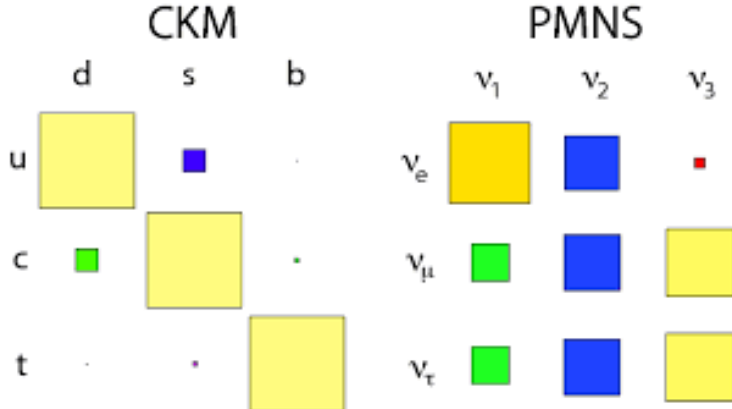




Particle Physics II

Lecture 6: The CKM Matrix and CP Violation

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CP Violation in the Early Universe

- very early in the universe might expect equal numbers of baryons and anti-baryons
- however, today the universe is matter dominated (no evidence for anti-galaxies, etc.)
- from “Big Bang Nucleosynthesis” obtain the matter/antimatter asymmetry

$$\xi = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \approx 10^{-9}$$

i.e. for every baryon in the universe today there are 10^9 photons

How did this happen?

CP Violation in the Early Universe

Early in the universe need to create a **very small** asymmetry between baryons and antibaryons:

- e.g. for every 10^9 antibaryons there were $10^9 + 1$ baryons
- baryons/antibaryons annihilate $\implies$
- 1 baryon + $\sim 10^9$ photons + no antibaryons

CP Violation in the Early Universe

To generate this initial asymmetry, three conditions must be met (Sakharov, 1967):

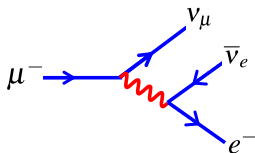
- 1 “Baryon number violation”, i.e. $n_B - n_{\bar{B}}$ is not constant
- 2 “C and CP violation”, if CP is conserved for a reaction which generates a net number of baryons over antibaryons there would be a CP conjugate reaction generating a net number of antibaryons
- 3 “Departure from thermal equilibrium”, in thermal equilibrium any baryon number violating process will be balanced by the inverse reaction

CP Violation in the Early Universe

- CP violation (CPV) is an essential aspect of our understanding of the universe
- a natural question is whether the SM can provide the necessary CPV
- there are two places in the SM where CP violation enters:
 - the **PMNS matrix** (neutrino sector) and
 - the **CKM matrix** (quark sector)
- first, CP violation has been observed in the quark sector
- since we are dealing with quarks, which are only observed as **bound states**, this is a fairly complicated subject. Here we will approach it in two steps:
 - 1 consider **particle – antiparticle oscillations** without CP violation
 - 2 then discuss the effects of **CP violation**
- many features in common with neutrino oscillations – except that we will be considering the oscillations of decaying particles (i.e. mesons)

Weak interaction of quarks

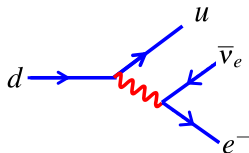
- ① slightly different values of G_F measured in μ decay and nuclear β decay:



$$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2}$$

1.2

1.17



$$G_F^\beta = (1.136 \pm 0.003) \times 10^{-5} \text{ GeV}^{-2}$$

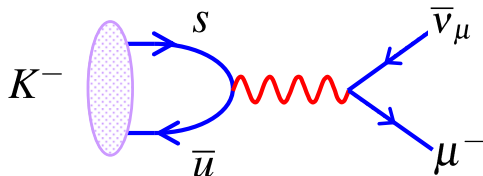
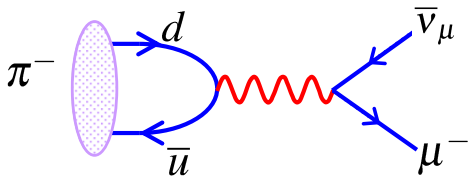
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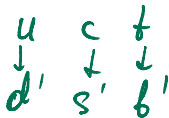
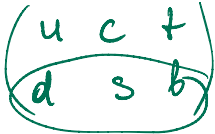
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Weak interaction of quarks

② in addition, certain decay modes are observed to be suppressed:

- e.g. compare $K^- \rightarrow \mu^- \bar{\nu}_\mu$ and $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$
- K decay rate is suppressed $\times 20$ compared to the expectation assuming a universal weak interaction of quarks





Weak interaction of quarks

Both observations explained by Cabibbo hypothesis (1963):

u, d, s

- weak eigenstates are different from mass eigenstates, i.e. weak interactions of quarks have same strength as for leptons but a u -quark couples to a linear combination of s and d quarks:

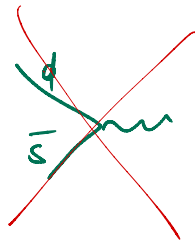
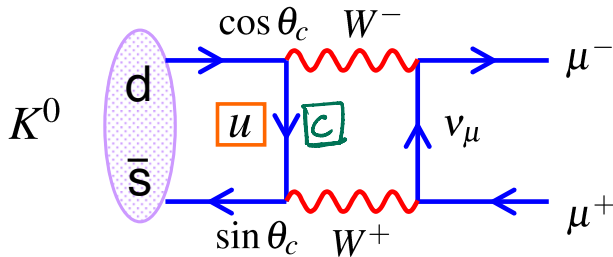
weak eigenstate

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$d' = d \cdot \cos \theta_C + s \cdot \sin \theta_C$

GIM mechanism

- in the weak interaction, have couplings between both ud and us which implies that neutral mesons can decay via box diagrams, e.g.:

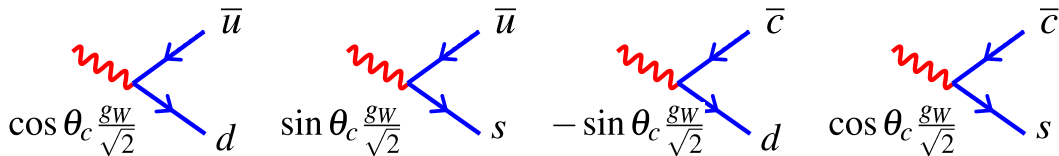


$$M_1 \propto g_W^4 \cos \theta_C \sin \theta_C$$

historically, the (not!) observed branching was much smaller than predicted

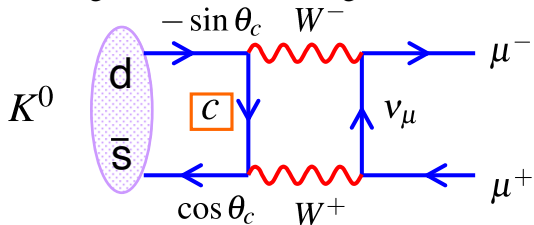
GIM mechanism

- led Glashow, Iliopoulos and Maiani to postulate existence of an extra quark
- before the discovery of charm quark in 1974!
- weak interaction couplings become in this case with u , d , s , and c quarks:



GIM mechanism

- given another box diagram for $K^0 \rightarrow \mu^+ \mu^-$:



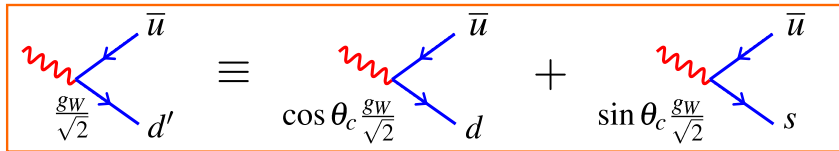
$$M_2 \propto -g_W^4 \cos \theta_C \sin \theta_C$$

same final state, so sum amplitudes:

$$|M|^2 = |M_1 + M_2|^2 \approx 0$$

cancellation not exact: $m_u \neq m_c$

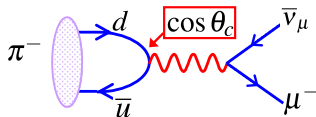
- i.e. weak interaction couples different generation of quarks between each other:



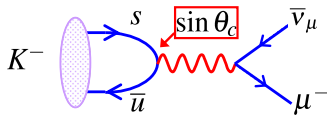
(the same is true for leptons, e.g. $e^- \nu_1$, $e^- \nu_2$, $e^- \nu_3$ couplings connect different generations)

GIM mechanism

- helps to explain the observations on the previous pages with $\theta_C = 13.1^\circ$
 - K decay suppressed by a factor of $\tan^2 \theta_C \approx 0.05$ relative to π decay

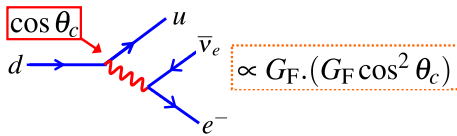
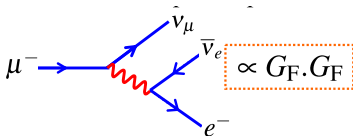


$$\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu) \propto |M|^2 \propto \cos^2 \theta_C$$



$$\Gamma(K^- \rightarrow \mu^- \bar{\nu}_\mu) \propto |M|^2 \propto \sin^2 \theta_C$$

- hence expect $G_F^\beta = G_F^\mu \cos \theta_C$:



CKM matrix

- extend ideas to three quark flavors (as in three flavor neutrino treatment):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak eigenstates
CKM Matrix
Mass Eigenstates

(Cabibbo, Kobayashi, Maskawa)

By convention CKM matrix defined as acting on quarks with charge $-\frac{1}{3}e$

- e.g. weak eigenstate d' is produced in weak decay of an up quark:

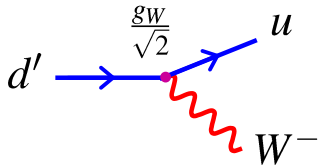
$$u \xrightarrow{\frac{g_W}{\sqrt{2}}} d' \quad \equiv \quad u \xrightarrow{V_{ud}^* \frac{g_W}{\sqrt{2}}} d + u \xrightarrow{V_{us}^* \frac{g_W}{\sqrt{2}}} s + u \xrightarrow{V_{ub}^* \frac{g_W}{\sqrt{2}}} b$$

W^+ W^+ W^+ W^+

- the CKM matrix elements V_{ij} are complex constants
- the CKM matrix is unitary
- the V_{ij} are not predicted by the SM – have to be determined from the experiment

Feynman rules

- depending on the order of the interaction, $u \rightarrow d$ or $d \rightarrow u$, the CKM matrix elements enter as either V_{ud} or V_{ud}^*
- writing the interaction in terms of **weak** eigenstates:



$$j_{d'u} = \bar{u} \left[-i \frac{g_W}{\sqrt{2}} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \right] \left[d' \right]$$

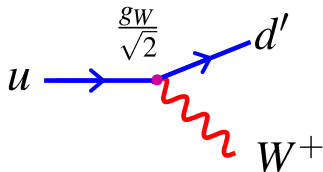
note: $\bar{u}$ is the adjoint spinor, not the antiup quark

$V_{ud} d + V_{us} s + V_{ub} b$

- giving the $d \rightarrow u$ weak current: $j_{du} = \bar{u} \left[-i \frac{g_W}{\sqrt{2}} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \right] V_{ud} d$

Feynman rules

- for $u \rightarrow d'$ the weak current is:

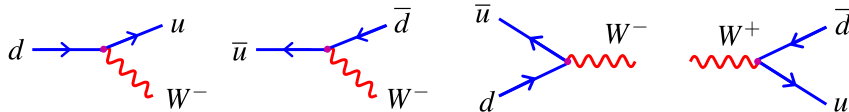


$$j_{ud'} = \bar{d}' \left[-i \frac{g_W}{\sqrt{2}} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \right] u$$

- for mass eigenstates: $\bar{d}' = d'^{\dagger} \gamma^0 \implies (V_{ud} d)^{\dagger} \gamma^0 = V_{ud}^* d'^{\dagger} \gamma^0 = V_{ud}^* \bar{d}'$
- giving the $u \rightarrow d$ weak current: $j_{ud} = \bar{d} V_{ud}^* \left[-i \frac{g_W}{\sqrt{2}} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \right] u$
- hence when the charge $-1/3$ quark enters as the adjoint spinor, the complex conjugate of the CKM matrix is used

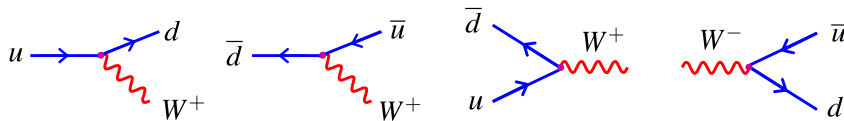
Feynman rules

- the vertex factor for the following diagrams:



is $-i \frac{g_W}{\sqrt{2}} V_{ud} \gamma^\mu \frac{1}{2} (1 - \gamma^5)$

- whereas, the vertex factor for:



is $-i \frac{g_W}{\sqrt{2}} V_{ud}^* \gamma^\mu \frac{1}{2} (1 - \gamma^5)$

Feynman rules

- experimentally determine:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ ? & ? & ? \end{pmatrix}$$

- currently, little direct experimental information on V_{td} , V_{ts} , V_{tb}
- assuming unitarity of CKM matrix, e.g. $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$:

$\theta_c \approx 13^\circ$ Cabibbo matrix

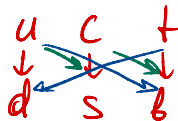
$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$

Near diagonal – very different from PMNS

$t \rightarrow Wb$; $t \rightarrow Ws, Wd$ ← limits

Feynman rules

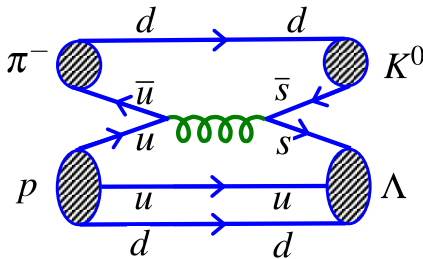
- Note: within the SM, the charged current $W^\pm$, weak interaction:
 - ① provides the only way to **change flavor**
 - ② only way to **change from one generation of quarks or leptons to another**
- the off-diagonal elements of the CKM matrix are relatively small:
 - weak interaction is largest between quarks of the same generation
 - coupling between first and third generation quarks is very small
- as for PMNS matrix – CKM matrix allows CP violation in the SM



The neutral kaon system

- neutral kaons are produced copiously in strong interactions, e.g.:

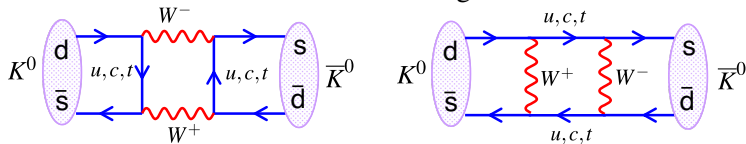
$$\pi^-(d\bar{u}) + p(uud) \rightarrow \Lambda(uds) + K^0(d\bar{s})$$



$$\pi^+(u\bar{d}) + p(uud) \rightarrow K^+(u\bar{s}) + \bar{K}^0(s\bar{d}) + p(uud)$$

The neutral kaon system

- neutral kaons decay via the weak interaction
- the weak interaction also allows mixing of neutral kaons via “box diagrams”



- this allows transitions between the strong eigenstates $K^0, \bar{K}^0$
- consequently, the neutral kaons propagate as eigenstates of the overall strong+weak interaction; i.e. as linear combinations of $K^0, \bar{K}^0$
- these neutral kaon states are called “K-short” K_S and “K-long” K_L
- they have approximately same mass $m(K_S) \approx m(K_L) \approx 498 \text{ MeV}$
- but very different lifetimes: $\tau(K_S) = 0.9 \times 10^{-10} \text{ s}$, $\tau(K_L) = 0.5 \times 10^{-7} \text{ s}$

CP eigenstates

- the K_S and K_L are closely related to CP-eigenstates
- the strong eigenstates $K^0(d\bar{s})$ and $\bar{K}^0(s\bar{d})$ have $J^P = 0^-$, i.e. are pseudoscalars with

$$\hat{P} |K^0\rangle = - |K^0\rangle, \hat{P} |\bar{K}^0\rangle = - |\bar{K}^0\rangle$$

- the charge conjugation operator changes particle into antiparticle and vice versa:
 - $\hat{C} |K^0\rangle = \hat{C} |d\bar{s}\rangle = + |s\bar{d}\rangle = |\bar{K}^0\rangle$
 - $\hat{C} |\bar{K}^0\rangle = |K^0\rangle$
 - here the “+” sign is purely conventional, could have used a “−” with no physical consequences;

CP eigenstates

- consequently:
 - $\hat{C}\hat{P}|K^0\rangle = -|\bar{K}^0\rangle,$
 - $\hat{C}\hat{P}|\bar{K}^0\rangle = -|K^0\rangle,$
 - i.e. neither K^0 or $\bar{K}^0$ are eigenstates of CP
- but can form CP eigenstates from linear combinations:
 - $|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle), \hat{C}\hat{P}|K_1\rangle = +|K_1\rangle$
 - $|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle), \hat{C}\hat{P}|K_2\rangle = -|K_2\rangle$

Decays of CP eigenstates

- neutral kaons often decay to pions (the lightest hadrons)
- the kaon masses are approximately 498 MeV and the pion masses are approximately 140 MeV
- hence neutral kaons can decay to either 2 or 3 pions

$$3 \times 140 = 420 < 498$$

Decays of CP eigenstates

Decays to Two Pions:

★ $K^0 \rightarrow \pi^0 \pi^0$ $J^P: 0^- \rightarrow 0^- + 0^-$

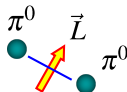
- Conservation of angular momentum $\rightarrow \vec{L} = 0$

$$\Rightarrow \hat{P}(\pi^0 \pi^0) = -1 \cdot -1 \cdot (-1)^L = +1$$

- The $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ is an eigenstate of $\hat{C}$

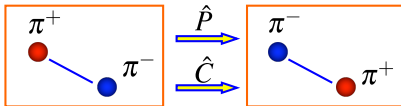
$$C(\pi^0 \pi^0) = C\pi^0 \cdot C\pi^0 = +1 \cdot +1 = +1$$

$$\Rightarrow CP(\pi^0 \pi^0) = +1$$



★ $K^0 \rightarrow \pi^+ \pi^-$ as before $\hat{P}(\pi^+ \pi^-) = +1$

★ Here the **C** and **P** operations have the identical effect



Hence the combined effect of $\hat{C}\hat{P}$ is to leave the system unchanged

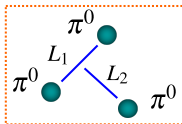
$$\hat{C}\hat{P}(\pi^+ \pi^-) = +1$$

Neutral kaon decays to two pions occur in CP even (i.e. +1) eigenstates

Decays of CP eigenstates

Decays to Three Pions:

★ $K^0 \rightarrow \pi^0 \pi^0 \pi^0$



$J^P : 0^- \rightarrow 0^- + 0^- + 0^-$

• Conservation of angular momentum:

$$L_1 \oplus L_2 = 0 \Rightarrow L_1 = L_2$$

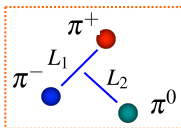
$$P(\pi^0 \pi^0 \pi^0) = -1 \cdot -1 \cdot -1 \cdot (-1)^{L_1} \cdot (-1)^{L_2} = -1$$

$$C(\pi^0 \pi^0 \pi^0) = +1 \cdot +1 \cdot +1$$

$$\Rightarrow CP(\pi^0 \pi^0 \pi^0) = -1$$

Remember L is magnitude of angular momentum vector

★ $K^0 \rightarrow \pi^+ \pi^- \pi^0$



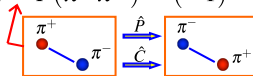
• Again $L_1 = L_2$

$$P(\pi^+ \pi^- \pi^0) = -1 \cdot -1 \cdot -1 \cdot (-1)^{L_1} \cdot (-1)^{L_2} = -1$$

$$C(\pi^+ \pi^- \pi^0) = +1 \cdot C(\pi^+ \pi^-) = P(\pi^+ \pi^-) = (-1)^{L_1}$$

Hence:

$$CP(\pi^+ \pi^- \pi^0) = -1 \cdot (-1)^{L_1}$$



- The small amount of energy available in the decay, $m(K) - 3m(\pi) \approx 70 \text{ MeV}$ means that the $L > 0$ decays are strongly suppressed by the angular momentum barrier effects (recall QM tunnelling in alpha decay)

Neutral kaon decays to three pions occur in CP odd (i.e. -1) eigenstates

Decays of CP eigenstates

- if CP were conserved in the weak decays of neutral kaons, would expect decays to pions to occur from states of definite CP (i.e. the CP eigenstates K_1, K_2):
 - $|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$, $\hat{C}\hat{P}|K_1\rangle = +|K_1\rangle$, $K_1 \rightarrow \pi\pi$: CP-even
 - $|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$, $\hat{C}\hat{P}|K_2\rangle = -|K_2\rangle$, $K_2 \rightarrow \pi\pi\pi$: CP-odd
- expect lifetimes of CP eigenstates to be very different
 - for two pion decay energy available: $m_K - 2m_\pi \approx 220 \text{ MeV}$
 - for three pion decay energy available: $m_K - 3m_\pi \approx 70 \text{ MeV}$
- expect decays to $\pi\pi$ to be more rapid than to $\pi\pi\pi$ due to increased phase space

Decays of CP eigenstates

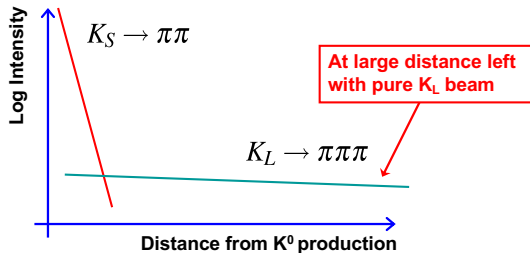
- this is exactly what is observed: a **short-lived** state “K-short” which decays to (mainly) to $\pi\pi$ and a **long-lived** state “K-long” which decays to $\pi\pi\pi$
- in the absence of CP violation we can identify
 - $|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$ with decays $K_S \rightarrow \pi\pi$
 - $|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$ with decays $K_L \rightarrow \pi\pi\pi$

Neutral kaons decays to pions

- consider the decays of a beam of K^0
- the decays to pions occur in states of definite CP
- if CP is conserved in the decay, need to express K^0 in terms of K_S and K_L :

$$|K^0\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$$

- from the point of view of decays to pions, a K^0 beam is a linear combination of CP eigenstates: a rapidly decaying CP-even and a long-lived CP-odd components
- expect to see $\pi\pi$ decays near start of beam and $\pi\pi\pi$ decays further downstream



Neutral kaons decays to pions

How this works algebraically:

- suppose at time $t = 0$ make a beam of pure K^0 : $|\psi(t = 0)\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$
- put in the time dependence of wave-function: $|K_S(t)\rangle = |K_S\rangle e^{-im_S t - \Gamma_S t/2}$
NOTE: the term $e^{-\Gamma_S t/2}$ ensures the K_S probability density decays exponentially, i.e.
 $|\psi_S|^2 = \langle K_S(t)|K_S(t)\rangle = e^{-\Gamma_S t} = e^{-t/\tau_S}$
- hence the wave-function evolves as:
$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} \left[|K_S\rangle e^{-(im_S + \frac{\Gamma_S}{2}t)} + |K_L\rangle e^{-(im_L + \frac{\Gamma_L}{2}t)} \right]$$
- writing $\theta_S(t) = e^{-(im_S + \frac{\Gamma_S}{2}t)}$ and $\theta_L(t) = e^{-(im_L + \frac{\Gamma_L}{2}t)}$,
$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} (\theta_S(t) |K_S\rangle + \theta_L(t) |K_L\rangle)$$
- the decay rate to two pions for a state which was produced as K^0 :
 $\Gamma(K_{t=0}^0 \rightarrow \pi\pi) \propto |\langle K_S|\psi(t)\rangle|^2 \propto |\theta_S(t)|^2 = e^{-\Gamma_S t} = e^{-t/\tau_S}$,
which is anticipated, i.e. decays of the short lifetime component K_S

Neutral kaons decays to leptons

- neutral kaons can also decay to leptons:
 - $\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e, \bar{K}^0 \rightarrow \pi^+ \mu^- \bar{\nu}_\mu$
 - $K^0 \rightarrow \pi^- e^+ \nu_e, K^0 \rightarrow \pi^- \mu^+ \nu_\mu$
 - note: the final states are not CP eigenstates which is why we express these decays in terms of $K^0, \bar{K}^0$
- neutral kaons propagate as combined eigenstates of weak + strong interaction i.e. the K_S, K_L . The main decay modes/branching fractions are:

K_S	$\rightarrow \pi^+ \pi^-$	$BR = 69.2\%$
	$\rightarrow \pi^0 \pi^0$	$BR = 30.7\%$
	$\rightarrow \pi^- e^+ \nu_e$	$BR = 0.03\%$
	$\rightarrow \pi^+ e^- \bar{\nu}_e$	$BR = 0.03\%$
	$\rightarrow \pi^- \mu^+ \nu_\mu$	$BR = 0.02\%$
	$\rightarrow \pi^+ \mu^- \bar{\nu}_\mu$	$BR = 0.02\%$

K_L	$\rightarrow \pi^+ \pi^- \pi^0$	$BR = 12.6\%$
	$\rightarrow \pi^0 \pi^0 \pi^0$	$BR = 19.6\%$
	$\rightarrow \pi^- e^+ \nu_e$	$BR = 20.2\%$
	$\rightarrow \pi^+ e^- \bar{\nu}_e$	$BR = 20.2\%$
	$\rightarrow \pi^- \mu^+ \nu_\mu$	$BR = 13.5\%$
	$\rightarrow \pi^+ \mu^- \bar{\nu}_\mu$	$BR = 13.5\%$

- leptonic decays are more likely for the K_L because the three pion decay modes have a lower decay rate than the two pion modes of the K_S

Strangeness oscillations (neglecting CP violation)

- the “semileptonic” decay rate to $\pi^- e^+ \nu_e$ occurs from the K^0 state
- to calculate the expected decay rate, need to know the K^0 component of the wave-function
- for a beam which was initially K^0 we have:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(\theta_S(t) |K_S\rangle + \theta_L(t) |K_L\rangle)$$

- writing K_S , K_L in terms of K^0 , $\bar{K}^0$:

$$|\psi(t)\rangle = \frac{1}{2} \left[\theta_S(t) (|K^0\rangle - |\bar{K}^0\rangle) + \theta_L(t) (|K^0\rangle + |\bar{K}^0\rangle) \right] \quad (1)$$

$$= \frac{1}{2}(\theta_S + \theta_L) |K^0\rangle + \frac{1}{2}(\theta_S - \theta_L) |\bar{K}^0\rangle \quad (2)$$

Strangeness oscillations (neglecting CP violation)

- because $\theta_S(t) \neq \theta_L(t)$, a state that was initially a K^0 evolves with time into a mixture of K^0 and $\bar{K}^0$ – “**strangeness oscillations**”
- the K^0 intensity (i.e. K^0 fraction):

$$\Gamma(K_{t=0}^0 \rightarrow K^0) = |\langle K^0 | \psi(t) \rangle|^2 = \frac{1}{4} |\theta_S + \theta_L|^2$$

- similarly

$$\Gamma(K_{t=0}^0 \rightarrow \bar{K}^0) = |\langle \bar{K}^0 | \psi(t) \rangle|^2 = \frac{1}{4} |\theta_S - \theta_L|^2$$

Strangeness oscillations (neglecting CP violation)

- using the identity: $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\Re(z_1 z_2^*)$

$$\begin{aligned}
 |\theta_S \pm \theta_L|^2 &= |e^{-(im_S + \frac{1}{2}\Gamma_S)t} \pm e^{-(im_L + \frac{1}{2}\Gamma_L)t}|^2 \\
 &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2\Re\{e^{-im_S t} e^{-\frac{1}{2}\Gamma_S t} \cdot e^{+im_L t} e^{-\frac{1}{2}\Gamma_L t}\} \\
 &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t} \Re\{e^{-i(m_S - m_L)t}\} \\
 &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t} \cos(m_S - m_L)t \\
 &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t} \cos \Delta m t
 \end{aligned}$$

- oscillations between neutral kaon states with frequency given by the mass splitting $\Delta m = m(K_L) - m(K_S)$
- as in neutrino oscillations! Only this time we have **decaying states**
- from previous equations:

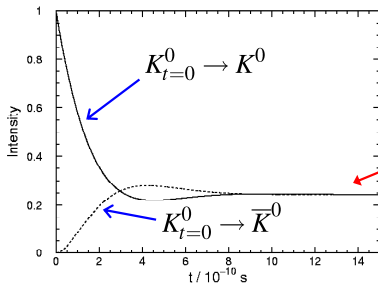
$$\begin{aligned}
 \Gamma(K_{t=0}^0 \rightarrow K^0) &= \frac{1}{4} [e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t] \\
 \Gamma(K_{t=0}^0 \rightarrow \bar{K}^0) &= \frac{1}{4} [e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t]
 \end{aligned}$$

Strangeness oscillations (neglecting CP violation)

- experimentally: $\tau(K_S) = 0.9 \times 10^{-10}$ s, $\tau(K_L) = 0.5 \times 10^{-7}$ s, and $\Delta m = (3.506 \pm 0.006) \times 10^{-15}$ GeV, i.e. the $m(K_L) > m(K_S)$ by **1 part in 10^{16}**
- the mass difference corresponds to an oscillation period of

$$T_{osc} = \frac{2\pi\hbar}{\Delta m} \approx 1.2 \times 10^{-9} \text{ s}$$

- the oscillation period is relatively long compared to the K_S lifetime and consequently do not observe very pronounced oscillations



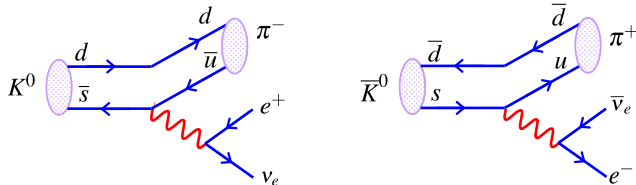
$$\Gamma(K_{t=0}^0 \rightarrow K^0) = \frac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

$$\Gamma(K_{t=0}^0 \rightarrow \bar{K}^0) = \frac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

After a few K_S lifetimes, left with a pure K_L beam which is half K^0 and half $\bar{K}^0$

Strangeness oscillations (neglecting CP violation)

- strangeness oscillations can be studied with semileptonic decays



- the charge of the observed pion (or lepton) tags the decay as from either a $\bar{K}^0$ or K^0 because:

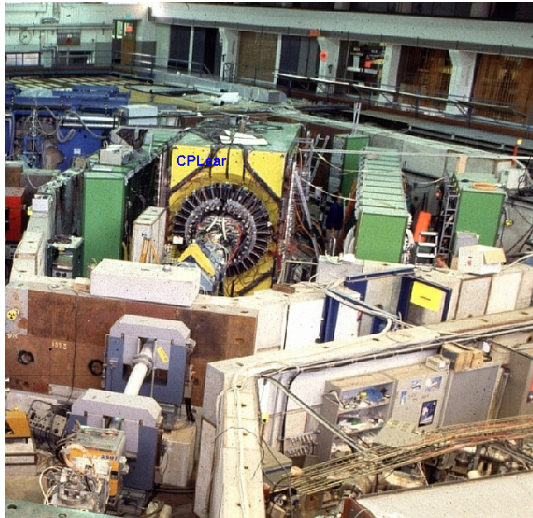
$$\begin{array}{lcl}
 K^0 \rightarrow \pi^- e^+ \nu_e & \text{but} & \bar{K}^0 \not\rightarrow \pi^- e^+ \nu_e \\
 \bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e & & K^0 \not\rightarrow \pi^+ e^- \bar{\nu}_e
 \end{array}
 \quad \text{NOT ALLOWED}$$

- so for an initial K^0 beam observe the decays to both charge combinations:

$$\begin{array}{ll}
 K_{t=0}^0 \rightarrow K^0 & K_{t=0}^0 \rightarrow \bar{K}^0 \\
 \quad \quad \quad \hookrightarrow \pi^- e^+ \nu_e & \quad \quad \quad \hookrightarrow \pi^+ e^- \bar{\nu}_e
 \end{array}$$

which provides a way of measuring strangeness oscillations

The CPLEAR experiment @ CERN (1990 – 1996)



- used a low energy $\bar{p}$ beam
- neutral kaons produced:

$$\bar{p}p \rightarrow K^- \pi^+ K^0$$

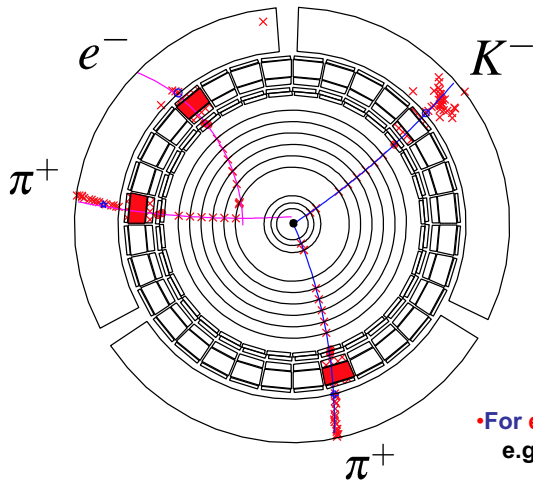
$$\bar{p}p \rightarrow K^+ \pi^- \bar{K}^0$$

- low energy, so particles produced almost at rest
- observe production and decay in the same detector
- charge of $K^\pm \pi^\mp$ in the production process tags the initial neutral kaon as either K^0 or $\bar{K}^0$

- charge of decay products tags the decay as being either K^0 or $\bar{K}^0$
- provides a direct probe of strangeness oscillations

The CPLEAR experiment

An example of a CPLEAR event



$$K^-(s\bar{u})$$

$$K^0(d\bar{s})$$

$$\bar{K}^0(s\bar{d})$$

Production:

$$\bar{p}p \rightarrow K^- \pi^+ K^0$$

Decay:

$$\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e$$

Mixing

- For each event know initial wave-function,
e.g. here: $|\psi(t=0)\rangle = |K^0\rangle$

The CPLEAR experiment

- can measure decay rates as a function of time for all combinations: e.g.

$$R^+ = \Gamma(K_{t=0}^0 \rightarrow \pi^- e^+ \bar{\nu}_e) \propto \Gamma(K_{t=0}^0 \rightarrow K^0)$$

- from previous equations:

$$\begin{aligned} R_+ &\equiv \Gamma(K_{t=0}^0 \rightarrow \pi^- e^+ \nu_e) = N_{\pi e \nu} \frac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right] \\ R_- &\equiv \Gamma(K_{t=0}^0 \rightarrow \pi^+ e^- \bar{\nu}_e) = N_{\pi e \nu} \frac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right] \\ \bar{R}_- &\equiv \Gamma(\bar{K}_{t=0}^0 \rightarrow \pi^+ e^- \bar{\nu}_e) = N_{\pi e \nu} \frac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right] \\ \bar{R}_+ &\equiv \Gamma(\bar{K}_{t=0}^0 \rightarrow \pi^- e^+ \nu_e) = N_{\pi e \nu} \frac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right] \end{aligned}$$

where $N_{\pi e \nu}$ is some overall normalization factor

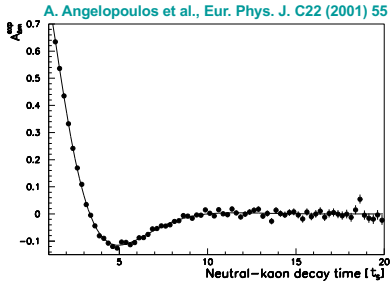
- express measurements as an “asymmetry” to remove dependence on $N_{\pi e \nu}$:

$$A_{\Delta m} = \frac{(R_+ + \bar{R}_-) - (R_- + \bar{R}_+)}{(R_+ + \bar{R}_-) + (R_- + \bar{R}_+)}$$

The CPLEAR experiment

- using the above expressions for R_+ etc obtain:

$$A_{\Delta m} = \frac{2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$



- ★ Points show the data
- ★ The line shows the theoretical prediction for the value of Δm most consistent with the CPLEAR data:

$$\Delta m = 3.485 \times 10^{-15} \text{ GeV}$$

- the sign of Δm is not determined here but is known from other experiments
- when the CPLEAR results are combined with experiments as Fermilab, obtain:

$$\Delta m = m(K_L) - m(K_S) = (3.506 \pm 0.006) \times 10^{-15} \text{ GeV}$$

CP violation in the kaon system

- so far we have ignored CP violation in the neutral kaon system
- identified the K_S as the CP-even state and K_L as the CP-odd state

$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$

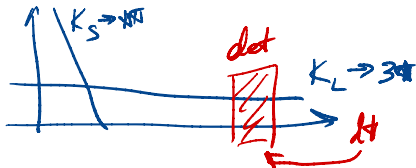
with decays: $K_S \rightarrow \pi\pi$

CP = +1

$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$

with decays: $K_L \rightarrow \pi\pi\pi$

CP = -1
- at a long distance from the production point, a beam of neutral kaons will be 100% K_L (the K_S component will have decayed away)
- hence, if CP is conserved, would expect to see only 3π decays



CP violation in the kaon system

- several attempts to find violation, e.g. [Phys. Rev. Lett. 6 \(1961\) 552](#) (27 citations)

VOLUME 6, NUMBER 10

PHYSICAL REVIEW LETTERS

MAY 15, 1961

DECAY PROPERTIES OF K_2^0 MESONS*

D. Neagu, E. O. Okonov, N. I. Petrov, A. M. Rosanova, and V. A. Rusakov

Joint Institute of Nuclear Research, Moscow, U.S.S.R.

(Received April 20, 1961)

- “We have recorded so far more than 500 V^0 events interpreted as decays of K_2^0 mesons.”
- “Neither the authors of the quoted paper nor we have found any event of K_2^0 -meson decay into two charged π mesons.”

CP violation in the kaon system

- more perseverance in [Phys. Rev. Lett. 13 \(1964\) 138](#) (3861 citation)

VOLUME 13, NUMBER 4

PHYSICAL REVIEW LETTERS

27 JULY 1964

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON^{*†}

J. H. Christenson, J. W. Cronin,[†] V. L. Fitch,[†] and R. Turlay[§]

Princeton University, Princeton, New Jersey

(Received 10 July 1964)

- observed 45 $K_L \rightarrow \pi^+ \pi^-$ decays in a sample of 22700 kaon decays a long distance from the production point
 $\implies$ **weak interactions violate CP**
- $22700/45 \approx 504$: people in Dubna just didn't get lucky to observe a possible two-pion event!
- Fitch & Cronin received a Nobel prize in Physics in 1980 for this discovery!

CP violation in the kaon system

- CP is violated in hadronic weak interactions, but only at the level of 2 parts in 1000:

K_L to pion BRs:

K_L	$\rightarrow \pi^+ \pi^- \pi^0$	$BR = 12.6\%$	$CP = -1$
	$\rightarrow \pi^0 \pi^0 \pi^0$	$BR = 19.6\%$	$CP = -1$
	$\rightarrow \pi^+ \pi^-$	$BR = 0.20\%$	$CP = +1$
	$\rightarrow \pi^0 \pi^0$	$BR = 0.08\%$	$CP = +1$

CP violation in the kaon system

★ Two possible explanations of CP violation in the kaon system:

i) The K_S and K_L do not correspond exactly to the CP eigenstates K_1 and K_2

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} [|K_1\rangle + \varepsilon|K_2\rangle] \quad |K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} [|K_2\rangle + \varepsilon|K_1\rangle]$$

with $|\varepsilon| \sim 2 \times 10^{-3}$

• In this case the observation of $K_L \rightarrow \pi\pi$ is accounted for by:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} [|K_2\rangle + \varepsilon|K_1\rangle]$$

$\swarrow$ $\pi\pi$ CP = +1
 $\searrow$ $\pi\pi\pi$ CP = -1

ii) and/or CP is violated in the decay

$$|K_L\rangle = |K_2\rangle$$

CP = -1
 $\swarrow$ $\pi\pi\pi$ CP = -1
 $\searrow$ $\pi\pi$ CP = +1

Parameterised by ε'

★ Experimentally both known to contribute to the mechanism for CP violation in the kaon system but i dominates: $\varepsilon'/\varepsilon = (1.7 \pm 0.3) \times 10^{-3}$ { NA48 (CERN)
{ KTeV (FermiLab)

CP violation in semileptonic decays

- if observe a neutral kaon beam a long time after production (i.e. at large distances) it will consist of a pure K_L component:

$$|K_L\rangle = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\varepsilon|^2}} \left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle \right]$$

$\downarrow$
 $\longrightarrow \pi^+ e^- \bar{\nu}_e$

$\downarrow$
 $\longrightarrow \pi^- e^+ \nu_e$

- decays to $\pi^- e^+ \nu_e$ must come from the $\bar{K}^0$ component, and decays to $\pi^+ e^- \bar{\nu}_e$ must come from the K^0 component:

$$\Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e) \propto |\langle \bar{K}^0 | K_L \rangle|^2 \propto |1 - \varepsilon|^2 \approx 1 - 2\Re\{\varepsilon\}$$

$$\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) \propto |\langle K^0 | K_L \rangle|^2 \propto |1 + \varepsilon|^2 \approx 1 + 2\Re\{\varepsilon\}$$

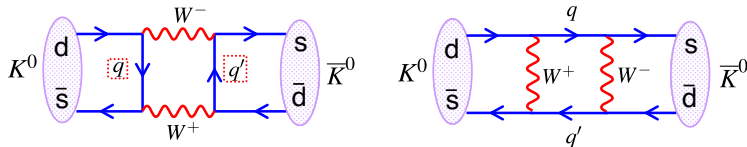
- results in a small difference in decay rates: the decay to $\pi^- e^+ \nu_e$ is **0.7% more likely** than the decay to $\pi^+ e^- \bar{\nu}_e$

CP violation in semileptonic decays

- this difference has been observed and thus provides the first direct evidence for an absolute difference between matter and antimatter
- it also provides an unambiguous definition of matter which could, for example, be transmitted to aliens in a distant galaxy:
“The electrons in our atoms have the same charge as those emitted least often in the decays of the long-lived neutral kaon”

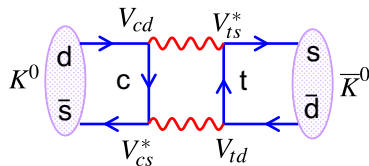
CP violation and the CKM matrix

- how we can explain $\Gamma(\bar{K}_{t=0}^0 \rightarrow K^0) \neq \Gamma(K_{t=0}^0 \rightarrow \bar{K}^0)$ in terms of the CKM matrix?
- consider the box diagrams responsible for mixing, i.e.:



where $q = \{u, c, t\}$, $q' = \{u, c, t\}$

- have to sum over all possible quark exchanges in the box. For simplicity, consider just one diagram:

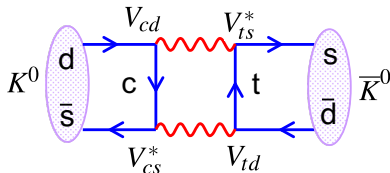


$$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^*$$

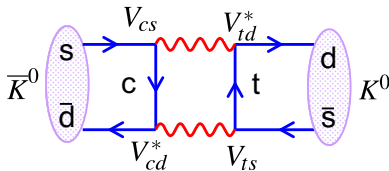
A constant related to integrating over virtual momenta

CP violation and the CKM matrix

- compare the equivalent box diagrams for $K^0 \rightarrow \bar{K}^0$ and $\bar{K}^0 \rightarrow K^0$:



$$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^*$$



$$M'_{fi} \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*$$

- therefore difference in rates:

$$\Gamma(K^0 \rightarrow \bar{K}^0) - \Gamma(\bar{K}^0 \rightarrow K^0) \propto M_{fi} - M_{fi}^* = 2\mathcal{I}\{M_{fi}\}$$

- hence the rates can only be different if the CKM matrix has imaginary component:

$$|\epsilon| \propto \mathcal{I}\{M_{fi}\}$$

- in the kaon system we can show:

$$|\epsilon| \propto A_{ut} \cdot \mathfrak{I}\{V_{ud} V_{us}^* V_{td} V_{ts}^*\} + A_{ct} \cdot \mathfrak{I}\{V_{cd} V_{cs}^* V_{td} V_{ts}^*\} + A_{tt} \cdot \mathfrak{I}\{V_{td} V_{ts}^* V_{td} V_{ts}^*\}$$

Shows that CP violation is related to the imaginary parts of the CKM matrix

Summary

- the weak interactions of quarks are described by the CKM matrix
- similar structure to the lepton sector, although unlike the PMNS matrix, the CKM matrix is nearly diagonal
- CP violation enters through via a complex phase in the CKM matrix
- a great deal of experimental evidence for CP violation in the weak interactions of quarks
- CP violation is needed to explain matter – antimatter asymmetry in the Universe
- HOWEVER, CP violation in the SM is not sufficient to explain the matter – antimatter asymmetry. There is probably another mechanism.