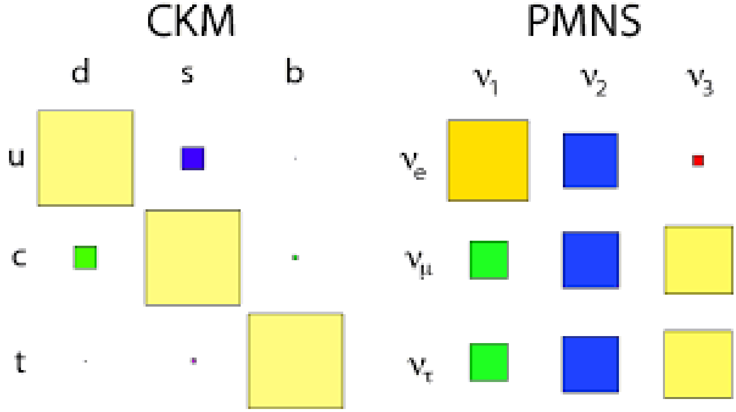

Particle Physics II
Lecture 6: The CKM Matrix and CP Violation

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CP Violation in the Early Universe

- very early in the universe might expect equal numbers of baryons and anti-baryons
- however, today the universe is matter dominated (no evidence for anti-galaxies, etc.)
- from "Big Bang Nucleosynthesis" obtain the matter/antimatter asymmetry

$$\xi = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \approx 10^{-9}$$

i.e. for every baryon in the universe today there are 10^9 photons

How did this happen?

CP Violation in the Early Universe

Early in the universe need to create a **very small** asymmetry between baryons and antibaryons:

- e.g. for every 10^9 antibaryons there were $10^9 + 1$ baryons
- baryons/antibaryons annihiliate $\Longrightarrow$
- 1 baryon + $\sim 10^9$ photons + no antibaryons

CP Violation in the Early Universe

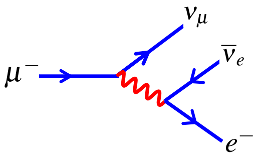
To generate this initial asymmetry, three conditions must be met (Sakharov, 1967):

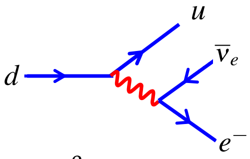
1. “Baryon number violation”, i.e. $n_B - n_{\bar{B}}$ is not constant
2. “C and CP violation”, if CP is conserved for a reaction which generates a net number of baryons over antibaryons there would be a CP conjugate reaction generating a net number of antibaryons
3. “Departure from thermal equilibrium”, in thermal equilibrium any baryon number violating process will be balanced by the inverse reaction

CP Violation in the Early Universe

- CP violation (CPV) is an essential aspect of our understanding of the universe
- a natural question is whether the SM can provide the necessary CPV
- there are two places in the SM where CP violation enters:
  - the PMNS matrix (neutrino sector) and
  - the CKM matrix (quark sector)
- first, CP violation has been observed in the quark sector
- since we are dealing with quarks, which are only observed as **bound states**, this is a fairly complicated subject. Here we will approach it in two steps:
  1. consider particle – antiparticle oscillations without CP violation
  2. then discuss the effects of CP violation
- many features in common with neutrino oscillations – except that we will be considering the oscillations of decaying particles (i.e. mesons)

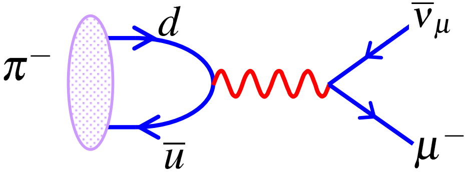
1. slightly different values of G_F measured in μ decay and nuclear β decay:

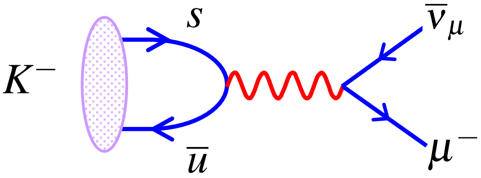




$$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\,\mathrm{GeV}^{-2} \qquad G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

2. in addition, certain decay modes are observed to be suppressed:
   - e.g. compare $K^- \to \mu^- \bar{\nu}_\mu$ and $\pi^- \to \mu^- \bar{\nu}_\mu$
   - K decay rate is suppressed $\times 20$ compared to the expectation assuming a universal weak interaction of quarks



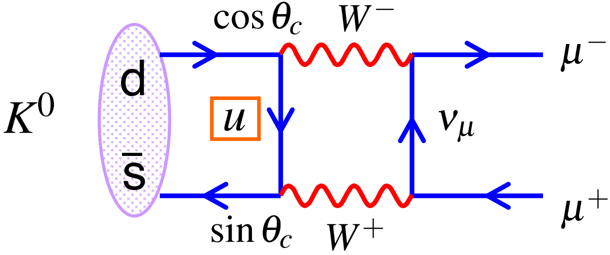


Both observations explained by Cabbibo hypothesis (1963):

- weak eigenstates are different from mass eigenstates, i.e. weak interactions of quarks have same strength as for leptons but a u-quark couples to a linear combination of s and d quarks:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

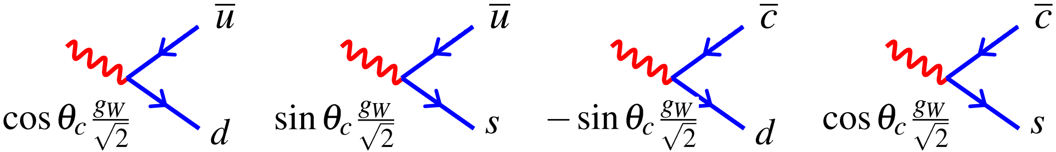
- in the weak interaction, have couplings between both ud and us which implies that neutral mesons can decay via box diagrams, e.g.:



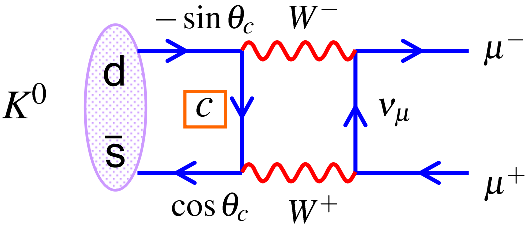
$$M_1 \propto g_W^4 \cos\theta_C \sin\theta_C$$

historically, the (not!) observed branching was much smaller than predicted

- led Glashow, Illiopoulos and Maiani to postulate existence of an extra quark
- before the discovery of charm quark in 1974!
- weak interaction couplings become in this case with u, d, s, and c quarks:



- given another box diagram for $K^0 \to \mu^+\mu^-$:



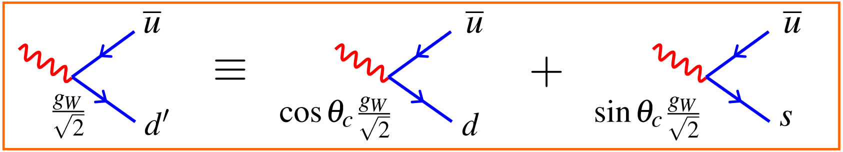
$M_2 \propto -g_W^4 \cos\theta_C \sin\theta_C$

same final state, so sum amplitudes:

$$|M|^2 = |M_1 + M_2|^2 \approx 0$$

cancellation not exact: $m_u \neq m_c$

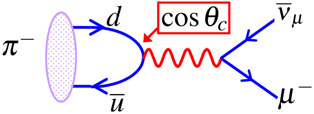
- i.e. weak interaction couples different generation of quarks between each other:

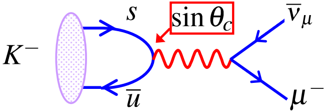


(the same is true for leptons, e.g. $e^-\nu_1$, $e^-\nu_2$, $e^-\nu_3$ couplings connect different generations)

GIM mechanism

- helps to explain the observations on the previous pages with $\theta_C = 13.1^\circ$
  - K decay suppressed by a factor of $\tan^2\theta_C \approx 0.05$ relative to π decay

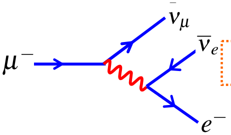




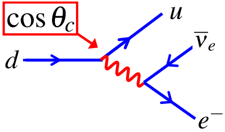
$$\Gamma(\pi^- \to \mu^-\overline{\nu}_\mu) \propto |M|^2 \propto \cos^2\theta_c$$

$$\Gamma(K^- \to \mu^-\overline{\nu}_\mu) \propto |M|^2 \propto \sin^2\theta_c$$

  - hence expect $G_F^\beta = G_F^\mu \cos\theta_C$:



$\propto G_F.G_F$



$\propto G_F.(G_F\cos^2\theta_c)$

CKM matrix

- extend ideas to three quark flavors (as in three flavor neutrino treatment):

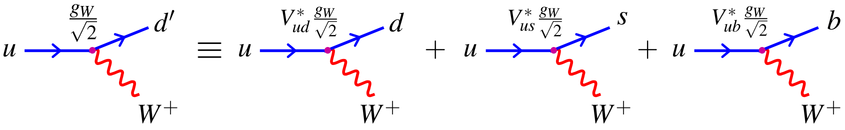
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

By convention CKM matrix defined as acting on quarks with charge $-\frac{1}{3}e$

Weak eigenstates | **CKM Matrix** | **Mass Eigenstates**

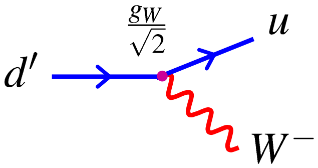
(**Cabibbo, Kobayashi, Maskawa**)

- e.g. weak eigenstate d' is produced in weak decay of an up quark:



- the CKM matrix elements V_{ij} are complex constants
- the CKM matrix is unitary
- the V_{ij} are not predicted by the SM – have to be determined from the experiment

- depending on the order of the interaction, $u \to d$ or $d \to u$, the CKM matrix elements enter as either V_{ud} or V_{ud}^*
- writing the interaction in terms of **weak** eigenstates:

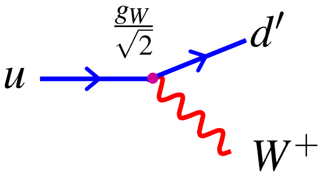


$$j_{d'u} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{1}{2}(1-\gamma^5)\right]d'$$

note: $\bar{u}$ is the adjoint spinor, not the antiup quark

- giving the $d \to u$ weak current: $j_{du} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{1}{2}(1-\gamma^5)\right]V_{ud}d$

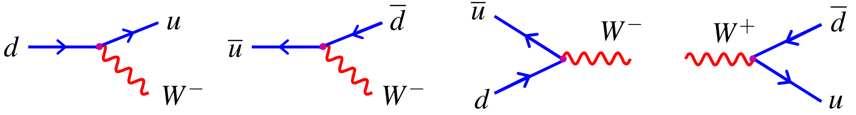
- for $u \to d'$ the weak current is:



$$j_{ud'} = \bar{d}' \left[-i \frac{g_W}{\sqrt{2}} \gamma^\mu \frac{1}{2}(1-\gamma^5) \right] u$$

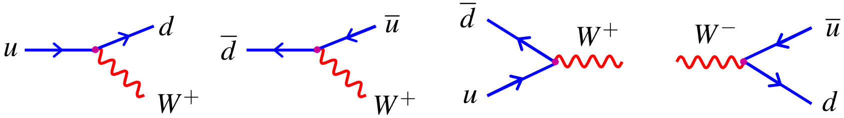
- for mass eigenstates: $\bar{d}' = d'^\dagger \gamma^0 \implies (V_{ud} d)^\dagger \gamma^0 = V_{ud}^* d^\dagger \gamma^0 = V_{ud}^* \bar{d}$
- giving the $u \to d$ weak current: $j_{ud} = \bar{d} V_{ud}^* \left[-i \frac{g_W}{\sqrt{2}} \gamma^\mu \frac{1}{2}(1-\gamma^5) \right] u$
- hence when the charge $-1/3$ quark enters as the adjoint spinor, the complex conjugate of the CKM matrix is used

- the vertex factor for the following diagrams:



is $-i\frac{g_W}{\sqrt{2}}V_{ud}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)$

- whereas, the vertex factor for:



is $-i\frac{g_W}{\sqrt{2}}V^*_{ud}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)$

- experimentally determine:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ ? & ? & ? \end{pmatrix}$$

- currently, little direct experimental information on V_{td}, V_{ts}, V_{tb}
- assuming unitarity of CKM matrix, e.g. $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$:

Cabibbo matrix

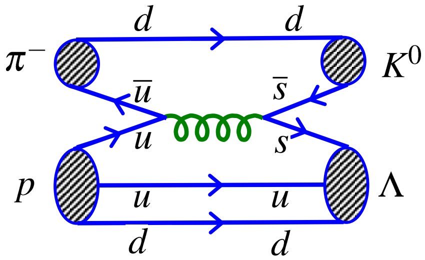
$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$

Near diagonal – very different from PMNS

- Note: within the SM, the charged current $W^{\pm}$, weak interaction:
  1. provides the only way to change flavor
  2. only way to change from one generation of quarks or leptons to another
- the off-diagonal elements of the CKM matrix are relatively small:
  - weak interaction is largest between quarks of the same generation
  - coupling between first and third generation quarks is very small
- as for PMNS matrix – CKM matrix allows CP violation in the SM

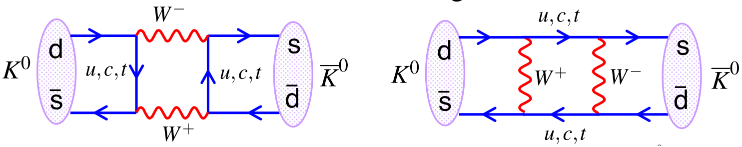
- neutral kaons are produced copiously in strong interactions, e.g.:

$$\pi^-(d\bar{u}) + p(uud) \to \Lambda(uds) + K^0(d\bar{s})$$



$$\pi^+(u\bar{d}) + p(uud) \to K^+(u\bar{s}) + \overline{K}^0(s\bar{d}) + p(uud)$$

- neutral kaons decay via the weak interaction
- the weak interaction also allows mixing of neutral kaons via "box diagrams"



- this allows transitions between the strong eigenstates states K^0, $\overline{K}^0$
- consequently, the neutral kaons propagate as eigenstates of the overall strong+weak interaction; i.e. as linear combinations of K^0, $\overline{K}^0$
- these neutral kaon states are called "K-short" K_S and "K-long" K_L
- they have approximately same mass $m(K_S) \approx m(K_L) \approx 498$ MeV
- but very different lifetimes: $\tau(K_S) = 0.9 \times 10^{-10}$ s, $\tau(K_L) = 0.5 \times 10^{-7}$ s

CP eigenstates

- the K_S and K_L are closely related to CP-eigenstates
- the strong eigenstates $K^0(d\bar{s})$ and $\overline{K}^0(s\bar{d})$ have $J^P = 0^-$, i.e. are pseudoscalars with

$$\widehat{P}\left|K^0\right\rangle = -\left|K^0\right\rangle, \widehat{P}\left|\overline{K}^0\right\rangle = -\left|\overline{K}^0\right\rangle$$

- the charge conjugation operator changes particle into antiparticle and vice versa:
  - $\widehat{C}\left|K^0\right\rangle = \widehat{C}\left|d\bar{s}\right\rangle = +\left|s\bar{d}\right\rangle = \left|\overline{K}^0\right\rangle$
  - $\widehat{C}\left|\overline{K}^0\right\rangle = \left|K^0\right\rangle$
  - here the "+" sign is purely conventional, could have used a "−" with no physical consequences;

- consequently:
  - $\widehat{C}\widehat{P}\left|K^0\right\rangle = -\left|\overline{K}^0\right\rangle$,
  - $\widehat{C}\widehat{P}\left|\overline{K}^0\right\rangle = -\left|K^0\right\rangle$,
  - i.e. neither K^0 or $\overline{K}^0$ are eigenstates of CP
- but can form CP eigenstates from linear combinations:
  - $\left|K_1\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\overline{K}^0\right\rangle\right)$, $\widehat{C}\widehat{P}\left|K_1\right\rangle = +\left|K_1\right\rangle$
  - $\left|K_2\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\overline{K}^0\right\rangle\right)$, $\widehat{C}\widehat{P}\left|K_2\right\rangle = -\left|K_2\right\rangle$

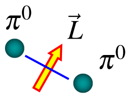
Decays of CP eigenstates

- neutral kaons often decay to pions (the lightest hadrons)
- the kaon masses are approximately 498 MeV and the pion masses are approximately 140 MeV
- hence neutral kaons can decay to either 2 or 3 pions

Decays of CP eigenstates

Decays to Two Pions:

☆ $K^0 \rightarrow \pi^0\pi^0 \qquad J^P: \quad 0^- \rightarrow 0^- + 0^-$



- Conservation of angular momentum ➞ $\vec{L} = 0$

$$\Rightarrow \quad \hat{P}(\pi^0\pi^0) = -1.-1.(-1)^L = +1$$

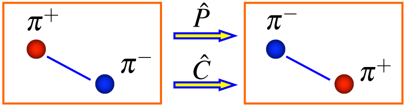
- The $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ is an eigenstate of $\hat{C}$

$$C(\pi^0\pi^0) = C\pi^0.C\pi^0 = +1.+1 = +1$$

$$\Rightarrow \quad CP(\pi^0\pi^0) = +1$$

☆ $K^0 \rightarrow \pi^+\pi^-$ as before $\hat{P}(\pi^+\pi^-) = +1$

★ Here the **C** and **P** operations have the identical effect



Hence the combined effect of $\hat{C}\hat{P}$ is to leave the system unchanged

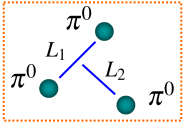
$$\hat{C}\hat{P}(\pi^+\pi^-) = +1$$

Neutral kaon decays to two pions occur in CP even (i.e. +1) eigenstates

Decays of CP eigenstates

Decays to Three Pions:

☆ $K^0 \rightarrow \pi^0\pi^0\pi^0$ $J^P: \quad 0^- \rightarrow 0^- + 0^- + 0^-$



Remember L is magnitude of angular momentum vector

- Conservation of angular momentum:

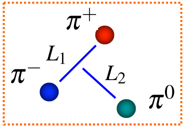
$$L_1 \oplus L_2 = 0 \quad \Rightarrow \quad L_1 = L_2$$

$$P(\pi^0\pi^0\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^0\pi^0\pi^0) = +1.+1.+1$$

$$\Rightarrow \quad CP(\pi^0\pi^0\pi^0) = -1$$

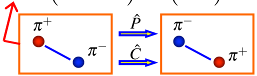
☆ $K^0 \rightarrow \pi^+\pi^-\pi^0$



- Again $L_1 = L_2$

$$P(\pi^+\pi^-\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^+\pi^-\pi^0) = +1.C(\pi^+\pi^-) = P(\pi^+\pi^-) = (-1)^{L_1}$$



Hence: $CP(\pi^+\pi^-\pi^0) = -1.(-1)^{L_1}$

- The small amount of energy available in the decay, $m(K) - 3m(\pi) \approx 70\,\text{MeV}$ means that the L>0 decays are strongly suppressed by the angular momentum barrier effects (recall QM tunnelling in alpha decay)

Neutral kaon decays to three pions occur in CP odd (i.e. -1) eigenstates

Decays of CP eigenstates

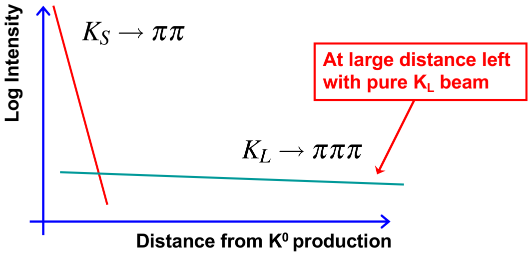
- if CP were conserved in the weak decays of neutral kaons, would expect decays to pions to occur from states of definite CP (i.e. the CP eigenstates K_1, K_2):
  - $|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - \left|\overline{K}^0\right\rangle\right)$, $\widehat{C}\widehat{P}\,|K_1\rangle = +\,|K_1\rangle$, $K_1 \to \pi\pi$: CP-even
  - $|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + \left|\overline{K}^0\right\rangle\right)$, $\widehat{C}\widehat{P}\,|K_2\rangle = -\,|K_2\rangle$, $K_2 \to \pi\pi\pi$: CP-odd
- expect lifetimes of CP eigenstates to be very different
  - for two pion decay energy available: $m_K - 2m_\pi \approx 220$ MeV
  - for three pion decay energy available: $m_K - 3m_\pi \approx 70$ MeV
- expect decays to $\pi\pi$ to be more rapid than to $\pi\pi\pi$ due to increased phase space

Decays of CP eigenstates

- this is exactly what is observed: a **short-lived** state "K-short" which decays to (mainly) to $\pi\pi$ and a **long-lived** state "K-long" which decays to $\pi\pi\pi$
- in the absence of CP violation we can identify
  - $|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right)$ with decays $K_S \to \pi\pi$
  - $|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right)$ with decays $K_L \to \pi\pi\pi$

Neutral kaons decays to pions

- consider the decays of a beam of K^0
- the decays to pions occur in states of definite CP
- if CP is conserved in the decay, need to express K^0 in terms of K_S and K_L:
  $\left|K^0\right\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$
- from the point of view of decays to pions, a K^0 beam is a linear combination of CP eigenstates: a rapidly decaying CP-even and a long-lived CP-odd components
- expect to see $\pi\pi$ decays near start of beam and $\pi\pi\pi$ decays further downstream



How this works algebraically:

- suppose at time $t = 0$ make a beam of pure K^0: $|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$
- put in the time dependence of wave-function: $|K_S(t)\rangle = |K_S\rangle\, e^{-im_S t - \Gamma_S t/2}$
  NOTE: the term $e^{-\Gamma_S t/2}$ ensures the K_S probability density decays exponentially, i.e. $|\psi_S|^2 = \langle K_S(t)|K_S(t)\rangle = e^{-\Gamma_S t} = e^{-t/\tau_S}$
- hence the wave-function evolves as:
  $$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|K_S\rangle\, e^{-(im_S + \frac{\Gamma_S}{2}t)} + |K_L\rangle\, e^{-(im_L + \frac{\Gamma_L}{2}t)}\right]$$
- writing $\theta_S(t) = e^{-(im_S + \frac{\Gamma_S}{2}t)}$ and $\theta_L(t) = e^{-(im_L + \frac{\Gamma_L}{2}t)}$,
  $$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(\theta_S(t)\,|K_S\rangle + \theta_L(t)\,|K_L\rangle)$$
- the decay rate to two pions for a state which was produced as K^0:
  $\Gamma(K^0_{t=0} \to \pi\pi) \propto |\langle K_S|\psi(t)\rangle|^2 \propto |\theta_S(t)|^2 = e^{-\Gamma_S t} = e^{-t/\tau_S}$,
  which is anticipated, i.e. decays of the short lifetime component K_S

Neutral kaons decays to leptons

- neutral kaons can also decay to leptons:
  - $\overline{K}^0 \to \pi^+ e^- \bar{\nu}_e$, $\overline{K}^0 \to \pi^+ \mu^- \bar{\nu}_\mu$
  - $K^0 \to \pi^- e^+ \nu_e$, $K^0 \to \pi^- \mu^+ \nu_\mu$
  - note: the final states are not CP eigenstates which is why we express these decays in terms of K^0, $\overline{K}^0$
- neutral kaons propagate as combined eigenstates of weak + strong interaction i.e. the K_S, K_L. The main decay modes/branching fractions are:

K_S	$\rightarrow$	$\pi^+\pi^-$	$BR = 69.2\%$
	$\rightarrow$	$\pi^0\pi^0$	$BR = 30.7\%$
	$\rightarrow$	$\pi^- e^+ \nu_e$	$BR = 0.03\%$
	$\rightarrow$	$\pi^+ e^- \bar{\nu}_e$	$BR = 0.03\%$
	$\rightarrow$	$\pi^- \mu^+ \nu_\mu$	$BR = 0.02\%$
	$\rightarrow$	$\pi^+ \mu^- \bar{\nu}_\mu$	$BR = 0.02\%$

K_L	$\rightarrow$	$\pi^+\pi^-\pi^0$	$BR = 12.6\%$
	$\rightarrow$	$\pi^0\pi^0\pi^0$	$BR = 19.6\%$
	$\rightarrow$	$\pi^- e^+ \nu_e$	$BR = 20.2\%$
	$\rightarrow$	$\pi^+ e^- \bar{\nu}_e$	$BR = 20.2\%$
	$\rightarrow$	$\pi^- \mu^+ \nu_\mu$	$BR = 13.5\%$
	$\rightarrow$	$\pi^+ \mu^- \bar{\nu}_\mu$	$BR = 13.5\%$

- leptonic decays are more likely for the K_L because the three pion decay modes have a lower decay rate than the two pion modes of the K_S

Strangeness oscillations (neglecting CP violation)

- the "semileptonic" decay rate to $\pi^- e^+ \nu_e$ occurs from the K^0 state
- to calculate the expected decay rate, need to know the K^0 component of the wave-function
- for a beam which was initially K^0 we have:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(\theta_S(t)\,|K_S\rangle + \theta_L(t)\,|K_L\rangle)$$

- writing K_S, K_L in terms of K^0, $\overline{K}^0$:

$$|\psi(t)\rangle = \frac{1}{2}\left[\theta_S(t)\Big(\left|K^0\right\rangle - \left|\overline{K}^0\right\rangle\Big) + \theta_L(t)\Big(\left|K^0\right\rangle + \left|\overline{K}^0\right\rangle\Big)\right] \quad (1)$$

$$= \frac{1}{2}(\theta_S + \theta_L)\left|K^0\right\rangle + \frac{1}{2}(\theta_S - \theta_L)\left|\overline{K}^0\right\rangle \quad (2)$$

Strangeness oscillations (neglecting CP violation)

- because $\theta_S(t) \neq \theta_L(t)$, a state that was initially a K^0 evolves with time into a mixture of K^0 and $\overline{K}^0$ – **"strangeness oscillations"**
- the K^0 intensity (i.e. K^0 fraction):

$$\Gamma(K^0_{t=0} \to K^0) = \left|\left\langle K^0 \middle| \psi(t) \right\rangle\right|^2 = \frac{1}{4}|\theta_S + \theta_L|^2$$

- similarly

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \left|\left\langle \overline{K}^0 \middle| \psi(t) \right\rangle\right|^2 = \frac{1}{4}|\theta_S - \theta_L|^2$$

Strangeness oscillations (neglecting CP violation)

- using the identity: $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\mathcal{R}(z_1 z_2^*)$

$$
\begin{aligned}
|\theta_S \pm \theta_L|^2 &= |e^{-(im_S+\frac{1}{2}\Gamma_S)t} \pm e^{-(im_L+\frac{1}{2}\Gamma_L)t}|^2 \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2\Re\{e^{-im_S t} e^{-\frac{1}{2}\Gamma_S t} . e^{+im_L t} e^{-\frac{1}{2}\Gamma_L t}\} \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\Re\{e^{-i(m_S-m_L)t}\} \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos(m_S-m_L)t \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t
\end{aligned}
$$

- oscillations between neutral kaon states with frequency given by the mass splitting $\Delta m = m(K_L) - m(K_S)$
- as in neutrino oscillations! Only this time we have decaying states
- from previous equations:

$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

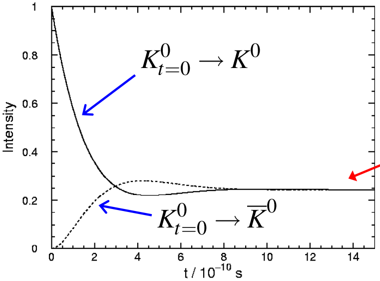
$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

Strangeness oscillations (neglecting CP violation)

- experimentally: $\tau(K_S) = 0.9 \times 10^{-10}$ s, $\tau(K_L) = 0.5 \times 10^{-7}$ s, and $\Delta m = (3.506 \pm 0.006) \times 10^{-15}$GeV, i.e. the $m(K_L) > m(K_S)$ by 1 part in 10^{16}
- the mass difference corresponds to an oscillation period of

$$T_{osc} = \frac{2\pi\hbar}{\Delta m} \approx 1.2 \times 10^{-9}\text{s}$$

- the oscillation period is relatively long compared to the K_S lifetime and consequently do not observe very pronounced oscillations



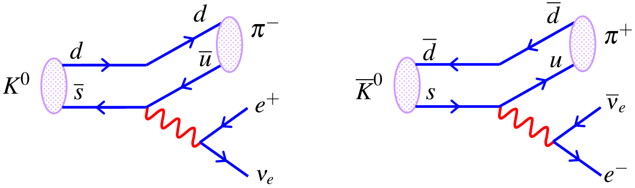
$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta mt\right]$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta mt\right]$$

After a few K_S lifetimes, left with a pure K_L beam which is half K^0 and half $\overline{K}^0$

Strangeness oscillations (neglecting CP violation)

- strangeness oscillations can be studied with semileptonic decays



- the charge of the observed pion (or lepton) tags the decay as from either a $\overline{K}^0$ pr K^0 because:

$$K^0 \rightarrow \pi^- e^+ \nu_e \qquad \overline{K}^0 \rightarrow \pi^+ e^- \overline{\nu}_e$$

but

$$\overline{K}^0 \not\rightarrow \pi^- e^+ \nu_e \qquad K^0 \not\rightarrow \pi^+ e^- \overline{\nu}_e$$

NOT ALLOWED

- so for an initial K^0 beam observe the decays to both charge combinations:

$$K^0_{t=0} \rightarrow K^0 \hookrightarrow \pi^- e^+ \nu_e \qquad K^0_{t=0} \rightarrow \overline{K}^0 \hookrightarrow \pi^+ e^- \overline{\nu}_e$$

which provides a way of measuring strangeness oscillations

The CPLEAR experiment @ CERN (1990 – 1996)

- used a low energy $\bar{p}$ beam
- neutral kaons produced:

$$\bar{p}p \to K^-\pi^+K^0$$

$$\bar{p}p \to K^+\pi^-\overline{K}^0$$

- low energy, so particles produced almost at rest
- observe production and decay in the same detector
- charge of $K^\pm\pi^\mp$ in the production process tags the initial neutral kaon as either K^0 or $\overline{K}^0$
- charge of decay products tags the decay as being either K^0 or $\overline{K}^0$
- provides a direct probe of strangeness oscillations

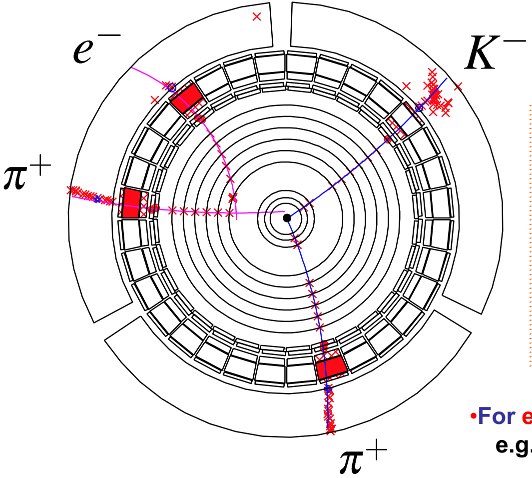
The CPLEAR experiment

$K^-(s\bar{u})$

$K^0(d\bar{s})$

$\overline{K}^0(s\bar{d})$

An example of a CPLEAR event



Production:

$$\bar{p}p \longrightarrow \boxed{K^-\pi^+} K^0$$

Mixing

Decay:

$$\overline{K}^0 \longrightarrow \boxed{\pi^+ e^-} \bar{\nu}_e$$

- **For each event know initial wave-function,**
  e.g. here: $|\psi(t=0)\rangle = |K^0\rangle$

- can measure decay rates as a function of time for all combinations: e.g.
  $R^{+} = \Gamma(K^0_{t=0} \to \pi^- e^+ \bar{\nu}_e) \propto \Gamma(K^0_{t=0} \to K^0)$
- from previous equations:

$$R_+ \equiv \Gamma(K^0_{t=0} \to \pi^- e^+ \nu_e) = N_{\pi e \nu} \frac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

$$R_- \equiv \Gamma(K^0_{t=0} \to \pi^+ e^- \overline{\nu}_e) = N_{\pi e \nu} \frac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

$$\overline{R}_- \equiv \Gamma(\overline{K}^0_{t=0} \to \pi^+ e^- \overline{\nu}_e) = N_{\pi e \nu} \frac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

$$\overline{R}_+ \equiv \Gamma(\overline{K}^0_{t=0} \to \pi^- e^+ \nu_e) = N_{\pi e \nu} \frac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

  where $N_{\pi e \nu}$ is some overall normalization factor
- express measurements as an "asymmetry" to remove dependence on $N_{\pi e \nu}$:

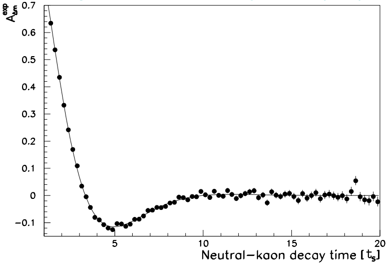
$$A_{\Delta m} = \frac{\left(R_+ + \bar{R}_-\right) - \left(R_- + \bar{R}_+\right)}{\left(R_+ + \bar{R}_-\right) + \left(R_- + \bar{R}_+\right)}$$

The CPLEAR experiment

- using the above expressions for R_+ etc obtain:

$$A_{\Delta m} = \frac{2e^{-(\Gamma_S+\Gamma_L)t/2}\cos \Delta mt}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$

A. Angelopoulos et al., Eur. Phys. J. C22 (2001) 55



★ **Points show the data**

★ **The line shows the theoretical prediction for the value of Δm most consistent with the CPLEAR data:**

$$\Delta m = 3.485 \times 10^{-15}\,\text{GeV}$$

- the sign of Δm is not determined here but is known from other experiments
- when the CPLEAR results are combined with experiments as Fermilab, obtain:

$$\Delta m = m(K_L) - m(K_S) = (3.506 \pm 0.006) \times 10^{-15}\text{GeV}$$

CP violation in the kaon system

- so far we have ignored CP violation in the neutral kaon system
- identified the K_S as the CP-even state and K_L as the CP-odd state

$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$ **with decays:** $K_S \rightarrow \pi\pi$ **CP = +1**

$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$ **with decays:** $K_L \rightarrow \pi\pi\pi$ **CP = -1**

- at a long distance from the production point, a beam of neutral kaons will be 100% K_L (the K_S component will have decayed away)
- hence, if CP is conserved, would expect to see only 3π decays

CP violation in the kaon system

- several attempts to find violation, e.g. Phys. Rev. Lett. 6 (1961) 552 (27 citations)

VOLUME 6, NUMBER 10 PHYSICAL REVIEW LETTERS MAY 15, 1961

DECAY PROPERTIES OF K_2^0 MESONS*

D. Neagu, E. O. Okonov, N. I. Petrov, A. M. Rosanova, and V. A. Rusakov
Joint Institute of Nuclear Research, Moscow, U.S.S.R.
(Received April 20, 1961)

- "We have recorded so far more than 500 V^0 events interpreted as decays of K_2^0 mesons."
- "Neither the authors of the quoted paper nor we have found any event of K_2^0-meson decay into two charged π mesons."

CP violation in the kaon system

- more perseverance in Phys. Rev. Lett. 13 (1964) 138 (3861 citation)

VOLUME 13, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JULY 1964

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

- observed 45 $K_L \to \pi^+\pi^-$ decays in a sample of 22700 kaon decays a long distance from the production point

  $\Longrightarrow$ weak interactions violate CP

- 22700/45 $\approx$ 504: people in Dubna just didn't get lucky to observe a possible two-pion event!

- Fitch & Cronin received a Nobel prize in Physics in 1980 for this discovery!

CP violation in the kaon system

- CP is violated in hadronic weak interactions, but only at the level of 2 parts in 1000:

K_L to pion BRs:

$$
\begin{array}{llll}
K_L \rightarrow & \pi^+\pi^-\pi^0 & BR = 12.6\% & CP = -1 \\
\rightarrow & \pi^0\pi^0\pi^0 & BR = 19.6\% & CP = -1 \\
\rightarrow & \pi^+\pi^- & BR = 0.20\% & CP = +1 \\
\rightarrow & \pi^0\pi^0 & BR = 0.08\% & CP = +1
\end{array}
$$

CP violation in the kaon system

★ **Two possible explanations of CP violation in the kaon system:**

i) The K_S and K_L do not correspond exactly to the CP eigenstates K_1 and K_2

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_1\rangle + \varepsilon|K_2\rangle\right] \qquad |K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]$$

with $|\varepsilon| \sim 2\times 10^{-3}$

• In this case the observation of $K_L \to \pi\pi$ is accounted for by:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]$$

- $|K_1\rangle \to \pi\pi$ — CP = +1
- $|K_2\rangle \to \pi\pi\pi$ — CP = -1

ii) and/or CP is violated in the decay

$$|K_L\rangle = |K_2\rangle$$ CP = -1

- $\to \pi\pi\pi$ — CP = -1
- $\to \pi\pi$ — CP = +1

Parameterised by ε'

★ Experimentally both known to contribute to the mechanism for CP violation in the kaon system but <u>i)</u> dominates: $\varepsilon'/\varepsilon = (1.7\pm 0.3)\times 10^{-3}$ { NA48 (CERN), KTeV (FermiLab)

CP violation in semileptonic decays

- if observe a neutral kaon beam a long time after production (i.e. at large distances) it will consist of a pure K_L component:

$$|K_L\rangle = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\varepsilon|^2}} \left[(1+\varepsilon)|K_0\rangle + (1-\varepsilon)|\overline{K}^0\rangle \right]$$

$|K_0\rangle \to \pi^+ e^- \overline{\nu}_e$, $\quad |\overline{K}^0\rangle \to \pi^- e^+ \nu_e$

- decays to $\pi^- e^+ \nu_e$ must come from the $\overline{K}^0$ component, and decays to $\pi^+ e^- \bar{\nu}_e$ must come from the K^0 component:

$$\Gamma(K_L \to \pi^+ e^- \overline{\nu}_e) \propto |\langle \overline{K}^0 | K_L \rangle|^2 \propto |1-\varepsilon|^2 \approx 1 - 2\Re\{\varepsilon\}$$

$$\Gamma(K_L \to \pi^- e^+ \nu_e) \propto |\langle K^0 | K_L \rangle|^2 \propto |1+\varepsilon|^2 \approx 1 + 2\Re\{\varepsilon\}$$

- results in a small difference in decay rates: the decay to $\pi^- e^+ \nu_e$ is 0.7% more likely then the decay to $\pi^+ e^- \bar{\nu}_e$

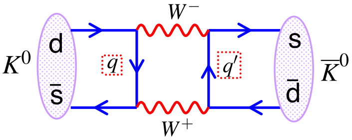
CP violation in semileptonic decays

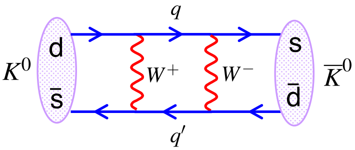
- this difference has been observed and thus provides the first direct evidence for an absolute difference between matter and antimatter
- it also provides an unambiguous definition of matter which could, for example, be transmitted to aliens in a distant galaxy:

  "The electrons in our atoms have the same charge as those emitted least often in the decays of the long-lived neutral kaon"

CP violation and the CKM matrix

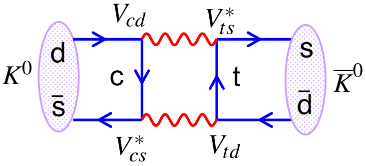
- how we can explain $\Gamma(\overline{K}^0_{t=0} \to K^0) \neq \Gamma(K^0_{t=0} \to \overline{K}^0)$ in terms of the CKM matrix?
- consider the box diagrams responsible for mixing, i.e.:





where $q = \{u, c, t\}$, $q' = \{u, c, t\}$

- have to sum over all possible quark exchanges in the box. For simplicity, consider just one diagram:

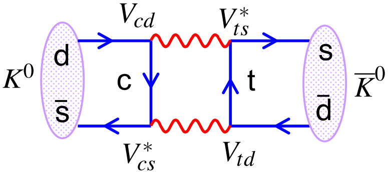


$$M_{fi} \propto A_{ct} V_{cd} V^*_{cs} V_{td} V^*_{ts}$$

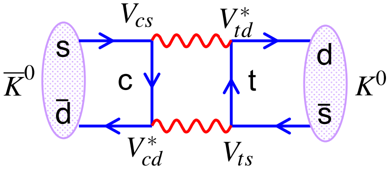
A constant related to integrating over virtual momenta

CP violation and the CKM matrix

- compare the equivalent box diagrams for $K^0 \to \overline{K}^0$ and $\overline{K}^0 \to K^0$:



$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^*$



$M'_{fi} \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*$

- therefore difference in rates:
  $\Gamma(K^0 \to \overline{K}^0) - \Gamma(\overline{K}^0 \to K^0) \propto M_{fi} - M_{fi}^* = 2\mathcal{I}\{M_{fi}\}$
- hence the rates can only be different if the CKM matrix has imaginary component:
  $|\epsilon| \propto \mathcal{I}\{M_{fi}\}$
- in the kaon system we can show:

$$|\varepsilon| \propto A_{ut}.\Im\{V_{ud}V_{us}^*V_{td}V_{ts}^*\} + A_{ct}.\Im\{V_{cd}V_{cs}^*V_{td}V_{ts}^*\} + A_{tt}.\Im\{V_{td}V_{ts}^*V_{td}V_{ts}^*\}$$

Shows that CP violation is related to the imaginary parts of the CKM matrix

Summary

- the weak interactions of quarks are described by the CKM matrix
- similar structure to the lepton sector, although unlike the PMNS matrix, the CKM matrix is nearly diagonal
- CP violation enters through via a complex phase in the CKM matrix
- a great deal of experimental evidence for CP violation in the weak interactions of quarks
- CP violation is needed to explain matter – antimatter asymmetry in the Universe
- HOWEVER, CP violation in the SM is not sufficient to explain the matter – antimatter asymmetry. There is probably another mechanism.