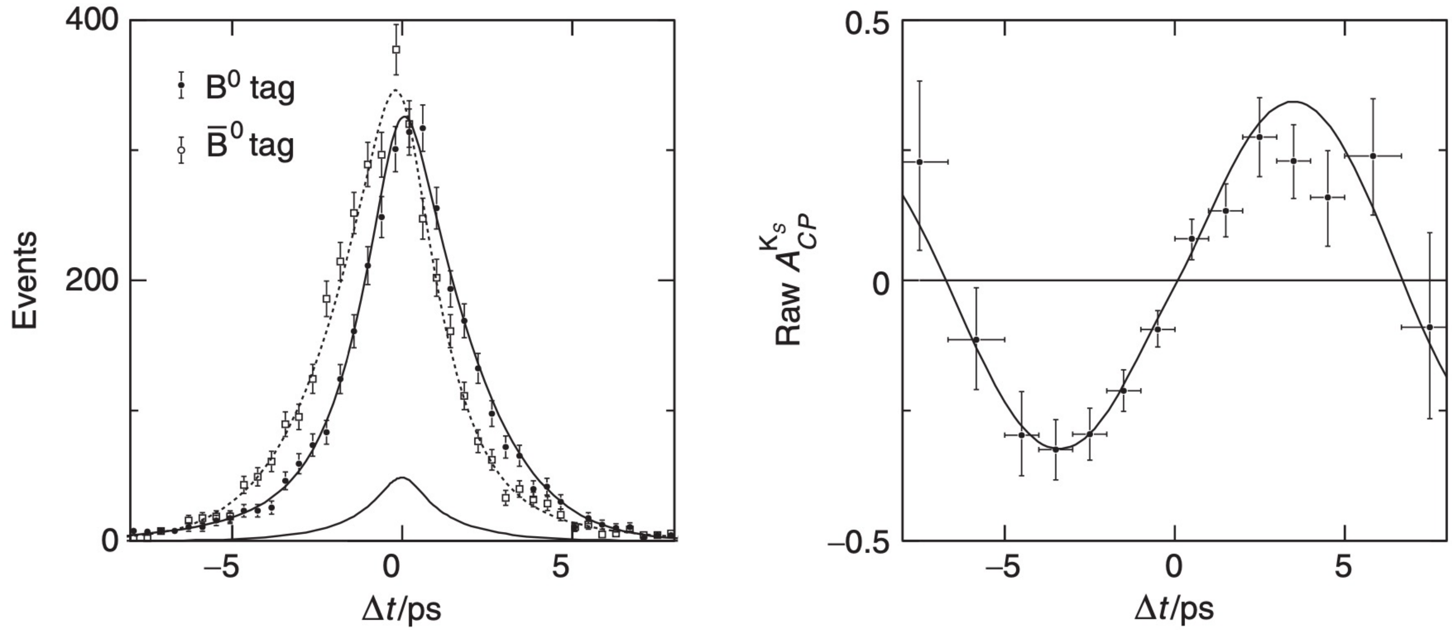


- **CP violation is measurable through the asymmetry**

$$A_{CP}^{K_S} = \frac{\Gamma(\overline{B}^0_{t=0} \to \psi K_S) - \Gamma(B^0_{t=0} \to \psi K_S)}{\Gamma(\overline{B}^0_{t=0} \to \psi K_S) + \Gamma(B^0_{t=0} \to \psi K_S)} = \sin(\Delta m_d t)\ \sin(2\beta).$$

CP violation in the interference: $B^0 \to J/\psi K_S$

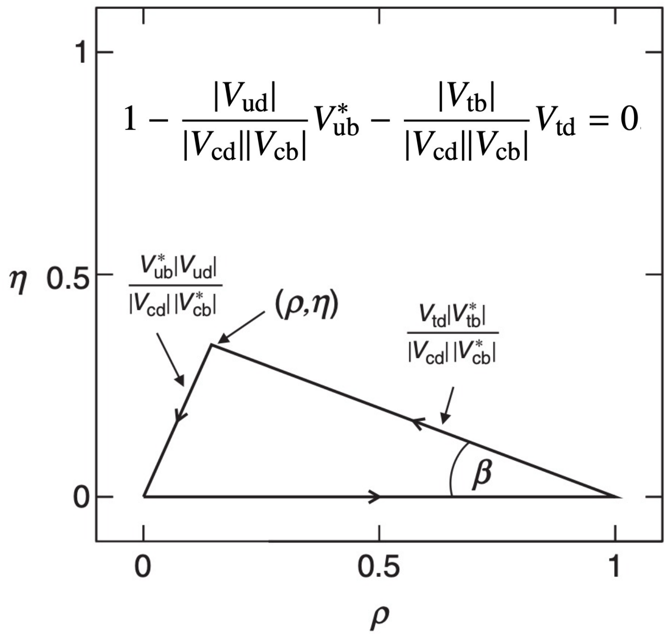
- **Results from Belle:**

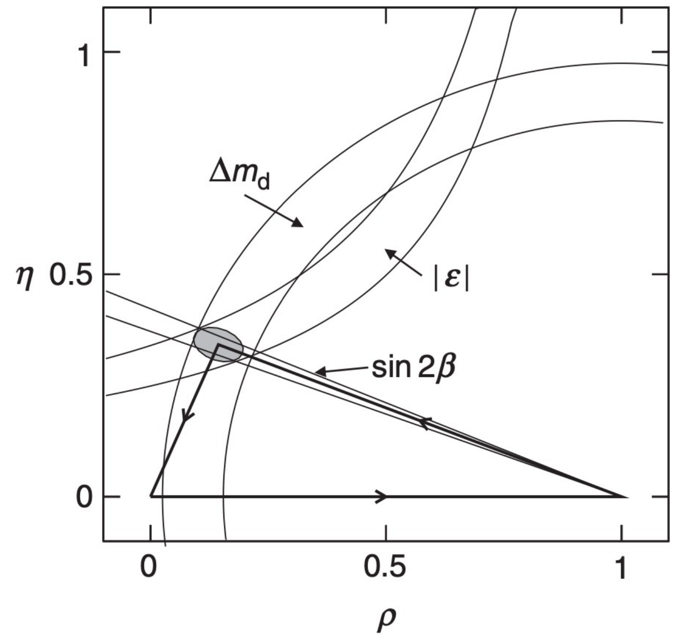


- **the observation of a non-zero value of sin(2β) is a direct manifestation of CP violation in the B-meson system**
- **Measured value: sin(2β) = 0.685 ± 0.032**

CP violation in the quark sector

- **Combination of various measurements:**





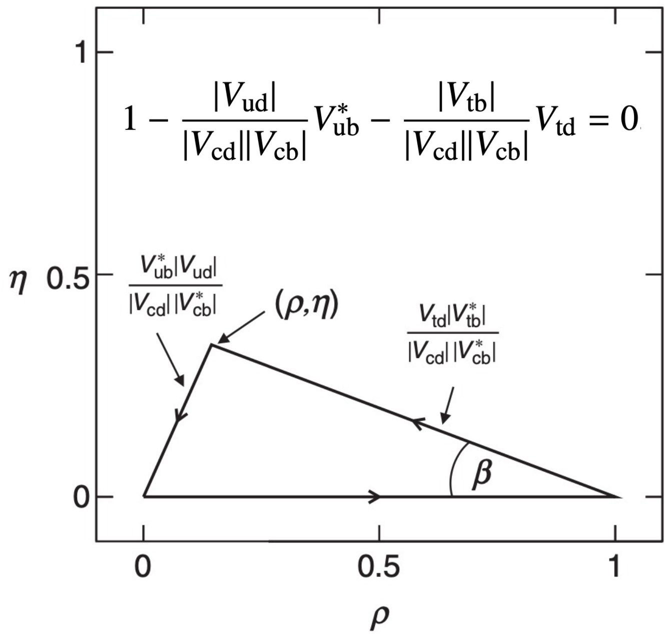
- **CP violation in the weak interactions of hadrons is described by the single irreducible complex phase in the CKM matrix**
- **In the Wolfenstein parametrisation, CP violation is associated with the parameter η. To O(λ⁴), the parameter η appears only in V_{ub} and V_{td}, with**

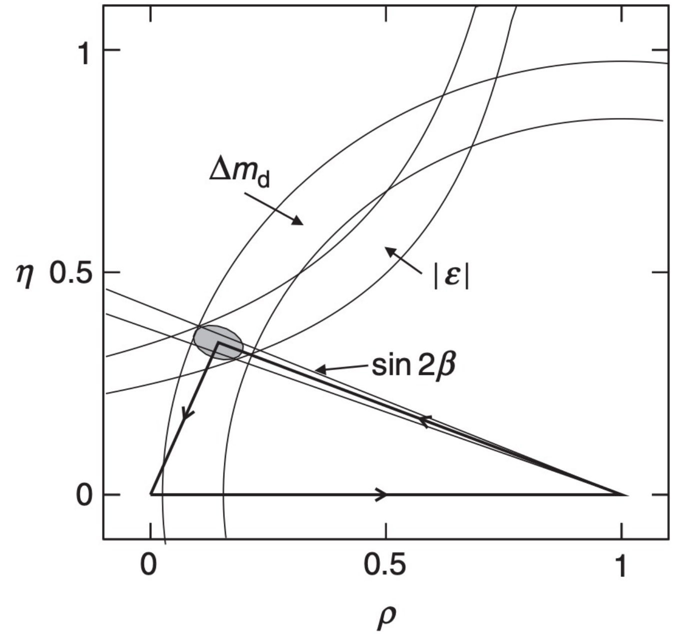
$$V_{ub} \approx A\lambda^3(\rho - i\eta) \quad \text{and} \quad V_{td} \approx A\lambda^3(1 - \rho - i\eta)$$

- **The measurements of non-zero values of |ε| and sin(2β) separately imply that η ≠ 0**
- **But ρ and η are determined from the measurements combinations**

CP violation in the quark sector

- **Combination of various measurements:**

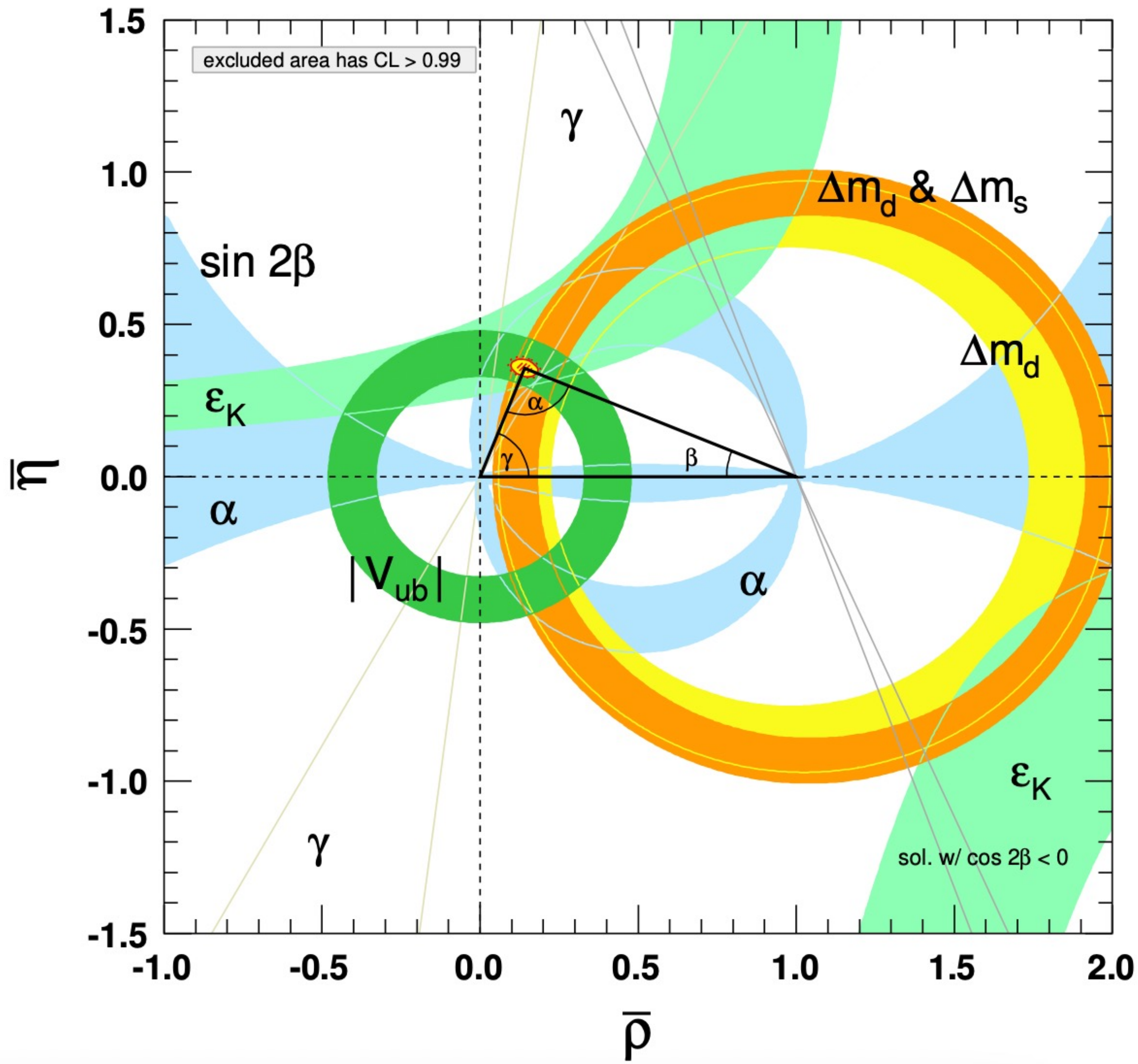




$$\lambda = 0.2253 \pm 0.0007,\ A = 0.811^{+0.022}_{-0.012},\ \rho = 0.13 \pm 0.02,\ \eta = 0.345 \pm 0.014.$$

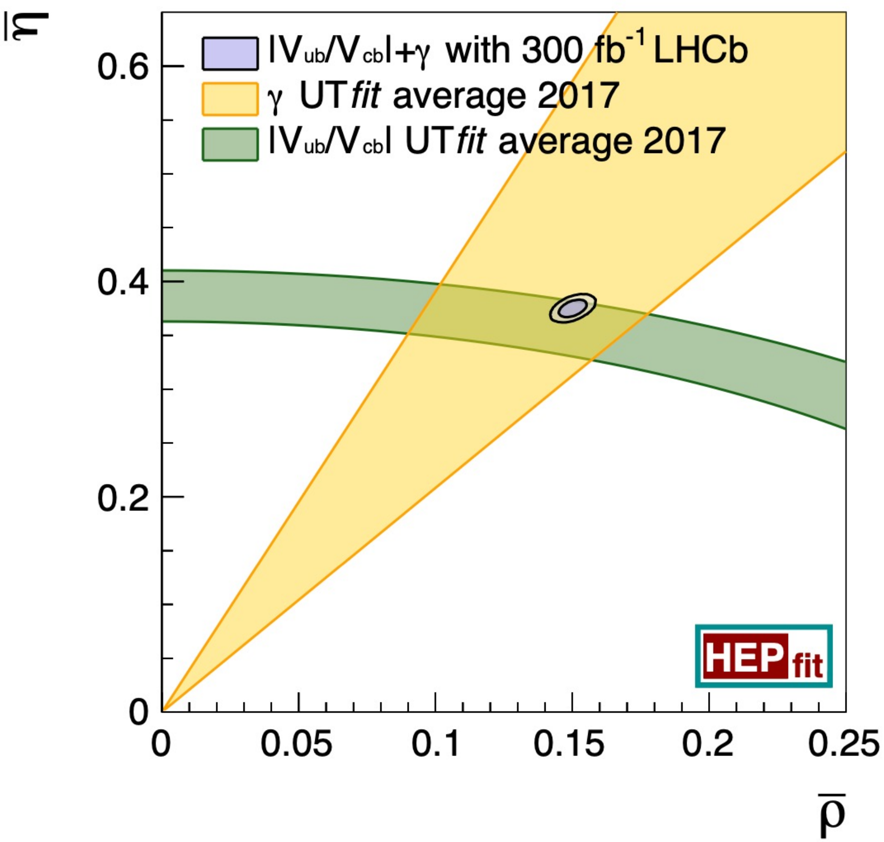
- **With all the measurements, the unitary triangle is closed**
- **A deviation from this could indicate a presence of a new physics**
- **Future experiments, LHCb in particular, will improve many measurements by orders of magnitude**

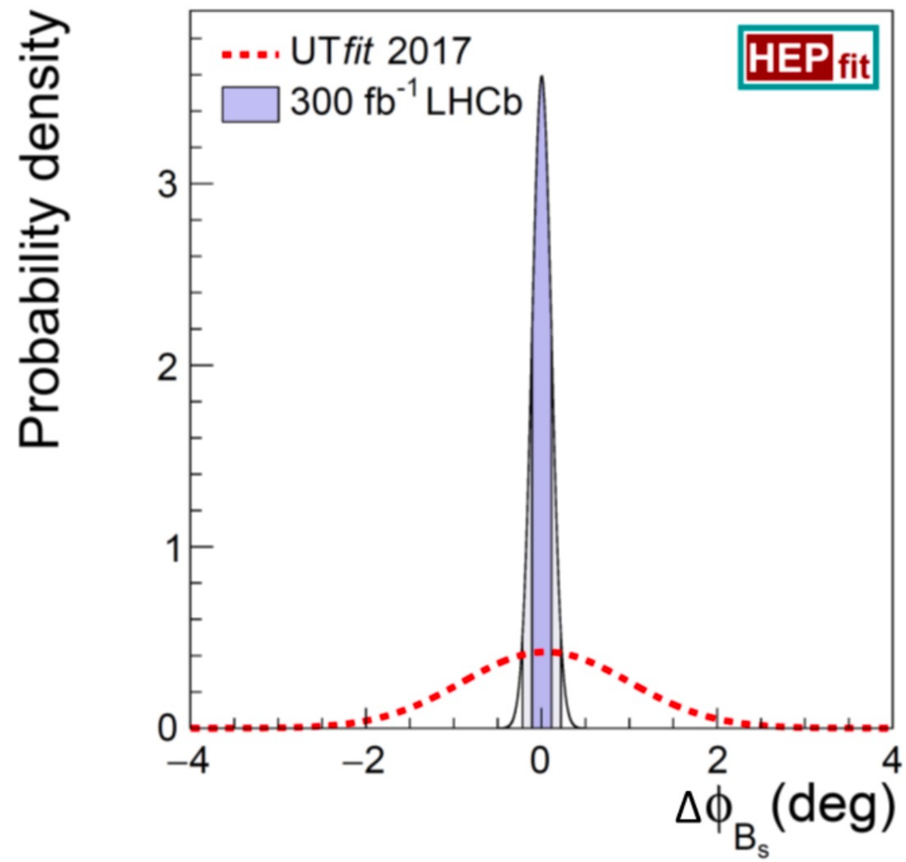
CKM triangle now



LHCb in the HL-LHC phase

- **In case the LHCb upgrade in 2030 happens:**





- **CKM triangle measurements will come on a new level**