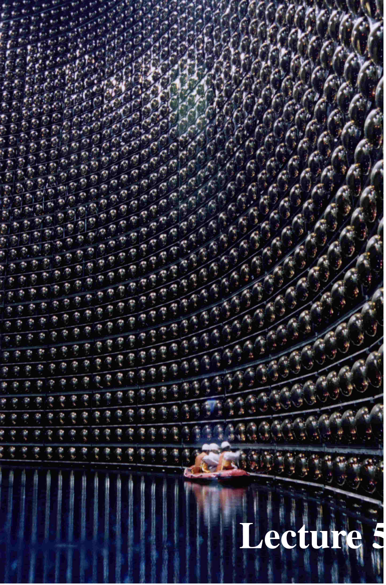

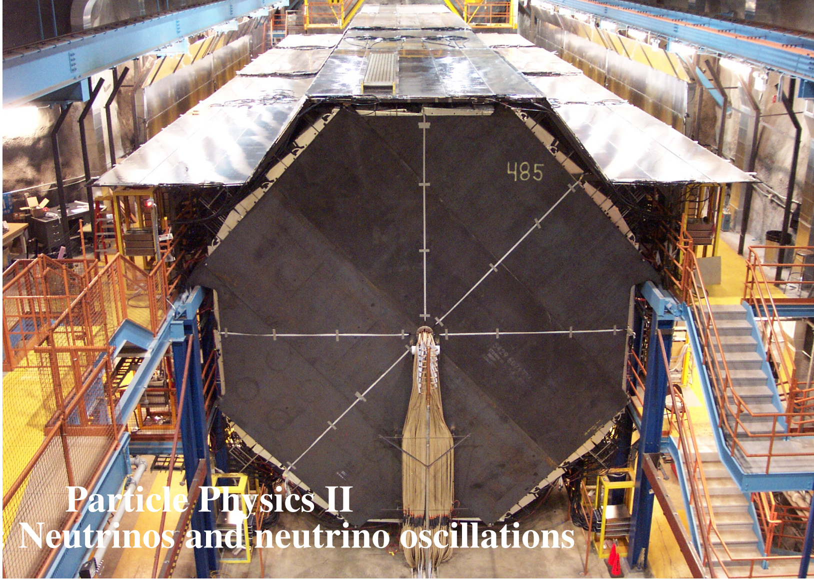

Particle Physics II
Lecture 5: Neutrinos and neutrino oscillations

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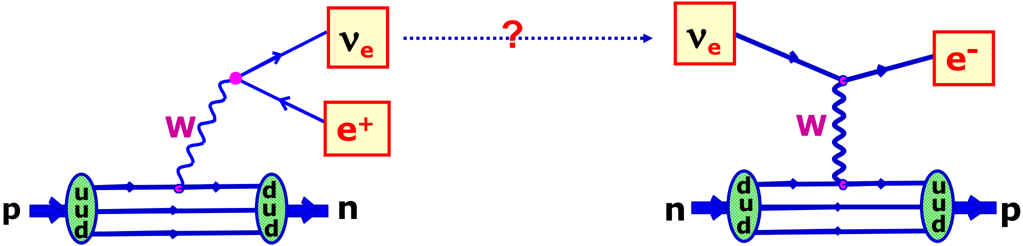
Neutrino flavors

- ν are never observed directly but detected via their weak interactions
- by definition, ν_e is the ν state produced along with e^+
- and vice versa: charged current weak interactions of the state ν_e produce e^-

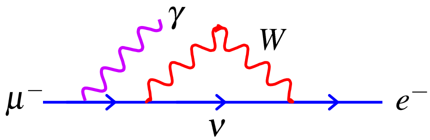
ν_e, ν_μ, ν_τ – weak eigenstates

- was assumed for many years: ν_e, ν_μ, ν_τ – massless fundamental particles

Experimental evidence: ν produced along with e^+ always lead to e^- in CC weak interactions, etc:

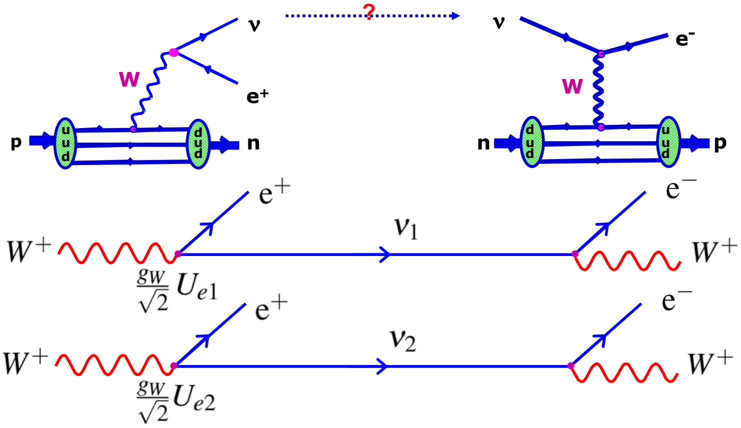


Experimental evidence: absence of $\mu^- \to e^-\gamma$ suggests that ν_e and ν_μ are distinct particles
$\mathcal{B}(\mu^- \to e^-\gamma) < 10^{-11}$



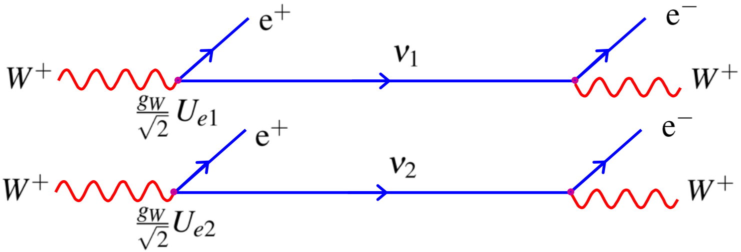
Mass eigenstates and weak eigenstates

- the essential feature in understanding the physics of neutrino oscillations is to understand what is meant by weak eigenstates and mass eigenstates ν_1, ν_2
- suppose the process below proceeds via two fundamental particle states



Mass eigenstates and weak eigenstates

- can't know which mass eigenstate (ν_1, ν_2) was involved



- in QM treat as a coherent state $\psi = \nu_e = U_{e1}\nu_1 + U_{e2}\nu_2$
- ν_e represents the wave-function of the coherent state produced along with e^+ in the weak interaction, i.e. the weak eigenstate

Neutrino oscillations for two flavors

- neutrinos are produced and interact as weak eigenstates, ν_e, ν_μ

- they are coherent linear combinations of the fundamental
  "mass eigenstates" ν_1, ν_2

- the mass eigenstates are the free particle solutions to the wave equation and will be taken to propagate as plane waves:

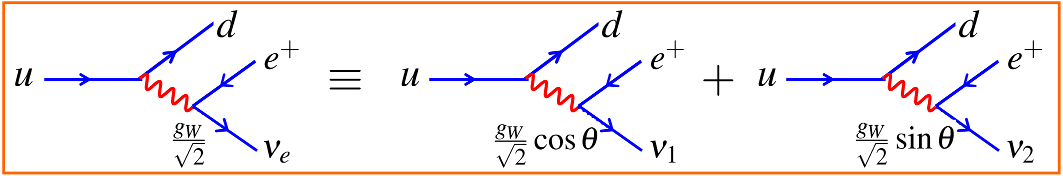
$$|\nu_1(t)\rangle = |\nu_1\rangle\, e^{i\vec{p}_1\cdot\vec{x}-iE_1 t}$$

$$|\nu_2(t)\rangle = |\nu_2\rangle\, e^{i\vec{p}_2\cdot\vec{x}-iE_2 t}$$

Neutrino oscillations for two flavors

- the weak and mass eigenstates are related by the unitary 2×2 matrix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$



- equation inversion leads to:

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Neutrino oscillations for two flavors

- suppose at time $t = 0$ a neutrino is produced in a pure ν_e state, e.g. in a decay $u \to de^+\nu_e$:

$$|\psi(0)\rangle = |\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

- take the z-axis to be along the neutrino direction

- the wave-function evolves according to the time evolution of the mass eigenstates (free particle solutions to the wave equation)

$$|\psi(t)\rangle = \cos\theta\,|\nu_1\rangle\,e^{-ip_1\cdot x} + \sin\theta\,|\nu_2\rangle\,e^{-ip_2\cdot x},$$

where $p_i \cdot x = E_i t - \vec{p}_i \cdot \vec{x} = E_i t - |\vec{p}_i| z$

- suppose the neutrino interacts in a detector at a distance L and a time T $\phi_i = p_i \cdot x = E_i T - |\vec{p}_i| L$, giving:

$$|\psi(L,T)\rangle = \cos\theta\,|\nu_1\rangle\,e^{-i\phi_1} + \sin\theta\,|\nu_2\rangle\,e^{-i\phi_2}$$

Neutrino oscillations for two flavors

- expressing the mass eigenstates, $|\nu_1\rangle$, $|\nu_2\rangle$, in terms of weak eigenstates:

$$|\psi(L,T)\rangle = \cos\theta\Big[\cos\theta\,|\nu_e\rangle - \sin\theta\,|\nu_\mu\rangle\Big]e^{-i\phi_1} + \sin\theta\Big[\sin\theta\,|\nu_e\rangle + \cos\theta\,|\nu_\mu\rangle\Big]e^{-i\phi_2}$$

$$|\psi(L,T)\rangle = |\nu_e\rangle\left[\cos^2\theta e^{-i\phi_1} + \sin^2\theta e^{-i\phi_2}\right] + |\nu_\mu\rangle \sin\theta\cos\theta\Big[-e^{-i\phi_1} + e^{-i\phi_2}\Big]$$

- if the masses of $|\nu_1\rangle$, $|\nu_2\rangle$ are the same, the mass eigenstates remain in phase, $\phi_1 = \phi_2$, and the state remains the linear combination corresponding to $|\nu_e\rangle$, and in a weak interaction will produce an electron

Neutrino oscillations for two flavors

- if the masses are different, the wave-function is no longer a pure $|\nu_e\rangle$:

$$P(\nu_e \to \nu_\mu) = |\langle\nu_\mu|\psi(L,T)\rangle|^2 \tag{1}$$

$$= \cos^2\theta \sin^2\theta\left(-e^{-i\phi_1} + e^{-i\phi_2}\right)\left(-e^{+i\phi_1} + e^{+i\phi_2}\right) \tag{2}$$

$$= \frac{1}{4}\sin^2 2\theta(2 - 2\cos(\phi_1 - \phi_2)) = \sin^2 2\theta \sin^2\left(\frac{\phi_1 - \phi_2}{2}\right) \tag{3}$$

Neutrino oscillations for two flavors

- let's look at the phase difference:

$$\Delta\phi_{12} = \phi_1 - \phi_2 = (E_1 - E_2)T - (|p_1| - |p_2|)L$$

- can assume that $|p_1| = |p_2| = p$ in which case

$$\Delta\phi_{12} = (E_1 - E_2)T = \left[\left(p^2 + m_1^2\right)^{1/2} - \left(p^2 + m_2^2\right)^{1/2}\right]L, \text{ as } L \approx (c)T$$

$$\Delta\phi_{12} = p\left[\left(1 + \frac{m_1^2}{p^2}\right)^{1/2} - \left(1 + \frac{m_2^2}{p^2}\right)^{1/2}\right]L \approx \frac{m_1^2 - m_2^2}{2p}L$$

- here we neglected that for the same momentum, different mass eigenstates propagate at different velocities and are observed at different times

- the full derivation requires a wave-packet treatment and gives the same result

- the phase difference can be written as:

$$\Delta\phi_{12} = (E_1 - E_2)T - \left(\frac{|p_1|^2 - |p_2|^2}{|p_1| + |p_2|}\right)L$$

$$\Delta\phi_{12} = (E_1 - E_2)\left[T - \left(\frac{E_1 + E_2}{|p_1| + |p_2|}\right)L\right] + \left(\frac{m_1^2 - m_2^2}{|p_1| + |p_2|}\right)L$$

- the first term of the RHS vanishes if we assume $E_1 = E_2$ or $\beta_1 = \beta_2$

- therefore in all cases:

$$\Delta\phi_{12} = \frac{m_1^2 - m_2^2}{2p}L = \frac{\Delta m^2}{2E}L$$

Neutrino oscillations for two flavors

- hence the two-flavor oscillation probability is:

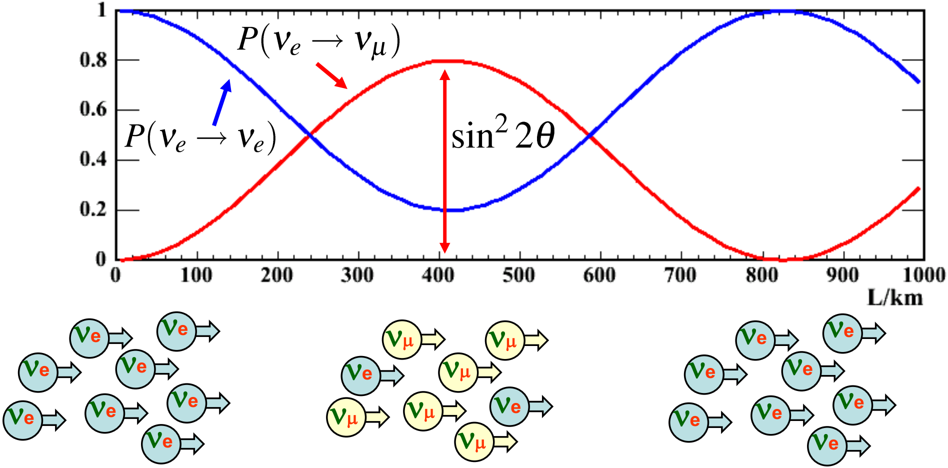
$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right), \text{ with } \Delta m^2_{21} = m^2_2 - m^2_1$$

- the corresponding two-flavor survival probability is:

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

Neutrino oscillations for two flavors

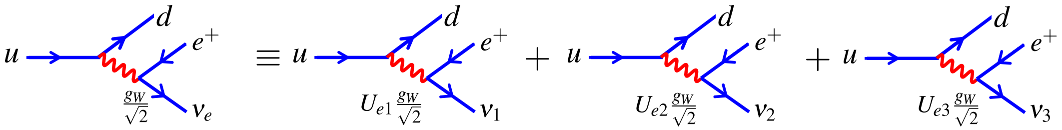
- e.g. $\Delta m^2 = 0.003\ \text{eV}^2$, $\sin^2 2\theta = 0.8$, $E_\nu = 1$ GeV, $\lambda_{\text{osc}} = \dfrac{4\pi E}{\Delta m^2}$



Neutrino oscillations for three flavors

- it is straightforward to extend this treatment to three generations of neutrinos
- in this case we have:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



- the 3×3 Unitary matrix U is known as the Pontecorvo-Maki-Nakagawa-Sakata matrix, usually abbreviated PMNS

- it has to be unitary to conserve probability

- using $U^\dagger U = 1 \implies U^{-1} = U^\dagger = (U^*)^T$ gives

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U^*_{e1} & U^*_{\mu 1} & U^*_{\tau 1} \\ U^*_{e2} & U^*_{\mu 2} & U^*_{\tau 2} \\ U^*_{e3} & U^*_{\mu 3} & U^*_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

- the unitarity of the PMNS matrix gives several useful relations:
  $UU^\dagger = I \implies$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} U^*_{e1} & U^*_{\mu 1} & U^*_{\tau 1} \\ U^*_{e2} & U^*_{\mu 2} & U^*_{\tau 2} \\ U^*_{e3} & U^*_{\mu 3} & U^*_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_{e1}U^*_{e1} + U_{e2}U^*_{e2} + U_{e3}U^*_{e3} = 1 \quad (4)$$

$$U_{\mu 1}U^*_{\mu 1} + U_{\mu 2}U^*_{\mu 2} + U_{\mu 3}U^*_{\mu 3} = 1 \quad (5)$$

$$U_{\tau 1}U^*_{\tau 1} + U_{\tau 2}U^*_{\tau 2} + U_{\tau 3}U^*_{\tau 3} = 1 \quad (6)$$

$$U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e3}U^*_{\mu 3} = 0 \quad (7)$$

$$U_{e1}U^*_{\tau 1} + U_{e2}U^*_{\tau 2} + U_{e3}U^*_{\tau 3} = 0 \quad (8)$$

$$U_{\mu 1}U^*_{\tau 1} + U_{\mu 2}U^*_{\tau 2} + U_{\mu 3}U^*_{\tau 3} = 0 \quad (9)$$

- to calculate the oscillation probability we can proceed as before

3-flavor oscillation probability

- consider a state produced at $t = 0$ as $|\nu_e\rangle$:

$$|\psi(t=0)\rangle = |\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle$$

- the wave-function evolves as:

$$|\psi(t)\rangle = |\nu_e\rangle = U_{e1}|\nu_1\rangle e^{-ip_1\cdot x} + U_{e2}|\nu_2\rangle e^{-ip_2\cdot x} + U_{e3}|\nu_3\rangle e^{-ip_3\cdot x},$$

  where $p_i \cdot x = E_i t - \vec{p}_i \cdot \vec{x} = E_i t - |\vec{p}|z$ (z-axis in direction of propagation)

- after traveling a distance L:

$$|\psi(L)\rangle = U_{e1}|\nu_1\rangle e^{-i\phi_1} + U_{e2}|\nu_2\rangle e^{-i\phi_2} + U_{e3}|\nu_3\rangle e^{-i\phi_3},$$

  where $\phi_i = p_i \cdot x = E_i t - |p|L = (E_i - |p|_i)L$

- as before we can approximate $\phi_i \approx \dfrac{m_i^2}{2E_i}L$

3-flavor oscillation probability

- expressing the mass eigenstates in terms of the weak eigenstates:

$$|\psi(L)\rangle = U_{e1}\Big[U^*_{e1}\,|\nu_e\rangle + U^*_{\mu 1}\,|\nu_\mu\rangle + U^*_{\tau 1}\,|\nu_\tau\rangle\Big]e^{-i\phi_1} \quad (10)$$

$$+ U_{e2}\Big[U^*_{e2}\,|\nu_e\rangle + U^*_{\mu 2}\,|\nu_\mu\rangle + U^*_{\tau 2}\,|\nu_\tau\rangle\Big]e^{-i\phi_2} \quad (11)$$

$$+ U_{e3}\Big[U^*_{e3}\,|\nu_e\rangle + U^*_{\mu 3}\,|\nu_\mu\rangle + U^*_{\tau 3}\,|\nu_\tau\rangle\Big]e^{-i\phi_3} \quad (12)$$

- which can be rearranged to give:

$$|\psi(L)\rangle = \Big[U_{e1}U^*_{e1}e^{-i\phi_1} + U_{e2}U^*_{e2}e^{-i\phi_2} + U_{e3}U^*_{e3}e^{-i\phi_3}\Big]\,|\nu_e\rangle \quad (13)$$

$$+ \Big[U_{e1}U^*_{\mu 1}e^{-i\phi_1} + U_{e2}U^*_{\mu 2}e^{-i\phi_2} + U_{e3}U^*_{\mu 3}e^{-i\phi_3}\Big]\,|\nu_\mu\rangle \quad (14)$$

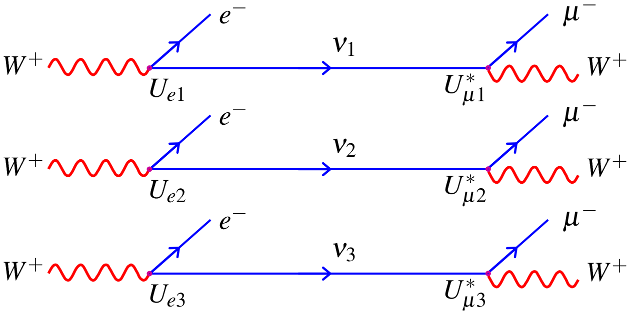
$$+ \Big[U_{e1}U^*_{\tau 1}e^{-i\phi_1} + U_{e2}U^*_{\tau 2}e^{-i\phi_2} + U_{e3}U^*_{\tau 3}e^{-i\phi_3}\Big]\,|\nu_\tau\rangle \quad (15)$$

- from there:

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu|\psi(L)\rangle|^2 \quad (16)$$

$$= \left| U_{e1}U_{\mu 1}^* e^{-i\phi_1} + U_{e2}U_{\mu 2}^* e^{-i\phi_2} + U_{e3}U_{\mu 3}^* e^{-i\phi_3} \right|^2 \quad (17)$$

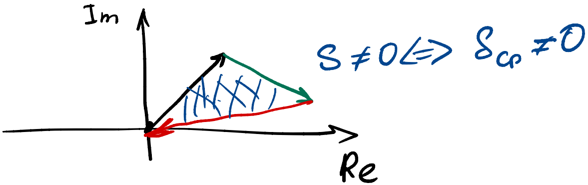
- the terms in this expression can be represented as:



- because of the unitarity of the PMNS matrix we have:

$$U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e2}U^*_{\mu 2} = 0:$$

unless the phases of the different components are different, the sum of these three diagrams is 0, i.e., **need different ν_i masses for oscillation**



3-flavor oscillation probability

- evaluate

$$P(\nu_e \to \nu_\mu) = \left|U_{e1}U^*_{\mu1}e^{-i\phi_1} + U_{e2}U^*_{\mu2}e^{-i\phi_2} + U_{e3}U^*_{\mu3}e^{-i\phi_3}\right|^2$$

using $|z_1 + z_2 + z_3|^2 \equiv |z_1|^2 + |z_2|^2 + |z_3|^2 + 2\mathcal{R}(z_1 z_2^* + z_1 z_3^* + z_2 z_3^*)$

$$P(\nu_e \to \nu_\mu) = \left|U_{e1}U^*_{\mu1}\right|^2 + \left|U_{e2}U^*_{\mu2}\right|^2 + \left|U_{e3}U^*_{\mu3}\right|^2 + \quad (18)$$

$$2\mathcal{R}(U_{e1}U^*_{\mu1}U^*_{e2}U_{\mu2}e^{-i(\phi_1-\phi_2)} + U_{e1}U^*_{\mu1}U^*_{e3}U_{\mu3}e^{-i(\phi_1-\phi_3)} + \quad (19)$$

$$U_{e2}U^*_{\mu2}U^*_{e3}U_{\mu3}e^{-i(\phi_2-\phi_3)}) \quad (20)$$

- can simplify the expression using $\left|U_{e1}U^*_{\mu1} + U_{e2}U^*_{\mu2} + U_{e3}U^*_{\mu3}\right|^2 = 0$

$$\implies \left|U_{e1}U^*_{\mu1}\right|^2 + \left|U_{e2}U^*_{\mu2}\right|^2 + \left|U_{e3}U^*_{\mu3}\right|^2 =$$

$$= -2\mathcal{R}\left(U_{e1}U^*_{\mu1}U^*_{e2}U_{\mu2} + U_{e1}U^*_{\mu1}U^*_{e3}U_{\mu3} + U_{e2}U^*_{\mu2}U^*_{e3}U_{\mu3}\right)$$

- substituting the last expression into Eq. 18:

$$P(\nu_e \to \nu_\mu) = 2\mathcal{R}\Big\{U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}\big[e^{-i(\phi_1-\phi_2)} - 1\big]\Big\} \tag{21}$$

$$+ 2\mathcal{R}\Big\{U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}\big[e^{-i(\phi_1-\phi_3)} - 1\big]\Big\} \tag{22}$$

$$+ 2\mathcal{R}\Big\{U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}\big[e^{-i(\phi_2-\phi_3)} - 1\big]\Big\} \tag{23}$$

- for electron survival probability:

$$P(\nu_e \to \nu_e) = |\langle \nu_e|\psi(L)\rangle|^2 \tag{24}$$

$$= \Big|U_{e1}U^*_{e1}e^{-i\phi_1} + U_{e2}U^*_{e2}e^{-i\phi_2} + U_{e3}U^*_{e3}e^{-i\phi_3}\Big|^2 \tag{25}$$

- using for it U unitarity $U_{e1}U_{e1}^* + U_{e2}U_{e2}^* + U_{e3}U_{e3}^* = 1$ get:

$$P(\nu_e \to \nu_e) = 1 + 2|U_{e1}|^2|U_{e2}|^2\mathcal{R}\Big\{e^{-i(\phi_1-\phi_2)} - 1\Big\} \quad (26)$$

$$+ 2|U_{e1}|^2|U_{e3}|^2\mathcal{R}\Big\{e^{-i(\phi_1-\phi_3)} - 1\Big\} \quad (27)$$

$$+ 2|U_{e2}|^2|U_{e3}|^2\mathcal{R}\Big\{e^{-i(\phi_2-\phi_3)} - 1\Big\} \quad (28)$$

3-flavor oscillation probability

- can simplify this expression using:

$$\mathcal{R}\{e^{-i(\phi_1-\phi_2)}-1\} = \cos(\phi_2-\phi_1)-1 = -2\sin^2\left(\frac{\phi_2-\phi_1}{2}\right) =$$

$$= -2\sin^2\left(\frac{(m_2^2-m_1^2)L}{4E}\right) \text{ with } \phi_i \approx \frac{m_i^2}{2E}L$$

- define

$$\Delta_{21} = \frac{(m_2^2-m_1^2)L}{4E} = \frac{\Delta m_{21}^2 L}{4E} \text{ with } \Delta m_{21}^2 = m_2^2 - m_1^2$$

note that $\Delta_{21} = (\phi_2-\phi_1)/2$ is a phase difference, i.e. dimensionless

- which gives electron neutrino survival probability:

$$P(\nu_e \to \nu_e) = 1 - 4|U_{e1}|^2|U_{e2}|^2\sin^2\Delta_{21} \quad (29)$$

$$- 4|U_{e1}|^2|U_{e3}|^2\sin^2\Delta_{31} \quad (30)$$

$$- 4|U_{e2}|^2|U_{e3}|^2\sin^2\Delta_{32} \quad (31)$$

- similar expressions can be obtained for the muon and tau neutrino survival probabilities
- note that since we only have three neutrino generations there are only two independent mass-squared differences, i.e.:

$$m_3^2 - m_1^2 = (m_3^2 - m_2^2) + (m_2^2 - m_1^2)$$

  and in the above equations only two of the Δ_{ij} are independent
- all expressions are in natural units
- converting to more practical units:

$$\Delta_{21} = 1.27 \frac{\Delta m_{21}^2(\mathrm{eV}^2) L(\mathrm{km})}{E(\mathrm{GeV})} \text{ and } \lambda_{\mathrm{osc}}(\mathrm{km}) = 2.47 \frac{E(\mathrm{GeV})}{\Delta m^2(\mathrm{eV}^2)}$$

CP and CPT in the weak interaction

- there are three important discrete symmetries:

$$\boxed{\text{Parity}} \qquad \hat{P}: \quad \vec{r} \to -\vec{r} \tag{32}$$

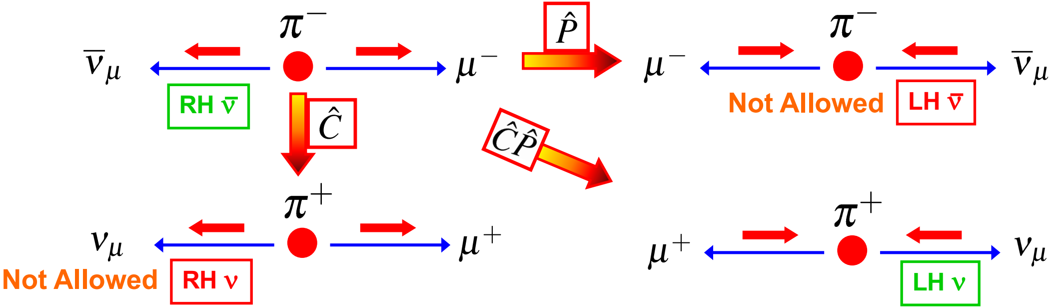
$$\boxed{\text{Time reversal}} \qquad \hat{T}: \quad t \to -t \tag{33}$$

$$\boxed{\text{Charge conjugation}} \qquad \hat{C}: \quad \text{Particle} \to \text{Antiparticle} \tag{34}$$

- the weak interaction violates parity conservation P, but also C

CP and CPT in the weak interaction

- consider pion decay remembering that the neutrino is ultra-relativistic and only left-handed ν and right-handed $\bar{\nu}$ participate in WI:



- hence weak interaction also violates charge conjugation symmetry but appears to be invariant under combined effect of C and P

CP and CPT in the weak interaction

CP transforms:

RH particles $\leftrightarrow$ LH antiparticles

LH particles $\leftrightarrow$ RH antiparticles

- if the weak interaction were invariant under CP, expect:

$$\Gamma\left(\pi^+ \to \mu^+ \nu_\mu\right) = \Gamma\left(\pi^- \to \mu^- \bar{\nu}_\mu\right)$$

- all Lorentz-invariant quantum field theories are invariant under $CPT \implies$ particles and antiparticles have identical mass, lifetime, magnetic moments etc

  Best current experimental test: $m_{K^0} - m_{\bar{K}^0} < 6 \times 10^{-19} m_{K^0}$
- since CPT holds:

  if CP invariance holds $\implies$ time reversal symmetry

  if CP is violated $\implies$ time reversal symmetry violated
- to account for the small excess of matter over antimatter that must have existed early in the universe, require CP violation in particle physics
- CP violation can arise in the weak interaction

CP and T violation in neutrino oscillations

- previously derived the oscillation probability for $\nu_e \to \nu_\mu$:

$$P(\nu_e \to \nu_\mu) = 2\mathcal{R}\Big\{U_{e1}U^*_{\mu1}U^*_{e2}U_{\mu2}\big[e^{-i(\phi_1-\phi_2)} - 1\big]\Big\} \tag{35}$$

$$+ 2\mathcal{R}\Big\{U_{e1}U^*_{\mu1}U^*_{e3}U_{\mu3}\big[e^{-i(\phi_1-\phi_3)} - 1\big]\Big\} \tag{36}$$

$$+ 2\mathcal{R}\Big\{U_{e2}U^*_{\mu2}U^*_{e3}U_{\mu3}\big[e^{-i(\phi_2-\phi_3)} - 1\big]\Big\} \tag{37}$$

- the oscillation probability for $\nu_\mu \to \nu_e$ can be obtained in the same manner or by simply exchanging the labels $(e) \leftrightarrow (\mu)$:

$$P(\nu_\mu \to \nu_e) = 2\mathcal{R}\Big\{U_{\mu1}U^*_{e1}U^*_{\mu2}U_{e2}\big[e^{-i(\phi_1-\phi_2)} - 1\big]\Big\} \tag{38}$$

$$+ 2\mathcal{R}\Big\{U_{\mu1}U^*_{e1}U^*_{\mu3}U_{e3}\big[e^{-i(\phi_1-\phi_3)} - 1\big]\Big\} \tag{39}$$

$$+ 2\mathcal{R}\Big\{U_{\mu2}U^*_{e2}U^*_{\mu3}U_{e3}\big[e^{-i(\phi_2-\phi_3)} - 1\big]\Big\} \tag{40}$$

- unless the elements of the PMNS matrix are real $P(\nu_e \to \nu_\mu) \neq P(\nu_\mu \to \nu_e)$

CP and T violation in neutrino oscillations

- unless the elements of the PMNS matrix are real $P(\nu_e \to \nu_\mu) \neq P(\nu_\mu \to \nu_e)$
- if any of the elements of the PMNS matrix are complex, neutrino oscillations are not invariant under time reversal $t \to -t$

CP and T violation in neutrino oscillations

- consider the effects of T, CP, and CPT on neutrino oscillations:

$$\boxed{\text{T}} \quad \nu_e \to \nu_\mu \quad \xrightarrow{\hat{T}} \quad \nu_\mu \to \nu_e \tag{41}$$

$$\boxed{\text{CP}} \quad \nu_e \to \nu_\mu \quad \xrightarrow{\hat{C}\hat{P}} \quad \bar{\nu}_e \to \bar{\nu}_\mu \tag{42}$$

$$\boxed{\text{CPT}} \quad \nu_e \to \nu_\mu \quad \xrightarrow{\hat{C}\hat{P}\hat{T}} \quad \bar{\nu}_\mu \to \bar{\nu}_e \tag{43}$$

  Note that C alone is not sufficient in Eq. 42 as it transforms LH neutrinos into LH antineutrinos (not involved in weak interaction)
- if the weak interactions are invariant under CPT:

  $P(\nu_e \to \nu_\mu) = P(\bar{\nu}_\mu \to \bar{\nu}_e)$

  $P(\nu_\mu \to \nu_e) = P(\bar{\nu}_e \to \bar{\nu}_\mu)$
- if the PMNS matrix is not purely real then

  $P(\nu_e \to \nu_\mu) \neq P(\nu_\mu \to \nu_e)$

  From above: $P(\nu_e \to \nu_\mu) \neq P(\bar{\nu}_e \to \bar{\nu}_\mu) \implies$ CP is violated in neutrino oscillations!

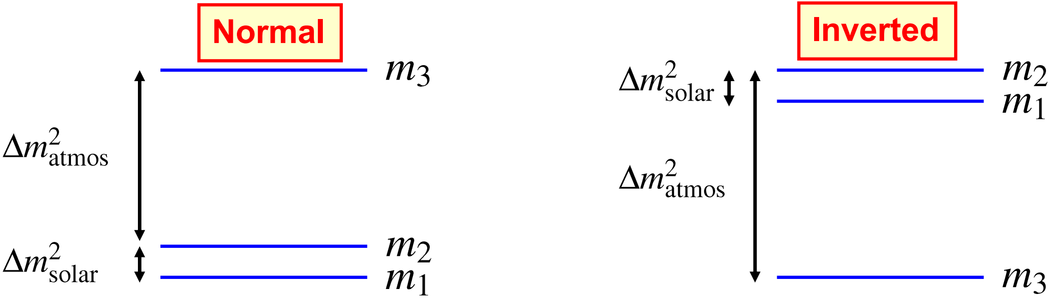
- to date, results on neutrino oscillations only determine

$$\left|\Delta m_{ij}^2\right| = \left|m_j^2 - m_i^2\right|$$

- there are two distinct and very different mass scales:
    - atmospheric neutrino oscillations: $\left|\Delta m^2\right|_{\text{atmos}} \sim 2.5 \times 10^{-3}\text{eV}^2$
    - solar neutrino oscillations: $\left|\Delta m^2\right|_{\text{solar}} \sim 8 \times 10^{-5}\text{eV}^2$

Neutrino mass hierarchy

- two possible assignments of mass hierarchy:



- in both cases: $\Delta m^2_{21} \sim 8 \times 10^{-5}\text{eV}^2$ (solar)
  $|\Delta m^2_{31}| \approx |\Delta m^2_{32}| \sim 2.5 \times 10^{-3}\text{eV}^2$ (atmospheric)
- hence we can approximate $\Delta m^2_{31} \approx \Delta m^2_{32}$

3-flavor oscillations neglecting CP violation

- neglecting CP violation considerably simplifies the algebra of 3-flavor neutrino oscillations
- taking the PMNS matrix to be real:

$$P(\nu_e \to \nu_\mu) = -4U_{e1}U_{\mu 1}U_{e2}U_{\mu 2}\sin^2\Delta_{21} \quad (44)$$
$$-4U_{e1}U_{\mu 1}U_{e3}U_{\mu 3}\sin^2\Delta_{31} \quad (45)$$
$$-4U_{e2}U_{\mu 2}U_{e3}U_{\mu 3}\sin^2\Delta_{32} \quad (46)$$

with $\Delta_{ij} = \frac{(m_j^2 - m_i^2)L}{4E} = \frac{\Delta m_{ij}^2 L}{4E}$

- using $\Delta_{31} \approx \Delta_{32}$

$$P(\nu_e \to \nu_\mu) = -4U_{e1}U_{\mu 1}U_{e2}U_{\mu 2}\sin^2\Delta_{21} \quad (47)$$
$$-4(U_{e1}U_{\mu 1} + U_{e2}U_{\mu 2})U_{e3}U_{\mu 3}\sin^2\Delta_{32} \quad (48)$$

- using $U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0$

$P(\nu_e \to \nu_\mu) \approx -4U_{e1}U_{\mu 1}U_{e2}U_{\mu 2}\sin^2\Delta_{21} + 4U_{e3}^2U_{\mu 3}^2\sin^2\Delta_{32}$

3-flavor oscillations neglecting CP violation

- can apply $\Delta_{31} \approx \Delta_{32}$ to the expression for ν_e survival probability:

$$P(\nu_e \to \nu_e) = 1 - 4U_{e1}^2 U_{e2}^2 \sin^2 \Delta_{21} - 4U_{e1}^2 U_{e3}^2 \sin^2 \Delta_{31} - 4U_{e2}^2 U_{e3}^2 \sin^2 \Delta_{32} \quad (49)$$

$$\approx 1 - 4U_{e1}^2 U_{e2}^2 \sin^2 \Delta_{21} - 4\left(U_{e1}^2 + U_{e2}^2\right) U_{e3}^2 \sin^2 \Delta_{32} \quad (50)$$

- which can be simplified using $U_{e1}U_{e1}^* + U_{e2}U_{e2}^* + U_{e3}U_{e3}^* = 1$:

$$\implies P(\nu_e \to \nu_\mu) \approx -4U_{e1}U_{\mu1}U_{e2}U_{\mu2} \sin^2 \Delta_{21} + 4U_{e3}^2 U_{\mu3}^2 \sin^2_{32}$$

3-flavor oscillations neglecting CP violation

- neglecting CP violation and taking $\left|\Delta m_{31}^2\right| \approx \left|\Delta m_{32}^2\right|$ obtain the following expressions for neutrino oscillation probabilities:

$$P(\nu_e \to \nu_e) \approx 1 - 4U_{e1}^2 U_{e2}^2 \sin^2\Delta_{21} - 4\left(1 - U_{e3}^2\right)U_{e3}^2 \sin^2\Delta_{32} \tag{51}$$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - 4U_{\mu1}^2 U_{\mu2}^2 \sin^2\Delta_{21} - 4\left(1 - U_{\mu3}^2\right)U_{\mu3}^2 \sin^2\Delta_{32} \tag{52}$$

$$P(\nu_\tau \to \nu_\tau) \approx 1 - 4U_{\tau1}^2 U_{\tau2}^2 \sin^2\Delta_{21} - 4\left(1 - U_{\tau3}^2\right)U_{\tau3}^2 \sin^2\Delta_{32} \tag{53}$$

$$P(\nu_e \to \nu_\mu) = P(\nu_\mu \to \nu_e) \approx -4U_{e1}U_{\mu1}U_{e2}U_{\mu2} \sin^2\Delta_{21} + 4U_{e3}^2 U_{\mu3}^2 \sin^2\Delta_{32} \tag{54}$$

$$P(\nu_e \to \nu_\tau) = P(\nu_\tau \to \nu_e) \approx -4U_{e1}U_{\tau1}U_{e2}U_{\tau2} \sin^2\Delta_{21} + 4U_{e3}^2 U_{\tau3}^2 \sin^2\Delta_{32} \tag{55}$$

$$P(\nu_\mu \to \nu_\tau) = P(\nu_\tau \to \nu_\mu) \approx -4U_{\mu1}U_{\tau1}U_{\mu2}U_{\tau2} \sin^2\Delta_{21} + 4U_{\mu3}^2 U_{\tau3}^2 \sin^2\Delta_{32} \tag{56}$$

3-flavor oscillations neglecting CP violation

- the wavelength associated with $\sin^2\Delta_{21}$ and $\sin^2\Delta_{32}$ are:

"SOLAR"

$$\lambda_{21} = \frac{4\pi E}{\Delta m^2_{21}}$$

"Long"-Wavelength

and

$$\lambda_{32} = \frac{4\pi E}{\Delta m^2_{32}}$$

"ATMOSPHERIC"

"Short"-Wavelength

- PMNS matrix is expressed in terms of 3 rotation angles θ_{12}, θ_{23}, θ_{13}, and a complex phase δ, using the notation $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$:

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

Dominates: "Atmospheric" "Solar"

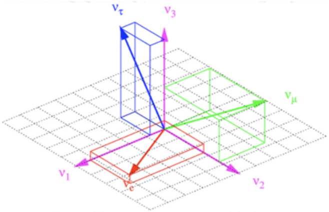
- writing out in full:

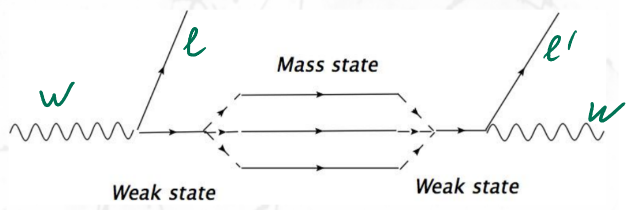
$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- there are six SM parameters that can be be measured in ν oscillation experiments

$\|\Delta m_{21}\|^2 = \|m_2^2 - m_1^2\|$	θ_{12}	**Solar and reactor neutrino experiments**
$\|\Delta m_{32}\|^2 = \|m_3^2 - m_2^2\|$	θ_{23}	**Atmospheric and beam neutrino experiments**
	θ_{13}	**Reactor neutrino experiments + future beam**
	δ	**Future beam experiments**

PMNS matrix: current picture





ν oscillation:

Atmospheric — Reactor — Solar

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$\theta_{23} \approx 45°$ — $\theta_{13} \approx 10°$ — $\theta_{12} \approx 35°$

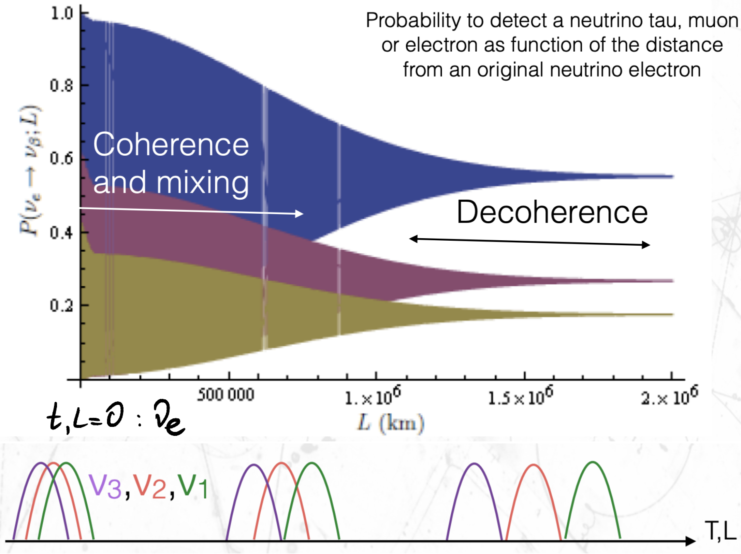
Atmospheric exp. — *Reactor* — *Solar exp.*

Reactor LBL

Accelerator LBL

LBL = Long BaseLine – new generation of the long baseline experiments

Quantum (de)coherence



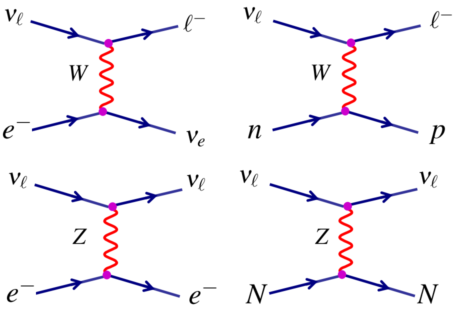
Neutrino experiments

Before discussing current experimental data, need to consider how ν interact in matter:

- two processes
  - charged current (CC) interactions (via a W boson) $\Longrightarrow$ charged lepton
  - neutral current (NC) interactions (via a Z boson)
- two possible “targets”: can have neutrino interactions with
  - atomic electrons
  - nucleons within the nucleus

CHARGED CURRENT

NEUTRAL CURRENT

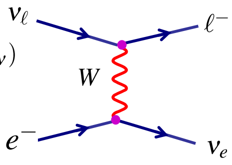


Neutrino interaction thresholds

- neutrino detection method depends on the neutrino energy and (weak) flavor
  - electron neutrinos from the sun and nuclear reactions have $E_\nu \sim 1$ MeV
  - muon atmospheric neutrinos have $E_\nu \sim 1$ GeV (from $\pi^\pm$ decays)
- these energies are relatively low and not all interactions are kinematically allowed, i.e. there is a threshold energy before an interaction can occur $\Longrightarrow$ need sufficient energy in the centre-of-mass frame to produce final state particles

1. Charged current interactions on atomic electrons (in lab. frame)

$p_\nu = (E_\nu, 0, 0, E_\nu)$
$p_e = (m_e, 0, 0, 0)$



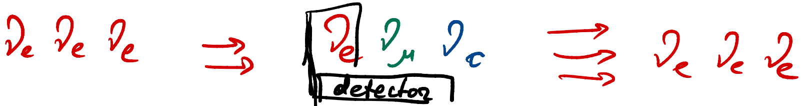
$$s = (p_\nu + p_e)^2 = (E_\nu + m_e)^2 - E_\nu^2$$

$= E_{tot}^2 - (\vec{p}_{tot})^2$; E_{tot}; $(\vec{p}_{tot})^2$

Require: $s > m_\ell^2$

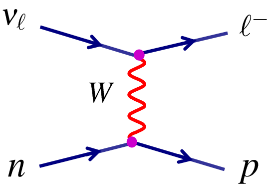
$$\Rightarrow \boxed{E_\nu > \left[\left(\frac{m_\ell}{m_e}\right)^2 - 1\right]\frac{m_e}{2}}$$

- putting in the numbers, for CC interactions with atomic electrons require: $E_{\nu_e} > 0$, $E_{\nu_\mu} > 11$ GeV, $E_{\nu_\tau} > 3090$ GeV
- for ν_μ, ν_τ high energy thresholds compared to typical energies considered here



Neutrino interaction thresholds

2 Charged current interactions on nucleons (in lab. frame):



$$s = (p_\nu + p_n)^2 = (E_\nu + m_n)^2 - E_\nu^2$$

Require: $s > (m_\ell + m_p)^2$

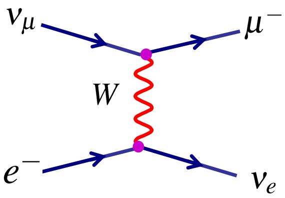
$$\Rightarrow \quad E_\nu > \frac{(m_p^2 - m_n^2) + m_\ell^2 + 2m_p m_\ell}{2m_n}$$

- for CC interactions with neutrons: $E_{\nu_e} > 0$, $E_{\nu_\mu} > 110$ MeV, $E_{\nu_\tau} > 3.5$ GeV
- ν_e from the sun and nuclear reactors $E_\nu \sim 1$ MeV which oscillate into ν_μ and ν_τ cannot interact via charged current interactions: "they effectively disappear"
- atmospheric ν_μ $E_\nu \sim 1$ GeV which oscillate into ν_τ cannot interact via charged current interactions: "disappear"
- to date, most experimental signatures for neutrino oscillation are a deficit of neutrino interactions (with the exception of SNO and OPERA) because below threshold for producing lepton of different flavor from original neutrino

with the exception of the new LBL experiments

Neutrino interaction thresholds

- previously derived CC νq cross sections in ultra-relativistic limit (neglecting $m(\nu/q)$)
- for high energy ν_μ can directly use previous results:



$$\sigma_{\nu_\mu e^-} = \frac{G_F^2 s}{\pi}$$

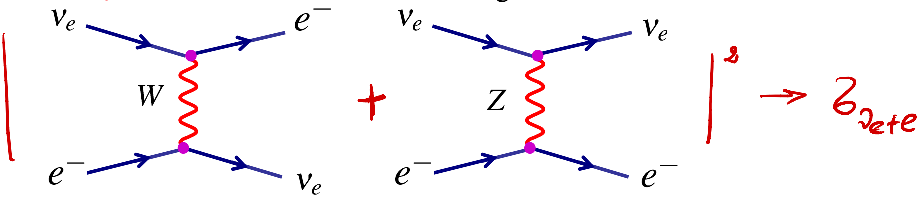
with $s = (E_\nu + m_e)^2 - E_\nu^2 \approx 2m_e E_\nu$

$$\boxed{\sigma_{\nu_\mu e^-} = \frac{2m_e G_F^2 E_\nu}{\pi}}$$

Cross section increases linearly with lab. frame neutrino energy

Neutrino interaction thresholds

- for ν_e there is another lowest order diagram with the same final state:

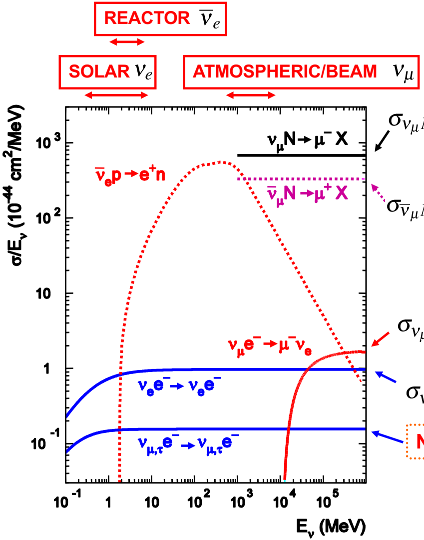


Total cross section is lower than the pure CC cross section due to negative interference $|M_{CC} + M_{NC}|^2 < |M_{CC}|^2$: $\sigma_{\nu_e e} \approx 0.6\sigma^{CC}_{\nu_e e}$

- in the high energy limit, the CC νN cross sections are larger due to the higher center-of-mass energy: $s = (E_\nu + m_n)^2 - E_\nu^2 \approx 2m_n E_\nu$

Neutrino detection

The detector technology/interaction process depends on neutrino type and energy:



$$\sigma_{\nu_\mu N} = 0.22\frac{2m_p G_F^2 E_\nu}{\pi}$$

$$\sigma_{\bar{\nu}_\mu N} = 0.33\frac{2m_p G_F^2 E_\nu}{3\pi}$$

Deep Inelastic Scattering (p. 332)

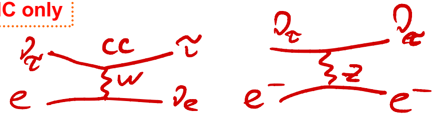
$$\sigma_{\nu_\mu e^-} = \frac{2m_e G_F^2 E_\nu}{\pi}$$

CC only
Threshold 11 GeV

$$\sigma_{\nu_e e^-} \approx 0.6\frac{2m_e G_F^2 E_\nu}{\pi}$$

NC + CC

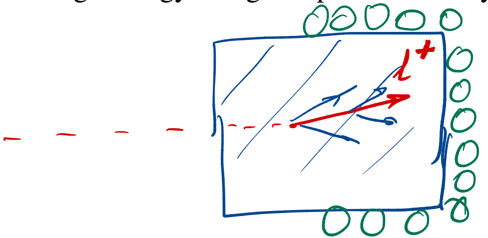
NC only



$\nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu$: $E_\nu > 1$ GeV

1. water Cherenkov: e.g. Super Kamiokande
2. Iron Calorimeters: e.g. MINOS, CDHS

Produce high energy charged lepton: relatively easy to detect



Solar neutrinos

ν_e: $E_\nu < 20$ MeV

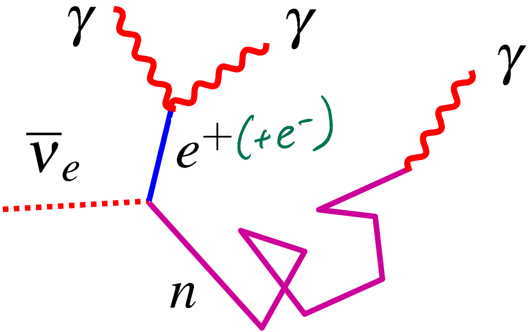
1. water Cherenkov: e.g. Super Kamiokande
   - detect Cherenkov light from electron produced in $\nu_e + e^- \to \nu_e + e^-$
   - because of background from natural radioactivity limited to $E_\nu > 5$ MeV
   - because Oxygen is a doubly magic nucleus don't get $\nu_e + n \to e^- + p$
2. Radio-Chemical: e.g. Homestake, SAGE, GALLEX
   - use inverse β-decay process, e.g. $\nu_e +^{71}\text{Ga} \to e^- +^{71}\text{Ge}$
   - chemically extract produced isotope and count decays (only gives a rate)

measure e (ν_e) energy

only count ν_e

$\bar{\nu}_e$: $E_{\bar{\nu}} < 5$ MeV

1. liquid scintillator: e.g. KamLAND
   - low energies $\Longrightarrow$ large radioactive background
   - dominant interaction: $\bar{\nu}_e + p \to e^+ + n$
   - prompt positron annihilation signal + delayed signal from n (space/time correlation reduces background)
   - electrons produced by photons excite the scintillator which produces light



Long baseline neutrino experiments

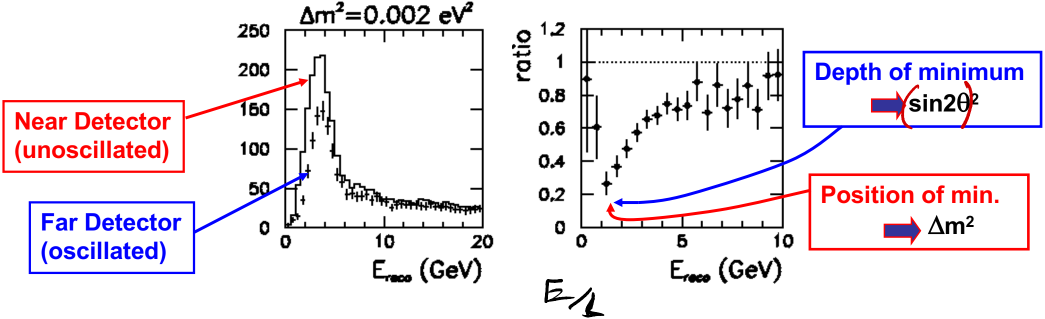
- initial studies of ν oscillations done with atmospheric and solar ν
- now the emphasis is on neutrino beam experiments
- allows to take control: design an experiment with specific goals
- many long baseline ν oscillation experiments were taking data:
  - K2K in Japan, MINOS in the US, CNGS in Europe
- and currently taking data:
  - T2K in Japan, NOvA in the US
- new ultimate long baseline experiments are currently under construction:
  - HyperK in Japan and DUNE in the US

Long baseline neutrino experiments

Basic idea:

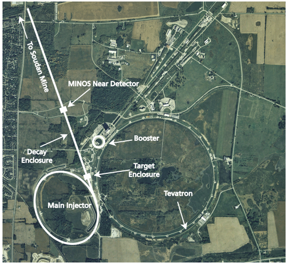
- intense ν beam

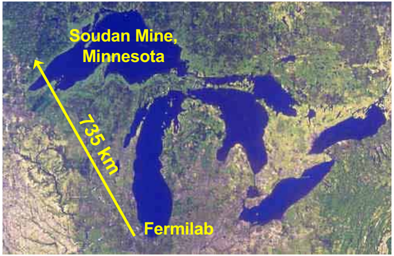
- two detectors: one close to beam, the other hundreds of km away
- measure ratio of the neutrino energy spectrum in the far detector (oscillated) to that in the near detector (unoscillated)
- partial cancellation of systematic biases



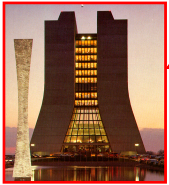
MINOS (2005 – 2016)

- 120 GeV protons extracted from the Main injector at Fermilab
- 2.5×10^{13} p/pulse hit target $\Longrightarrow$ very intense beam 0.3 MW on target





Two detectors:

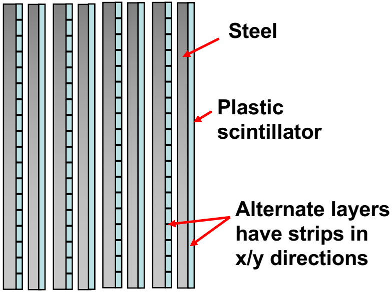


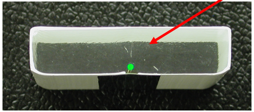
- ★ 1000 ton, NEAR Detector at Fermilab : 1 km from beam
- ★ 5400 ton FAR Detector, 720m underground in Soudan mine, N. Minnesota: 735 km from beam

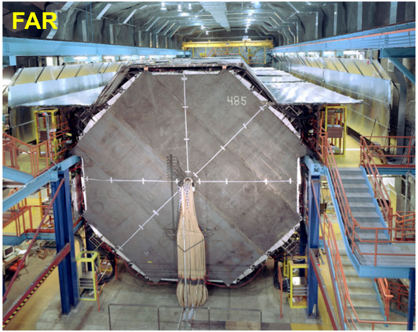
https://www-numi.fnal.gov/

The MINOS Detectors:

- Dealing with high energy neutrinos $E_\nu > 1\,\mathrm{GeV}$
- The muons produced by ν_μ interactions travel several metres
- Steel-Scintillator sampling calorimeter
- Each plane: 2.54 cm steel +1 cm scintillator
- A charged particle crossing the scintillator produces light – detect with PMTs

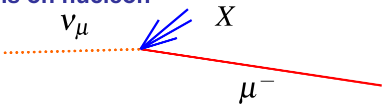




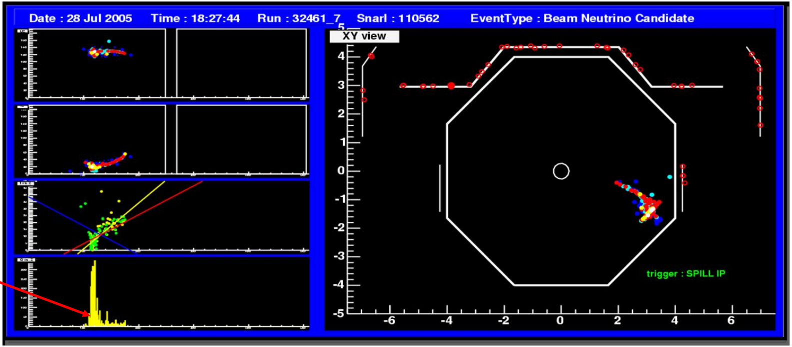


- Neutrino detection via CC interactions on nucleon

$$\nu_\mu + N \to \mu^- + X$$



Example event:



Signal from hadronic shower

- The main feature of the MINOS detector is the very good neutrino energy resolution

$$E_\nu = E_\mu + E_X$$

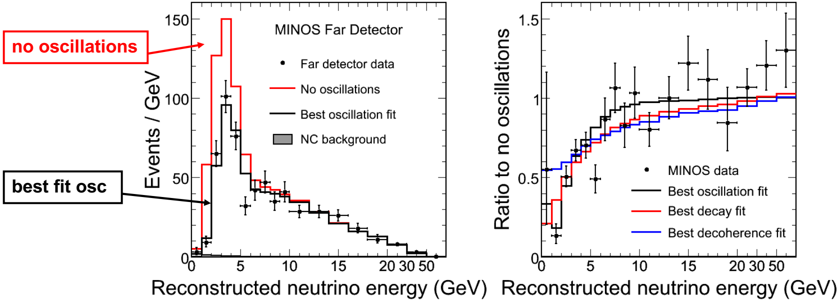
- Muon energy from range/curvature in B-field
- Hadronic energy from amount of light observed

$D_s \to \tau \nu_\tau$;

MINOS results

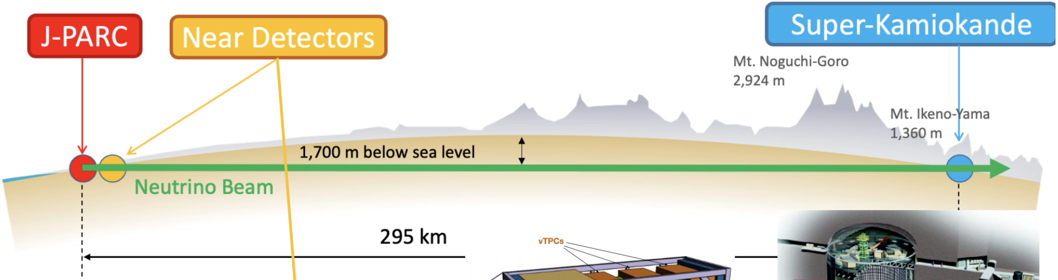
- for the MINOS experiment, L is fixed and observe oscillations as function of E_ν
- for $|\Delta m^2_{32}| \sim 2.5 \times 10^{-3}$ eV^2 first oscillation minimum at $E_\nu = 1.5$ GeV
- to a very good approximation can use 2-flavor case as oscillations corresponding to $|\Delta m^2_{21}| \sim 8 \times 10^{-5}$ eV^2 occur at $E_\nu = 50$ MeV, beam contains very few neutrinos at this energy + well below detection threshold

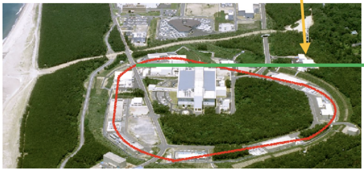
MINOS Collaboration, Phys. Rev. Lett. 101, 131802, 2008

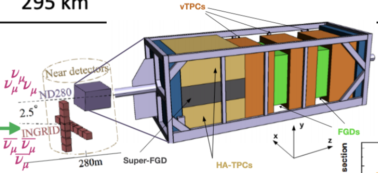


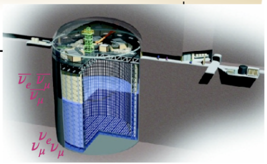
$$|\Delta m^2_{32}| = (2.43 \pm 0.12) \times 10^{-3}\,\text{eV}^2$$

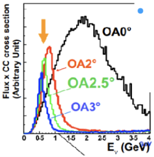
T2K experiment











- **Beam:** 30 GeV protons on a graphite target, producing Kaons and Pions

- **ND site:** *on and off axis detectors:*
  - Beam monitoring
  - Tune flux & interaction models predictions f
  - ν cross-sections
  - Magnetized 0.2T
  - Upgraded ND280 completed

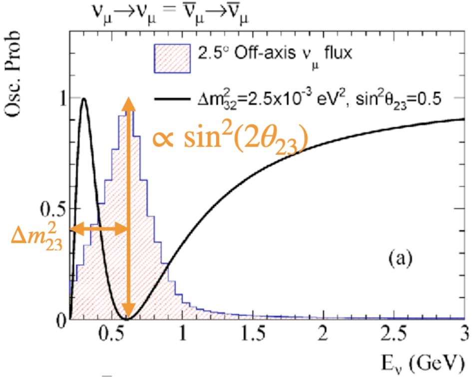
- **FD site:**
  - Super-Kamiokande
  - 50kton water Cherenkov detector

Oscillation at long baseline

Main idea: produce $\nu_\mu/\bar{\nu}_\mu$ beam and perform the measurement of rate, energy and flavor before and after oscillation

- Disappearance channel

$$P(\nu_\mu \to \nu_\mu) = P\left(\frac{L}{E}, \theta_{23}, \Delta m^2_{23}\right)$$

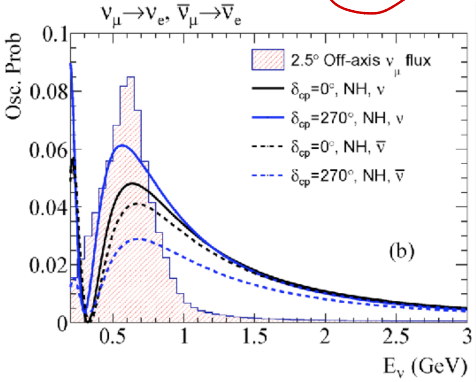


Oscillation at long baseline

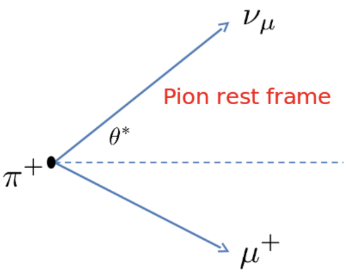
Main idea: produce $\nu_\mu/\bar{\nu}_\mu$ beam and perform the measurement of rate, energy and flavor before and after oscillation

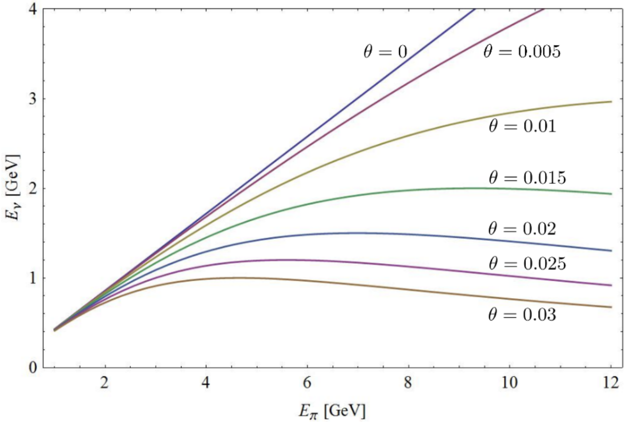
- Appearance channel

$$P(\nu_\mu \to \nu_e/\bar{\nu}_\mu \to \bar{\nu}_e) = P\left(\frac{L}{E}, \theta_{23}, \theta_{13}, \Delta m^2_{21}, \Delta m^2_{23}, \pm \sin\delta_{CP}\right)$$

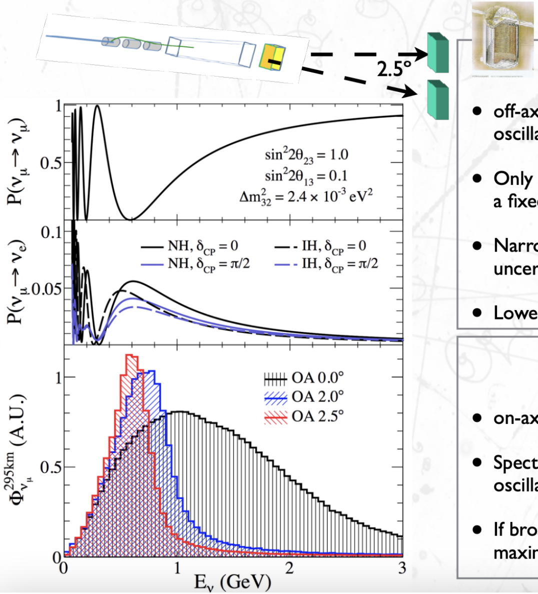


Off(On)-axis beam





Off(On)-axis beam



Off-axis

- off-axis optimises the flux at the maximum of the oscillation.
- Only one oscillation maximum can be measured at a fixed distance.
- Narrow beam less dependent on beam uncertainties but more on beam pointing.
- Lower energies achieved.

On-axis

- on-axis optimises the total integrated flux.
- Spectrum with higher neutrino energy (longer oscillation distances)
- If broad enough, more than one oscillation maximum can be measured at a fixed distance.

T2K-only oscillation results

First presented at Neutrino 2024 : https://doi.org/10.5281/zenodo.12704703

δ_{CP}:

- Preference for $\delta_{CP} \approx -\frac{\pi}{2}$
- Jarlskog-invariant gives a parametrized independent way to measure CP violation
- CP conservation excluded at >2σ in case of IO and <2σ for NO

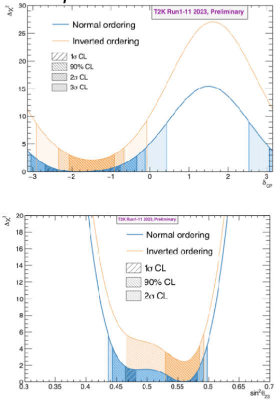
θ_{23} **and mass ordering:**

- Preference for NO and upper octant but not significant

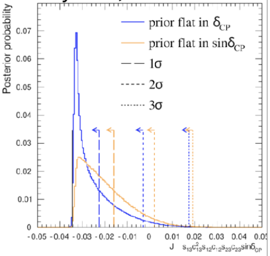
Best fit:

	Normal ordering	Inverted ordering
$\sin^2(\theta_{13})/10^{-3}$	$(21.9^{+0.9}_{-0.5})$	$(22.0^{+1.0}_{-0.4})$
δ_{CP}	$-2.08^{+1.33}_{-0.61}$	$-1.41^{+0.64}_{-0.82}$
Δm^2_{32} (NO)/Δm^2_{31} (IO)	$(2.521^{+0.037}_{-0.050})10^{-3}\mathrm{eV}^2/c^4$	$(-2.486^{+0.043}_{-0.044})10^{-3}\mathrm{eV}^2/c^4$
$\sin^2(\theta_{23})$	$0.568^{+0.024}_{-0.125}$ (90%)	$0.567^{+0.021}_{-0.048}$ (90%)

Frequentist



Bayesian, both MO



$U^\dagger U = 1$

$\chi^2 = \sum_{\text{points}} \frac{(y_i - f(x_i))^2}{(\delta(y_i))^2}$

13

T2K-NOvA joint fit

Challenges :

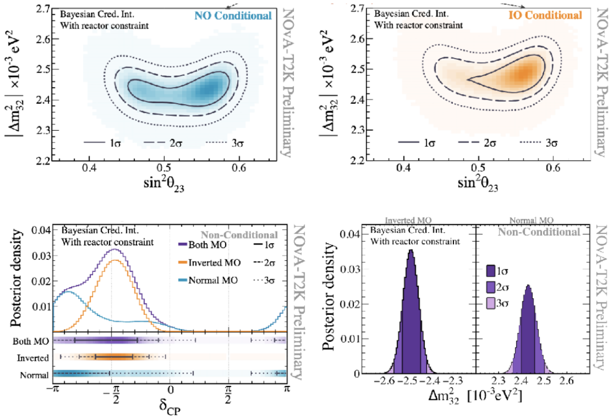
- Main source of correlations: Cross-section model
  - Studied artificial scenarios to see possible correlations
  - Evaluate the robustness of the fit against various models
  - Cross-experiment models after ND constraint

θ_{23} and $|\Delta m^2_{32}|$:

- Results still consistent with maximal mixing of θ_{23}

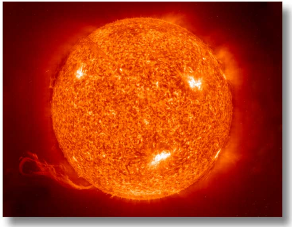
δ_{CP}:

- $\delta_{CP} = \frac{\pi}{2}$ excluded at 3σ for both mass ordering
- In case of IO, CP-conservation is excluded at 3σ

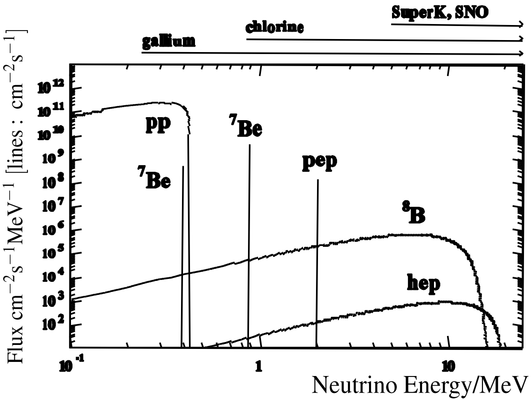


First presented at Fermilab on February, 16th, 2024 : https://indico.fnal.gov/event/62062/contributions/279004/attachments/175258/237774/021624_NOvAT2K_JointFitResults_ZV.pdf

Solar neutrinos



The Sun is powered by the weak interaction:
producing a very large flux of ν_e
$2 \times 10^{38} \nu_e s^{-1}$



- different nuclear reactions in the sun $\Longrightarrow$ complex E_ν spectrum

  $p + p \rightarrow d + e^+ + \nu_e$ $E_\nu < 0.5$ MeV

  $^8B \rightarrow ^8 Be^* + e^+ + \nu_e$ $E_\nu \sim 5$ MeV

  $p + e^- + p \rightarrow d + \nu_e$

  $^7Be + e^- \rightarrow ^7 Li + \nu_e$

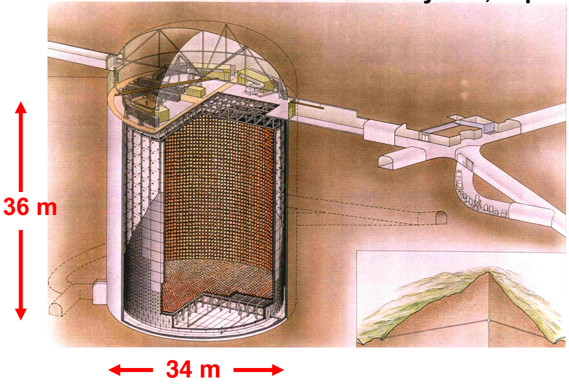
  $^3He + p \rightarrow ^4 He + e^+ + \nu_e$

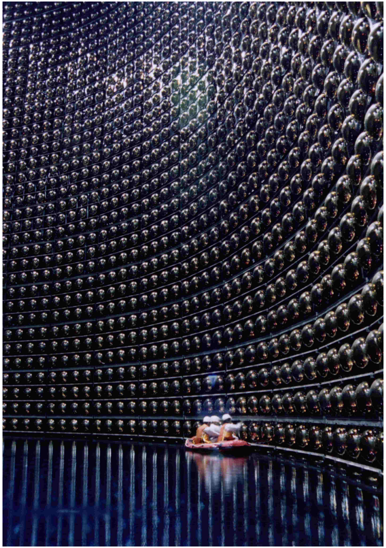
- all experiments saw a deficit of ν_e compared to prediction: the solar neutrino problem

Solar neutrinos I: Super Kamiokande

- 50 kton water Cherenkov detector
- water viewed by 11146 PMT
- deep underground to filter out cosmic rays otherwise too much background

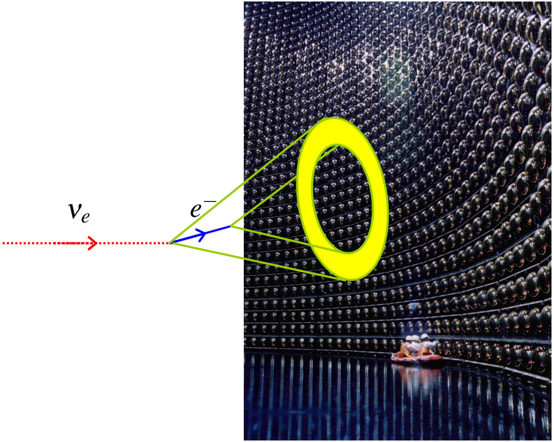
Mt. Ikenoyama, Japan

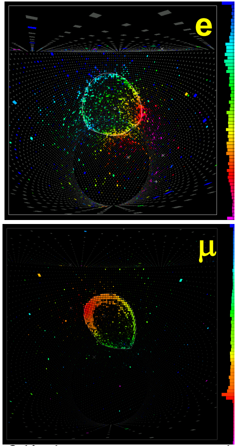




Solar neutrinos I: Super Kamiokande

- detect neutrinos by observing Cherenkov radiation from charged particles which travel faster than speed of light in water c/n



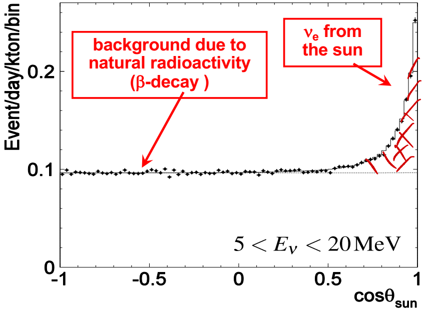


- can distinguish electrons from muons from pattern of light: muons produce clean rings whereas electrons produce more diffuse “fuzzy” rings

Solar neutrinos I: Super Kamiokande

- sensitive to solar neutrinos with $E_\nu > 5$ MeV
- for lower energies too much background from natural radioactivity (β-decays)
- hence detect mostly neutrinos from ${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$
- detect electron Cherenkov rings from $\nu_e + e^- \rightarrow \nu_e + e^-$
- in lab frame the e^- is produced preferentially along the ν_e direction

S.Fukada et al., Phys. Rev. Lett. 86 5651-5655, 2001



Results:

- clear signal of ν from the sun
- too few neutrinos:

$$\text{Data/SSM} = 0.45 \pm 0.02$$

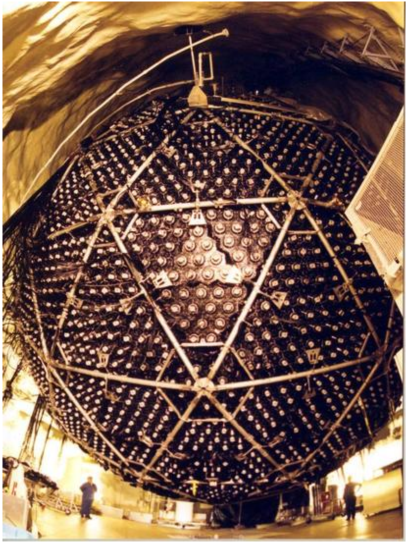
SSM="Standard Solar Model" prediciton

The solar Neutrino "Problem"

Solar neutrinos II: SNO

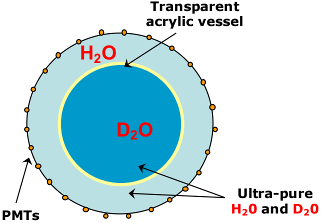
Sudbury Neutrino Observatory located in a deep mine in Ontario, Canada

D=(n p)



- 1 kton heavy water (D_2O) Cherenkov detector
- D_2O inside a 12m diameter acrylic vessel
- surrounded by 3 kton of H_2O
- main experimental challenge: need for very low background from radioactivity
- ultra-pure H_2O and D_2O
- surrounded by 9546 PMTs

H =(p)



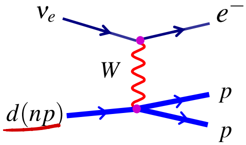
Solar neutrinos II: SNO

Detect Cherenkov light from three different reactions:

CHARGE CURRENT

- Detect Čerenkov light from electron
- Only sensitive to ν_e (thresholds)
- Gives a measure of ν_e flux

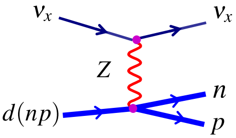
$$\text{CC Rate} \propto \phi(\nu_e)$$



NEUTRAL CURRENT

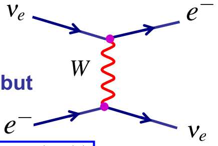
- Neutron capture on a deuteron gives **6.25 MeV**
- Detect Čerenkov light from electrons scattered by γ
- Measures total neutrino flux

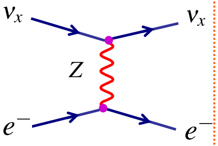
$$\text{NC Rate} \propto \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau)$$



ELASTIC SCATTERING

- Detect Čerenkov light from electron
- Sensitive to all neutrinos (NC part) – but larger cross section for ν_e



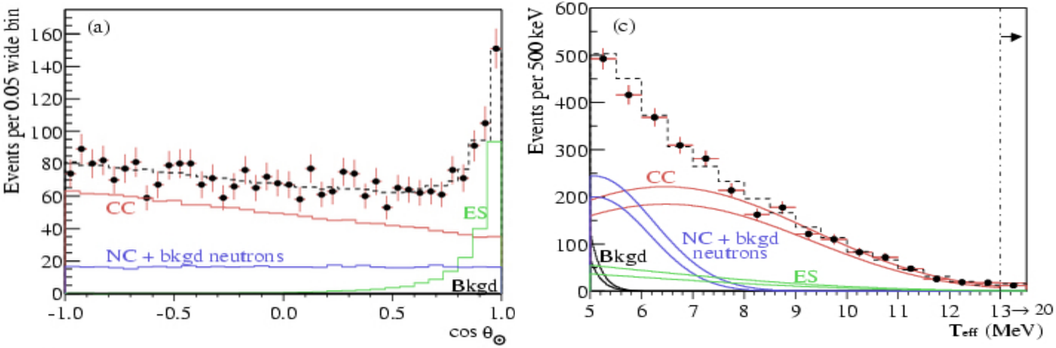


$$\text{ES Rate} \propto \phi(\nu_e) + 0.154(\phi(\nu_\mu) + \phi(\nu_\tau))$$

Solar neutrinos II: SNO

- experimentally can determine rates for different interactions from:
  - angle with respect to sun: electrons from ES point back to sun
  - energy: NC events have lower energy – 6.25 MeV γ from n capture
  - radius from center of detector: gives a measure of background from neutrinos

SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89:011301, 2002



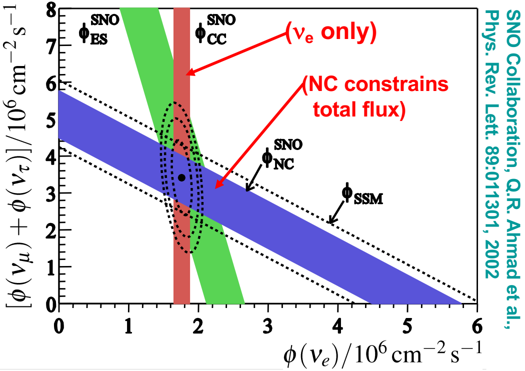
Solar neutrinos II: SNO

- using different distributions measure number of events of each type:
  - CC: $1968 \pm 61 \propto \phi(\nu_e)$
  - ES: $264 \pm 26 \propto \phi(\nu_e) + 0.154[\phi(\nu_\mu) + \phi(\nu_\tau)]$
  - NC: $576 \pm 50 \propto \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau)$

$\Longrightarrow$ Measure of electron neutrino flux + total flux!

Solar neutrinos II: SNO

- using known cross sections can convert observed number of events into fluxes
- the different processes impose different constraints
- where constraints meet gives separate measurements of ν_e and $\nu_\mu + \nu_\tau$ fluxes



SNO results:

$\phi(\nu_e) = (1.8 \pm 0.1) \times 10^{-6}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

$\phi(\nu_\mu) + \phi(\nu_\tau) = (3.4 \pm 0.6) \times 10^{-6}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

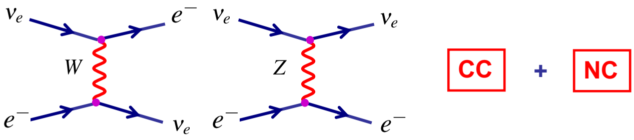
SSM prediction:

$\phi(\nu_e) = 5.1 \times 10^{-6}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

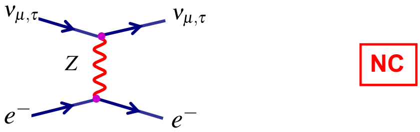
- clear evidence for a flux of ν_μ and/or ν_τ from the sun
- total neutrino flux is consistent with expectation from SSM
- clear evidence of $\nu_e \to \nu_\mu$ and/or $\nu_e \to \nu_\tau$ neutrino transitions

Interpretation of solar neutrino data

- the interpretation of the solar neutrino data is complicated by matter effects
  - the quantitative treatment is nontrivial and is not discussed
  - basic idea is that as ν leaves the sun it crosses a region of high electron density
  - the coherent forward scattering process ($\nu_e \to \nu_e$) for an electron neutrino



is different to that for a muon or tau neutrino



  - can enhance oscillations - “MSW effect”
- a combined analysis of all solar neutrino data gives:

$$\Delta m^2_{\mathrm{solar}} \approx 8 \times 10^{-5}\mathrm{eV}^2, \sin^2 2\theta_{\mathrm{solar}} \approx 0.85$$

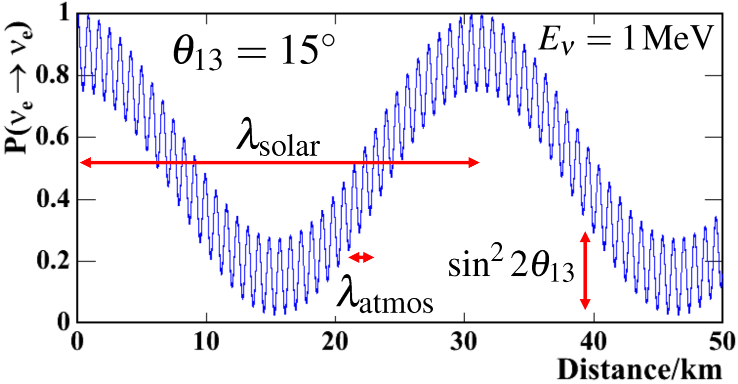
- to explain reactor neutrino experiments we need full three neutrino expression for the electron neutrino survival probability which depends on U_{e1}, U_{e2}, U_{e3}
- substituting these PMNS matrix elements in the expression:

$$\begin{aligned} P(\nu_e \to \nu_e) &\approx 1 - 4U_{e1}^2 U_{e2}^2 \sin^2\Delta_{21} - 4\big(1 - U_{e3}^2\big)U_{e3}^2 \sin^2\Delta_{32} && (57) \\ &= 1 - 4(c_{12}c_{13})^2 (s_{12}c_{13})^2 \sin^2\Delta_{21} - 4(1 - s_{13}^2)s_{13}^2 \sin^2\Delta_{32} && (58) \\ &= 1 - c_{13}^4 (2s_{12}c_{12})^2 \sin^2\Delta_{21} - (2c_{13}s_{13})^2 \sin^2\Delta_{32} && (59) \\ &= 1 - \cos^4\theta_{13} \sin^2 2\theta_{12} \sin^2\Delta_{21} - \sin^2 2\theta_{13} \sin^2\Delta_{32} && (60) \end{aligned}$$

- contributions with short (atmospheric) and long (solar) wavelengths

Reactor experiments

For a 1 MeV neutrino:



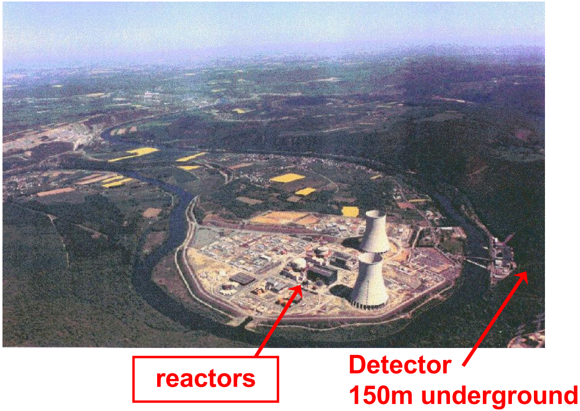
$$\lambda_{osc}(\mathrm{km}) = 2.47 \frac{E(\mathrm{GeV})}{\Delta m^2(\mathrm{eV}^2)} \implies \lambda_{21} = 30.0\mathrm{km},\ \lambda_{32} = 0.8\mathrm{km}$$

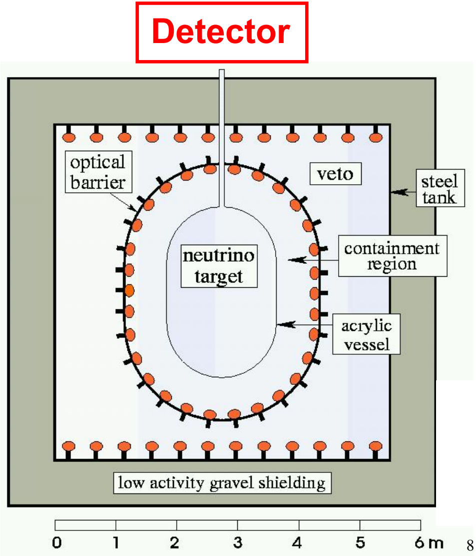
Amplitude of short wavelength oscillations given by $\sin^2 2\theta_{13}$

Reactor experiments I: CHOOZ

France

- two nuclear reactors, each 4.2 GW
- detector is 1 km from reactor cores
- reactors produce intense flux of $\bar{\nu}_e$





Reactor experiments I: CHOOZ

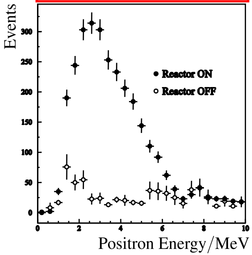
- antineutrinos interact via inverse β-decay: $\bar{\nu}_e + p \to e^+ + n$
- detector: liquid scintillator with Gd (large n capture cross section)
- detect γ from e^+ annihilation and a delayed signal from γ from n capture on Gd: $e^+ + e^- \to \gamma + \gamma, n + Gd \to Gd^* \to Gd + \gamma + \gamma + \ldots$

Reactor experiments I: CHOOZ

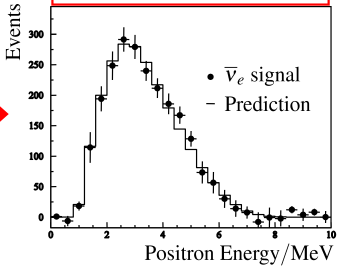
- at 1km and energies > 1 MeV, only short wavelength component matters:

$$P(\nu_e \to \nu_e) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{32} \approx 1 - \sin^2 2\theta_{13}\sin^2\Delta_{32}$$

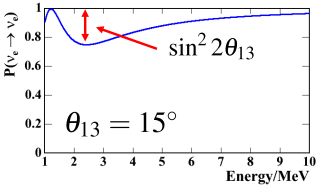
CHOOZ Raw Data



Background subtracted



Compare to effect of oscillations



- data agree with unoscillated prediction both in terms of rate and energy spectrum:

$$N_{\rm data}/N_{\rm expect} = 1.01 \pm 0.04$$

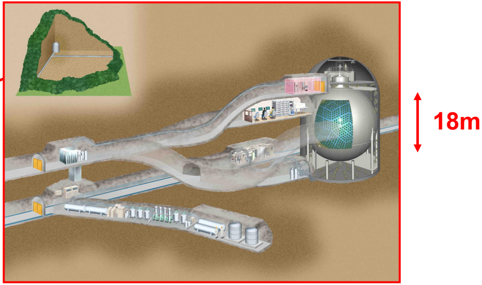
Reactor experiments I: CHOOZ

- hence $\sin^2 2\theta_{13}$ must be small: $\Longrightarrow$ $\sin^2 2\theta_{13} < 0.12 - 0.2$ (exact limit depends on $\left|\Delta m^2_{32}\right|$)
- from atmospheric neutrinos can exclude $\theta_{13} \sim \dfrac{\pi}{2}$
- hence the CHOOZ limit $\sin^2 2\theta_{13} < 0.2$ can be interpreted as $\sin^2 \theta_{13} < 0.05$

Reactor experiments II: KamLAND

- 70 GW from nuclear power (7% of World total) from reactors within 130-240 km

- Detector located in same mine as Super Kamiokande

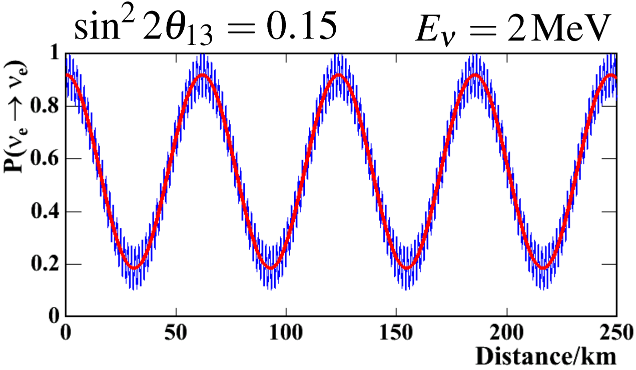


- liquid scintillator detector, 1789 PMTs
- detection via inverse β-decay: $\nu_e + p \to e^+ + n$ followed by

$e^+ = e^- \to \gamma\gamma$ – prompt

$n + p \to d + \gamma(2.2\text{ MeV})$ – delayed

Reactor experiments II: KamLAND



- for MeV neutrinos at a distance of 130-240 km oscillations due to Δm^2_{32} are very rapid
- experimentally only see average effect $\langle \sin^2 \Delta_{32} \rangle = 0.5$

Reactor experiments II: KamLAND

$$P(\nu_e \to \nu_e) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{32} \quad (61)$$

$$\approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \frac{1}{2}\sin^2 2\theta_{13} \quad (62)$$

$$= \cos^4\theta_{13} + \sin^4\theta_{13} - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \quad (63)$$

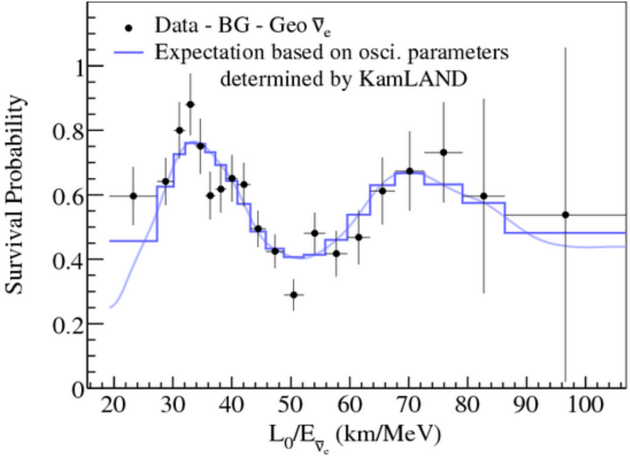
$$\approx \cos^4\theta_{13}(1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}) \quad (64)$$

- got two-flavor oscillation formula multiplied by $\cos^4\theta_{13}$
- from CHOOZ $\cos^4\theta_{13} > 0.9$

Reactor experiments II: KamLAND results

Observed: 1609 events
Expected: 2179 ± 89 events (if no oscillations)

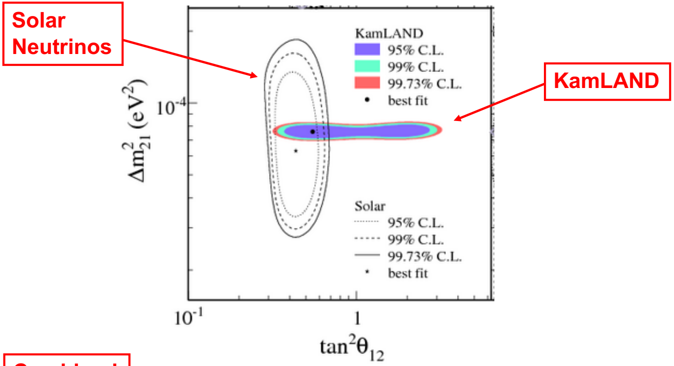
KamLAND Collaboration, Phys. Rev. Lett., 221803, 2008



- clear evidence of $\bar{\nu}_e$ oscillations consistent with the results from solar neutrinos
- oscillatory structure clearly visible
- compare data with expectations for different osc. parameters and perform χ^2 fit to extract measurement

Combined solar neutrino and KamLAND results

- KamLAND data provide strong constraints on $\left|\Delta m^2_{21}\right|$
- solar neutrino data (especially SNO) provide a strong constraint on θ_{12}



Combined

$$|\Delta m^2_{21}| = (7.59 \pm 0.21) \times 10^{-5}\,\mathrm{eV}^2$$

$$\tan^2\theta_{12} = 0.47^{+0.06}_{-0.05}$$

Summary of current knowledge

Solar neutrinos/KamLAND

- KamLAND + Solar: $\left|\Delta m^2_{21}\right| \approx (7.6 \pm 0.2) \times 10^{-5}$ eV^2
- SNO + KamLAND + Solar: $\tan^2\theta_{12} \approx 0.47 \pm 0.05$

  $\implies \sin\theta_{12} \approx 0.56; \cos\theta_{12} \approx 0.82$

Atmospheric neutrinos/Long baseline experiments

- MINOS: $\left|\Delta m^2_{32}\right| \approx (2.4 \pm 0.1) \times 10^{-3}$eV^2
- Super Kamiokande: $\sin^2\theta_{23} \approx 0.512^{+0.019}_{-0.022}$
- Super Kamiokande: $\delta_{CP} = 1.37^{+0.18}_{-0.16}$

DayaBay, CHOOZ + atmospheric

- $\sin^2\theta_{13} \approx (2.18 \pm 0.07) \times 10^{-2}$

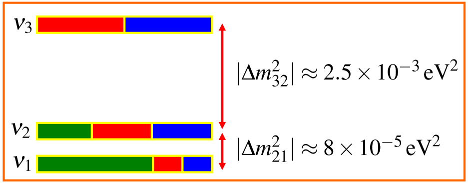
Summary of current knowledge

- have approximate expressions for mass eigenstates in terms of weak eigenstates:

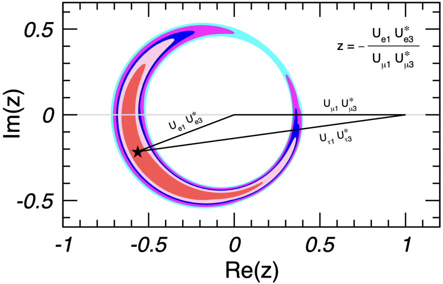
$$|\nu_3\rangle \approx \frac{1}{\sqrt{2}}(|\nu_\mu\rangle + |\nu_\tau\rangle)$$

$$|\nu_2\rangle \approx 0.53|\nu_e\rangle + 0.60(|\nu_\mu\rangle - |\nu_\tau\rangle)$$

$$|\nu_1\rangle \approx 0.85|\nu_e\rangle - 0.37(|\nu_\mu\rangle - |\nu_\tau\rangle)$$



- graphic representation of the mixing matrix measurements (unitarity triangle):



- neutrino oscillations require non-zero neutrino masses
- but only determine mass-squared differences – not the masses themselves
- no direct measure of neutrino mass – only mass limits:

  $m_\nu(e) < 0.8$ eV, $m_\nu(\mu) < 0.17$ MeV, $m_\nu(\tau) < 18.2$ MeV

  Note that e, μ, τ refer to charged lepton flavor in the experiment, e.g. $m_\nu(e) < 2$ eV refers to the limit from tritium β-decay
- also from cosmological evolution infer that the sum

  $\sum_i m_{\nu_i} <$ few eV
- 20 years ago: assumed massless neutrinos + hints that neutrinos might oscillate
- now, know a lot about massive neutrinos
- but many unknowns: mass hierarchy, absolute values of neutrino masses
- measurement of these SM parameters is the focus of the next generation of experiments